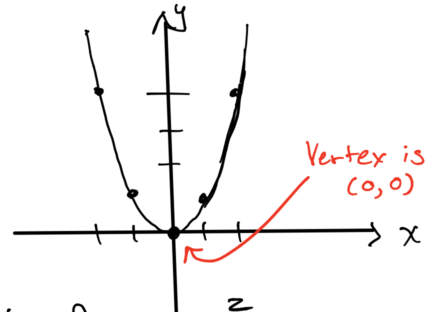
Parabolas: The basic equation: $y = x^2$

As with all equations, solutions are ordered pairs (a,b) such that $b = a^2$. We will find several solutions and graph them:

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

Note that there is symmetry: $(-a)^2 = a^2$.



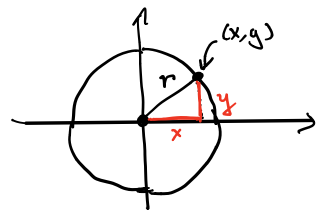
Note the shape of the graph of $y = x^2$.

The "turning point" is called the vertex. Here, the vertex is $(0,0)$.

Remark: There is a geometric description of the parabola, but we won't discuss this here.

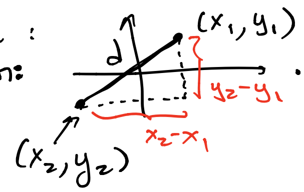
Circles: The basic circle:

The basic circle of radius r is the set of points a distance r from $(0,0)$; $(0,0)$ is called the center.



Let (x,y) be any point on the circle. Then, using the Pythagorean Theorem ($\Rightarrow a^2+b^2=c^2$) we see $x^2+y^2=r^2$.

Note: Then:



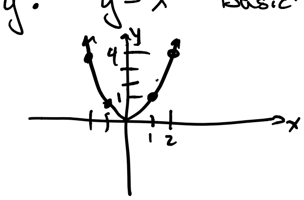
Let d = distance between (x_1,y_1) & (x_2,y_2).

So, $(x_2-x_1)^2 + (y_2-y_1)^2 = d^2$

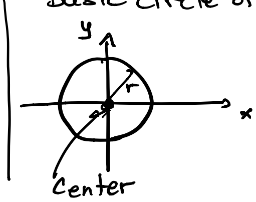
$$\Rightarrow d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}.$$

This is called the distance formula.

Summary: $y = x^2$ basic parabola



basic circle of radius r:



$x^2+y^2=r^2$.

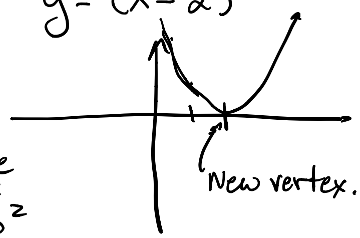
Transformations:

Examples:

$y = (x-2)^2$

(0,0) is the vertex of $y=x^2$.

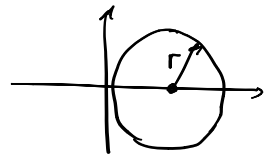
(2,0) is the vertex of $y=(x-2)^2$



At the vertex:
The left & right hand sides of $y=x^2$ are 0.
" " $y=(x-2)^2$ are 0

Try: What does the graph of $y=(x+2)^2$ look like? What is the vertex?

$(x-2)^2+y^2=r^2$



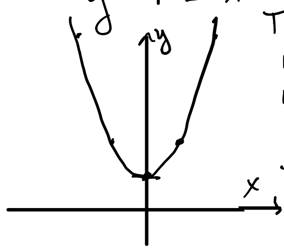
(0,0) is the center of $x^2+y^2=r^2$.
At the center the left side of its equation is 0.

(2,0) is the center of $(x-2)^2+y^2=r^2$.

Try: What does the graph of $(x+2)^2+y^2=25$ look like? What is the center? What is the radius?

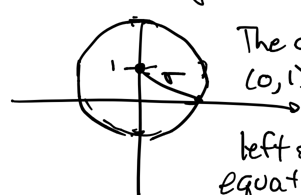
Example:

$y-1=x^2$



The vertex is (0,1), which is where both sides of the equation $y=x^2$ are zero.

$x^2+(y-1)^2=r^2$



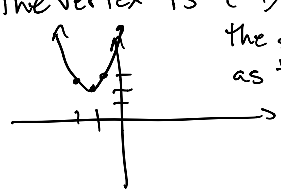
The center is (0,1), which is where the left side of its equation is 0.

Try: What does the graph of $y+2=x^2$ look like? What is the vertex?

What does the graph of $x^2+(y+2)^2=9$ look like? What is the center?

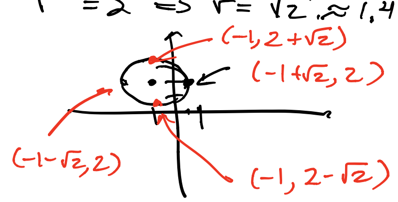
Example Graph $y-2=(x+1)^2$

The vertex is (−1,2); but is the same shape as the graph of $y=x^2$.



Graph: $(x+1)^2+(y-2)^2=2$.

The center is (−1,2).
$r^2=2 \Rightarrow r=\sqrt{2} \approx 1.4$

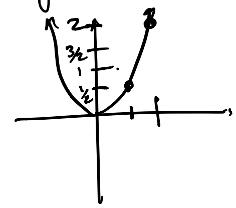


Example

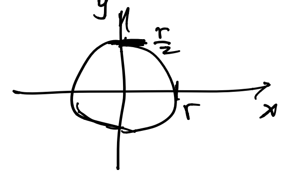
Consider

y	$2y$	x
2	4	-2
$\frac{1}{2}$	1	-1
0	0	0
$\frac{1}{2}$	1	1
2	4	2

The graph of $2y=x^2$ is



Note
$x^2+(2y)^2=r^2$
or $x^2+4y^2=r^2$



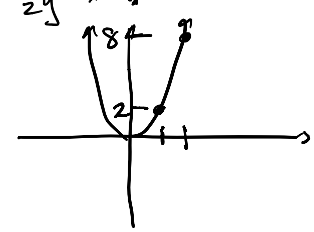
The difference between this graph and the graph of $y=x^2$ is the scale on the y-axis: If the unit for $y=x^2$ is 1 cm then the unit for $2y=x^2$ is $\frac{1}{2}$cm ($2\cdot\frac{1}{2}=1$)!

Graph these: $y=x^2$ & $2y=x^2$ on Desmos.

Compared to the graph of $y=x^2$, the graph of $2y=x^2$ is compressed by a factor of 2.

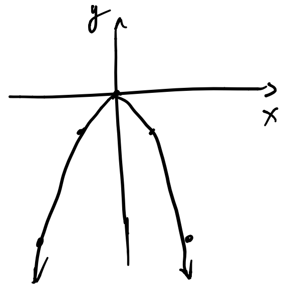
View $x^2+y^2=4$
& $x^2+(2y)^2=4$
on Desmos!

Example:
$\frac{1}{2}y=x^2$:



Example $-y=x^2$

y	$-y$	x
-4	4	-2
-1	1	-1
0	0	0
-1	1	1
-4	4	2

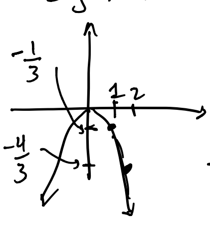


A similar transform for a circle produces no change! Why?.

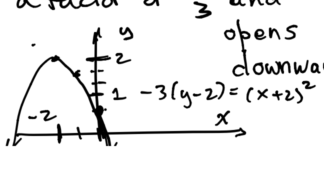
Graph: $-3(y-2)=(x+2)^2$:

Vertex is $(-2,2)$.
Compared to $y=x^2$, it is compressed along the y-axis by a factor of $\frac{1}{3}$ and opens downward.

$-3y=x^2$:



shift to new vertex



Try: Graph
$\frac{1}{2}(y+3)=(x-1)^2$
What is the vertex?

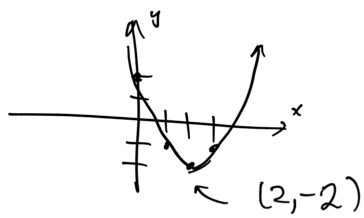
Example **Graph:** $y - x^2 + 4x - 2 = 0$

Let's complete the square to put it in form $a(y-k) = (x-h)^2$

$y - 2 = x^2 - 4x + 4 - 4$

$\Rightarrow y - 2 = (x-2)^2 - 4$

$\Rightarrow y + 2 = (x-2)^2$



Try: Graph:

$y + 2x^2 - 8x - 3 = 0$

Example:

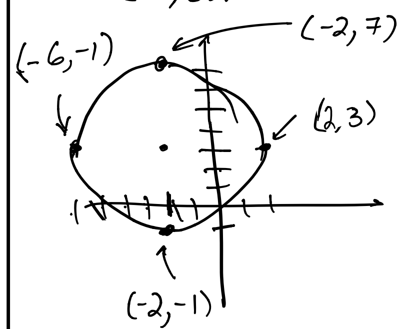
$x^2 + y^2 + 4x - 6y - 7 = 0$

$\Rightarrow x^2 + 4x + 4 - 4 + y^2 - 6y + 9 - 9 - 7 = 0$

$\Rightarrow (x+2)^2 + (y-3)^2 - 9 - 7 = 0$

$\Rightarrow (x+2)^2 + (y-3)^2 = 16$

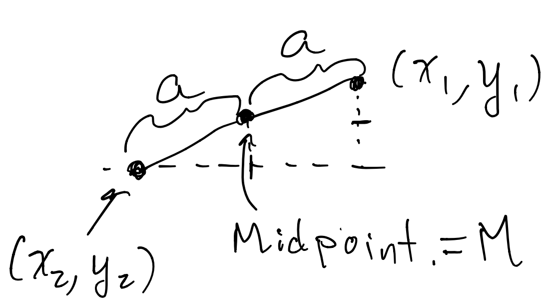
So $r = 4$ & the vertex is $(-2, 3)$.



Try: Graph

$x^2 + y^2 - 4x + 2y + 1 = 0$

Remark: for some problems you will need the distance formula and the midpoint formula:



$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$