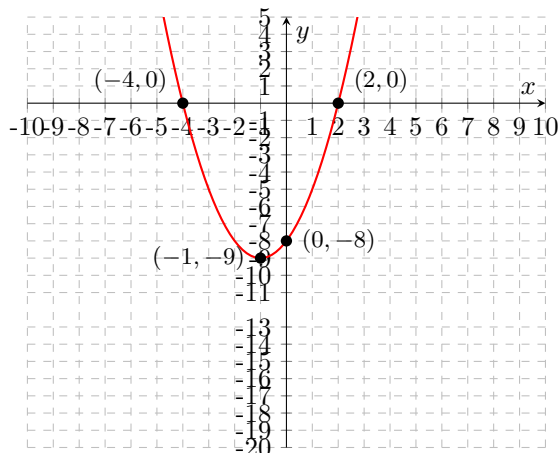


## Final Exam Review- MAT 1275 - Fall 2023

The following are the types of problems you may see on the final exam (they are not necessarily the exact problems).

### 1 Questions requiring supportive work

1. Simplify  $\frac{x^2}{x^2 - 4} - \frac{x - 2}{x + 2}$ .
2. Simplify  $\left(\frac{4x^3y^{-2}}{16x^{-3}y^4}\right)^{-1/2}$ .
3. Write in the form  $a + bi$ :  $\frac{2 - 5i}{3 - 4i}$ .
4. Simplify  $(3\sqrt{5} + 2\sqrt{7})(3\sqrt{5} - 2\sqrt{7})$ .
5. Simplify  $\frac{9x^4y^3 - 12x^2y^4 + 3xy^3}{3xy^3}$ .
6. Simplify  $\frac{\frac{x^2 - 4}{x^2 - x}}{x^2 - 3x + 2}$ .
7. Write using one radical  $\sqrt[3]{\sqrt{x^5}}$ .
8. Solve for  $x$  and simplify the answer:  $2x^2 - 2x - 3 = 0$ .
9. Solve  $\frac{x}{x - 2} = -\frac{1}{3} + \frac{10}{x}$ .
10. Solve for  $y$  and simplify the answer:  $\sqrt{y - 2} - 4 = -y$ .
11. Evaluate  $x^3 - 2x^2 - 5x + 6$  at  $x = 1$ . What does this tell you about  $x^3 - 2x^2 - 5x + 6$ ? Use long division to rewrite this polynomial as a product of two factors and then find all solutions to the equation  $x^3 - 2x^2 - 5x + 6 = 0$ .
12. Find all solutions to the equation  $3x^3 - 27x = 0$ .
13. Find an equation for a line which passes through  $(1, -5)$  which is perpendicular to  $y = \frac{1}{3}x + 4$ . Graph your line.
14. Given the following quadratic equation, sketch and label the graph:  $y^2 = -x^2 - 4x + 5$ .
15. By completing the square, find the vertex and the axis of symmetry of the parabola whose equation is the following:  
 $y = x^2 - 4x + 5$ .
16. (This problem doesn't require work but the answer is included below.) Consider the following graph of an equation of the form  $y = ax^2 + bx + c$ .



Mark true or false.

- (a) There is a solution for which  $y = -9$ .
- (b) There is exactly one solution for which  $x = \sqrt{3}$ .
- (c) The value of  $c$  is  $-9$ .
- (d) The value of  $a$  is negative.
- (e) The system  $\begin{cases} y = ax^2 + bx + c \\ y = 3 \end{cases}$  has exactly two solutions.
- (f) The axis of symmetry is given by the equation  $y = -9$ .

17. Find all solutions to

$$\begin{cases} 2x - 3y^2 = 1 \\ -x + 2y = -4 \end{cases}$$

18. Find all solutions to

$$\begin{cases} 2x - 3y = 7 \\ -x + 2y = -4 \end{cases}$$

19. A 10-foot ladder is leaning up against a wall. If the base of the ladder is situated 4 feet away from the base of the wall, what is the angle of elevation of the ladder? Draw a picture that depicts the situation and label the relevant information. Round your answer to the nearest tenth of a degree.

20. For each of the two expressions  $\cos(-210^\circ)$  and  $\tan\left(\frac{7\pi}{4}\right)$ :

- (a) identify the quadrant in which the angle is located,
- (b) find the reference angle,
- (c) find the exact value.

21. Given  $\sin(x) = -\frac{2}{5}$  and  $\cos(x) > 0$ , find the other 5 trigonometric values.

22. Given a  $\triangle ABC$  with  $\angle C = 40^\circ$ ,  $c = 3$  feet and  $b = 4$  feet, find  $\angle A$ . Draw a picture of the triangle and label it with the information provided. Round each answer to the nearest tenth. You may use either the law of sines:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

or the law of cosines:

$$\begin{aligned} c^2 &= a^2 + b^2 - 2ab \cos(C) \\ a^2 &= c^2 + b^2 - 2cb \cos(A) \\ b^2 &= a^2 + c^2 - 2ac \cos(B). \end{aligned}$$

23. Solve for  $x$  and round your answer to the nearest tenth:

$$3^x = 12.$$

24. Evaluate (without using a calculator):

$$\log_5(25\sqrt[4]{5}).$$

25. Evaluate  $-2^{-4} + 3^0 - (2/3)^{-1}$  and indicate where it is on a number line.

26. Simplify  $\frac{\sqrt[4]{x^3}}{\sqrt{x}}$ .

## 2 True/False questions

The following is just a sample of the types of problems you may find. Label the following True or False.

1. There could be 5 solutions to the system of equations:  $\begin{cases} x^2 + y^2 = 6 \\ x^2 - y = 8 \end{cases}$
2. The product of two polynomials of degree 3 has degree 9.
3. If 7 is a root of a polynomial, then  $(x - 7)$  is a factor of that polynomial.
4. The lines  $y = \frac{1}{2}x - 9$  and  $y = 2x - \frac{1}{2}$  are perpendicular.
5. If  $\cos \theta < 0$ , then  $\theta$  is either in the second or the third quadrant.
6. The equation  $3^x = 12$  has one solution and it is between 2 and 3.
7.  $\log 7$  is bigger than 2.
8. The expression  $\sqrt{9x^9y^4}$  is equal to  $3x^3y^2$  for all positive  $x$  and  $y$ .
9. The equations  $\frac{1}{2}x^2 + \frac{1}{3}x = \frac{1}{6}$  and  $3x^2 + 2x = 1$  are equivalent (they have the same solutions).
10.  $(a + b)^2 = a^2 + b^2$
11.  $(2\sqrt{7} - 3\sqrt{5})(2\sqrt{7} + 3\sqrt{5}) = 4 \cdot 7 - 9 \cdot 5$ .
12.  $x^{-1} = -x$ .
13.  $(2\sqrt{7})^2 = 2 \cdot 7 = 14$ .

## 3 Give example questions

The following are just examples of the types of examples you may be asked to provide. Give an example of:

- (a) an equation of a line that passes through  $(2, 5)$
- (b) a radical expression
- (c) a rational equation in one variable for which  $-3$  cannot be a solution
- (d) a polynomial with leading coefficient  $-7$ , degree 5, and 4 terms
- (e) a polynomial of degree 2 with roots  $-3$  and 5
- (f) an equation of a horizontal line that passes through  $(8, -\sqrt{3})$
- (g) an equation of a circle with center in the second quadrant
- (h) trigonometric ratio which has a different sign than tangent of an angle in quadrant III.
- (i) an equation that has no real solution
- (j) an equation with one variable that has two solutions
- (k) Give an example of how to add fractions with unequal denominators.

## 4 Answers to questions from part 1

1.  $\frac{4(x-1)}{(x+2)(x-2)}$

2.  $\sqrt{\frac{3y^6}{2x^6}}$

3.  $\frac{26}{25} - \frac{7}{25}i$

4. 17

5.  $3x^3 - 4xy + 1$

6.  $\frac{x+2}{(x-1)^2}, x \neq 2$

7.  $\sqrt[6]{x^5}$

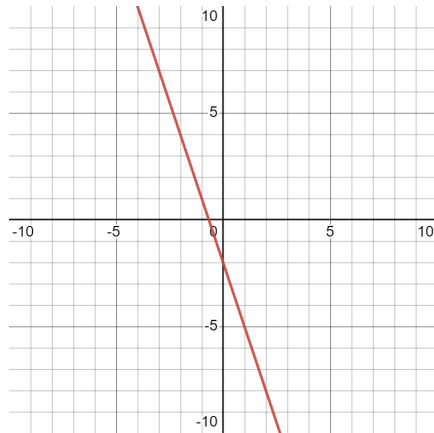
8.  $\frac{1}{2} \pm \frac{\sqrt{7}}{2}$

9.  $x = 3$  or  $5$

10.  $y = 3$

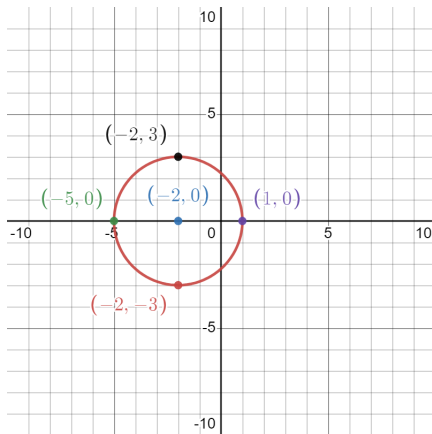
11.  $x = -2, 1,$  or  $3$

12.  $x = 0, 3,$  or  $-3$



13.  $y + 5 = -3(x - 1)$

14. This equation is equivalent to  $(x + 2)^2 + y^2 = 9$  and so the solutions form a circle centered at  $(-2, 0)$  with radius 3:



15.  $y - 1 = (x - 2)^2$ , The vertex is  $(2, 1)$  and the axis of symmetry is  $x = 2$ .

16. True, True, False, False, True, False

17.  $(2, -1), (\frac{26}{3}, \frac{7}{3})$

18.  $(2, -1)$

19.  $\cos^{-1} \frac{4}{10} \approx 66.4^\circ$

20. II,  $30^\circ$ ,  $\frac{-\sqrt{3}}{2}$ ; IV,  $\frac{\pi}{4}$ , -1

21.  $\cos(x) = \frac{\sqrt{21}}{5}$ ,  $\tan(x) = \frac{-2}{\sqrt{21}}$ ,  $\csc(x) = \frac{-5}{2}$ ,  $\sec(x) = \frac{5}{\sqrt{21}}$ ,  $\cot(x) = \frac{-\sqrt{21}}{2}$

22.  $81^\circ$

23. 2.3

24.  $\frac{9}{4}$

25.  $-\frac{9}{16}$ , which is between  $-1$  and  $0$  on the number line.

26.  $\sqrt[4]{x}$ .