Non linear sytems

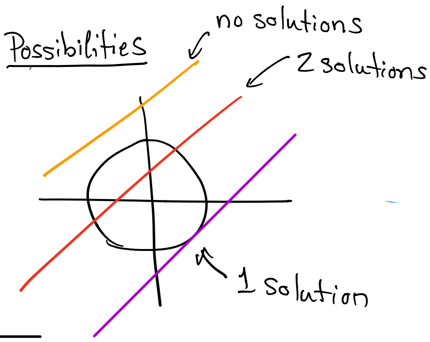
Consider

$$\begin{cases} x^2+y^2=4 \\ y=-x+1 \end{cases}$$

circle

line

Possibilities



$$y=-x+1 \implies x^2+(-x+1)^2=4$$

$$\implies x^2+x^2-2x+1=4$$

$$\implies 2x^2-2x-3=0$$

$$\implies x=\frac{2\pm\sqrt{4-4(2)(-3)}}{2\cdot 2}$$

$$=\frac{2\pm 2\sqrt{7}}{4}$$

$$=\frac{1}{2}\pm\frac{\sqrt{7}}{2}.$$

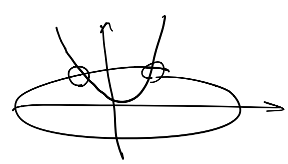
If $\left(\frac{1}{2}+\frac{\sqrt{7}}{2}, b\right)$ is a solution, then $b=-\frac{1}{2}-\frac{\sqrt{7}}{2}+1=\frac{1}{2}-\frac{\sqrt{7}}{2}$.
So $\left(\frac{1}{2}+\frac{\sqrt{7}}{2}, \frac{1}{2}-\frac{\sqrt{7}}{2}\right)$ is a solution.
If $\left(\frac{1}{2}-\frac{\sqrt{7}}{2}, b\right)$ is a solution, then $b=-\frac{1}{2}+\frac{\sqrt{7}}{2}+1=\frac{1}{2}+\frac{\sqrt{7}}{2}$
So $\left(\frac{1}{2}-\frac{\sqrt{7}}{2}, \frac{1}{2}+\frac{\sqrt{7}}{2}\right)$

There are 2 solutions: $\left(\frac{1}{2}+\frac{\sqrt{7}}{2}, \frac{1}{2}-\frac{\sqrt{7}}{2}\right)$

and. $\left(\frac{1}{2}-\frac{\sqrt{7}}{2}, \frac{1}{2}+\frac{\sqrt{7}}{2}\right)$

Example

$$\begin{cases} 2x^2+3y^2=5 \\ y=x^2 \end{cases}$$



$$2(y)+3y^2=5 \implies 3y^2+2y-5=0$$

$$\implies (y-1)(3y+5)=0$$

$$\implies y=1 \text{ or } y=-\frac{5}{3}.$$

It is clear from the second equation

Factor:

$AC=-15$, $B=2$

$5+-3=2$

$3y^2+5y-3y-5$

$=y(3y+5)+(-1)(3y+5)$

$=(y-1)(3y+5)$

that $y \geq 0$. So $y = 1$ ($y \neq -5/3$).
So our solutions are of the form $(a, 1)$.
Substituting into the second equation gives
$1 = a^2$ so $a = \pm 1$. The solutions are
therefore $(1, 1)$ and $(-1, 1)$.

<u>Example</u>

$$\begin{cases} 3x^2 + 2y^2 = 5 \\ x^2 + 2y^2 = 3 \end{cases}$$

We will multiply the second equation by -1 and add to the first equation:

$$\begin{cases} 3x^2 + 2y^2 = 5 \\ -x^2 - 2y^2 = -3 \end{cases}$$

$\Rightarrow \quad 2x^2 = 2 \quad \Rightarrow x^2 = 1 \Rightarrow x = 1$ or -1.

If $(1, b)$ is a solution, then $1 + 2b^2 = 3 \Rightarrow b^2 = 1$ and so $b = \pm 1$.
So $(1, 1)$ and $(1, -1)$ are solutions.

If $(-1, b)$ is a solution, then $1 + 2b^2 = 3 \Rightarrow b = 1$ or -1

So $(-1, 1)$ and $(-1, -1)$ are solutions.

There are 4 solutions to the system:

$(1,1), (1,-1), (-1, 1)$ and $(-1,-1)$