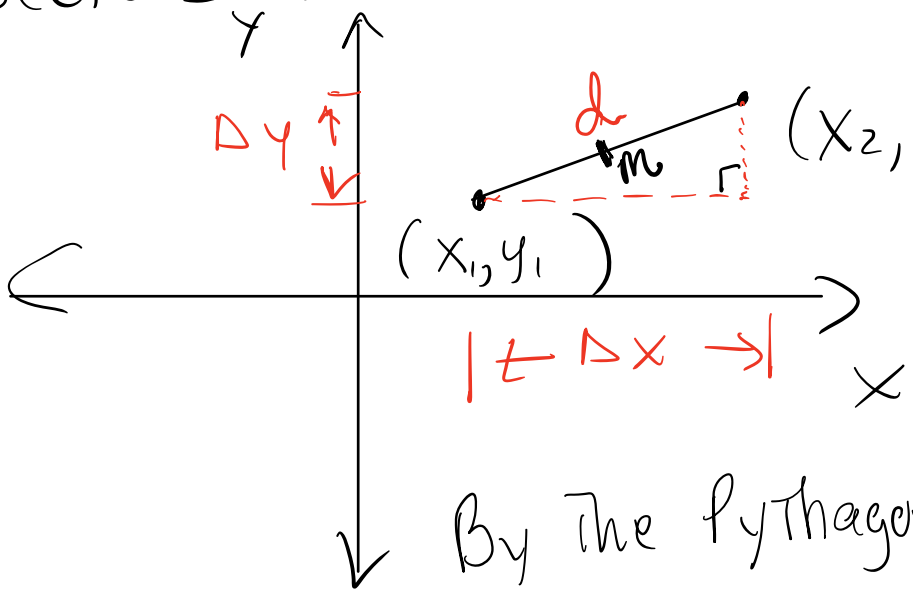
3/25/2022

Class Agenda 20: Midpoint & Distance Formulas, Perpendicular Bisectors & Circles



By the Pythagorean Thm

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

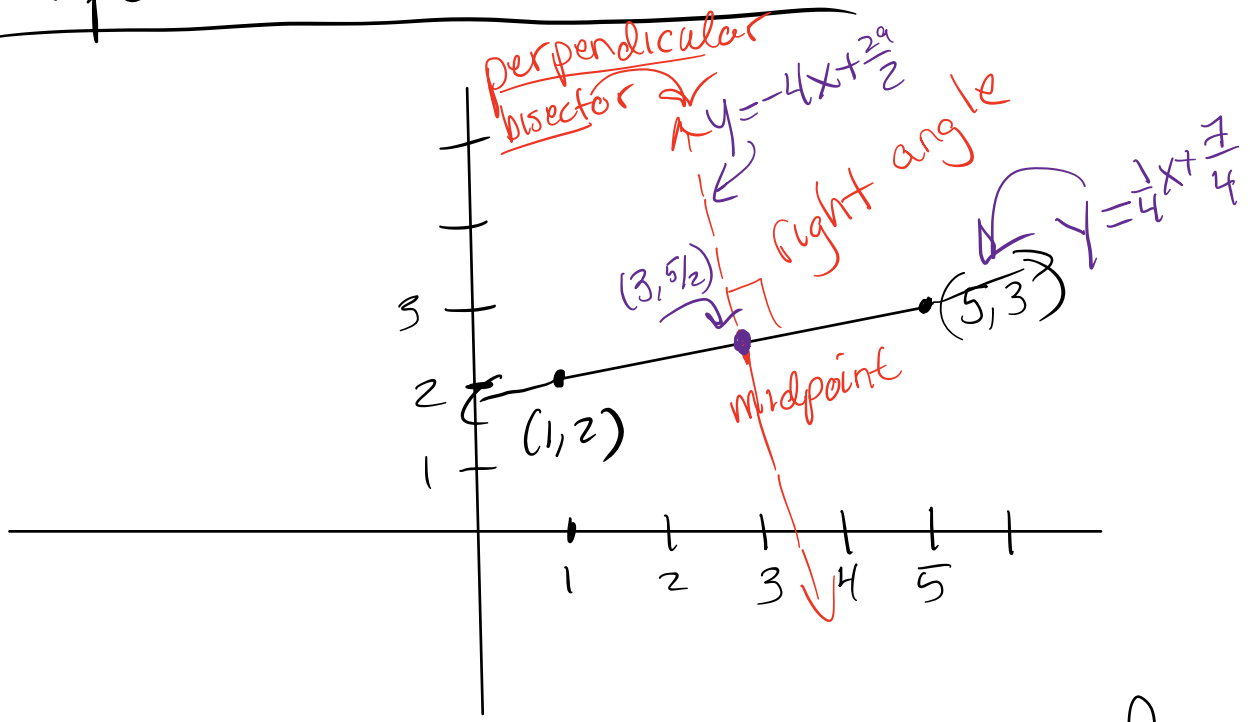
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

↑ the distance from (x_1, y_1) to (x_2, y_2)

The midpoint of the points (x_1, y_1) & (x_2, y_2):

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Perpendicular Bisector:



Step 1: What is the equation of the line passing through the points:

$(1, 2)$ & $(5, 3)$

(x_1, y_1) (x_2, y_2)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-2}{5-1} = \boxed{\frac{1}{4}}$$

Now use the point-slope formula

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{1}{4}(x - 1)$$

+2 +2

Rewrite this in slope-intercept form ($y = mx + b$)

$$y = \frac{1}{4}x - \frac{1}{4} + 2$$

$$= \frac{1}{4}x - \frac{1}{4} + \frac{8}{4}$$

$$\boxed{y = \frac{1}{4}x + \frac{7}{4}}$$

Step 2) Compute the midpoint of $(1,2)$ & $(5,3)$

(x_1, y_1) (x_2, y_2)

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{1+5}{2}, \frac{2+3}{2}\right)$$

$$= \left(\frac{6}{2}, \frac{5}{2}\right) = \left(3, \frac{5}{2}\right)$$

Step 3) Find the slope of the perpendicular bisector: negative reciprocal of $m = \frac{1}{4}$ → $m_2 = -4$

Step 4) Use the point-slope formula to find the equation of the perpendicular bisector

point: $(3, \frac{5}{2})$ slope $m_2 = -4$

$y - y_1 = m(x - x_1)$

$y - \frac{5}{2} = -4(x - 3)$

$+5/2$ $+5/2$

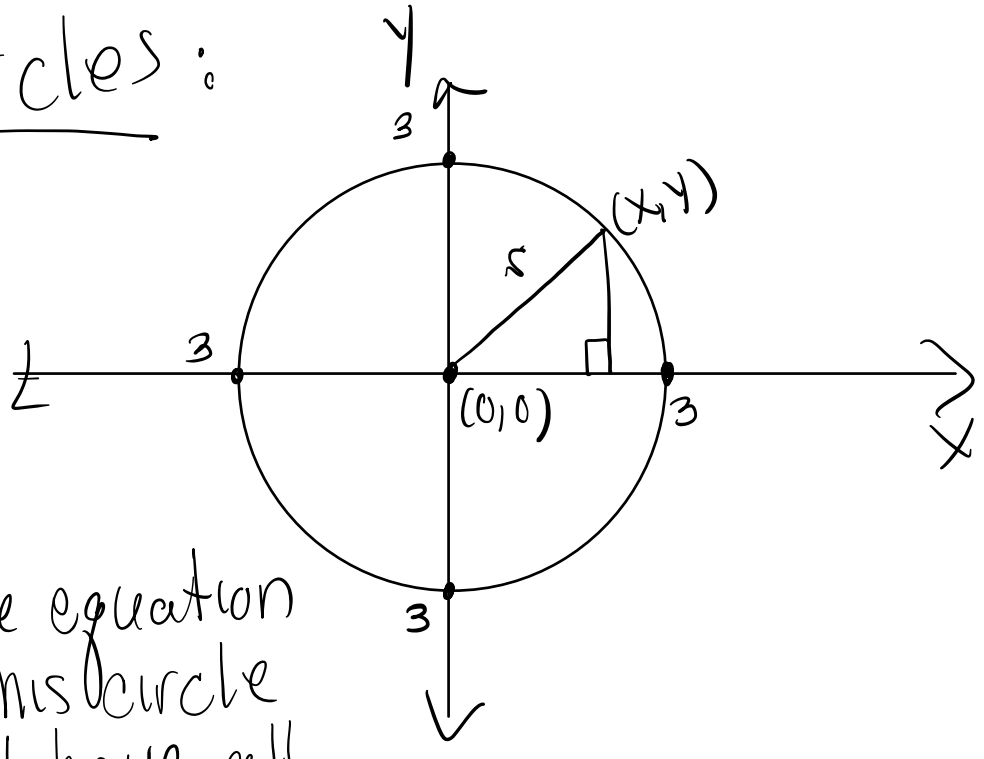
$y = -4x + (12 + 5/2)$

$y = -4x + \frac{24}{2} + \frac{5}{2}$

$y = -4x + \frac{29}{2}$

Reformat to $y = mx + b$ form.

Circles:



The equation of this circle must have all points (x,y) of distance $r=3$ from the origin.

$$\sqrt{(x-0)^2+(y-0)^2}=3$$

$$\sqrt{x^2+y^2}=3$$

$$\boxed{x^2+y^2=3^2}$$

standard form for the equation of a circle centered at $(0,0)$ with radius 3.

In general: $x^2+y^2=r^2$

(r: radius)

What if the center is not at the origin?

$$(x-h)^2+(y-k)^2=r^2$$

where (h,k) is the center.

WeBWorK set: Square Root Property

#3) $(x-1)^2=6$

$x-1=\pm\sqrt{6}$

$+\quad +1$

$$X = 1 \pm \sqrt{6}$$

2 solutions

$1+\sqrt{6}, \quad 1-\sqrt{6}$

WebWork Set: Quadratic Formula

#2) $x^2 - 10x + 34$

$a = 1$ $\quad c = 34$

$b = -10$

$$X = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(34)}}{2(1)}$$

$$34 \times 4 = 136$$

$$= \frac{10 \pm \sqrt{100 - 136}}{2}$$

$$= \frac{10 \pm \sqrt{-36}}{2} \rightarrow \frac{10 \pm 6i}{2} \rightarrow 5 \pm 3i$$