3/23/2023

Class Agenda 19 - Graphing Quadratic Functions - using The vertex formula and transformations of functions

So far: Quadratic Equations

$$ax^2+bx+c=0$$

When we solve This equation, looking for The value(s) of x which make The statement above true

Today: Quadratic Functions

$$f(x)=ax^2+bx+c$$

Ex Given $f(x) = x^2 + 2x + 1$

find $f(1) = 1^2 + 2 \cdot 1 + 1 = 4$

plot the point $(1, 4) \rightarrow$ on the graph of

$$f(x) = x^2 + 2x + 1$$

$$\xrightarrow[\text{input}]{x} \boxed{f(x)} \xrightarrow[\text{output}]{y}$$

$$\xrightarrow{1} \boxed{1^2 + 2 \cdot 1 + 1} \longrightarrow 4$$

Note: Graphs of quadratic functions are parabolas.

Ex $f(x) = x^2$

↑

Experimented in Desmos!

Note: Where the quadratic function is $= 0 \rightarrow$ cross the x-axis, These are called the "roots"

Tranformations

1

1) $f(x)=x^2+K$ if K is positive
The graph of x^2 is shifted <u>up</u>
K units

2) $f(x)=x^2+K$ if K is negative
" " " " " " <u>down</u>
K units

3) $f(x)=(x-K)^2$ if K is positive
The graph of x^2 is shifted
<u>right</u> K units

3) $f(x)=(x+K)^2$ if K is positive
" " " " " "
<u>left</u> K units

4) Graphing $f(x)=ax^2$
If a is <u>positive</u> the parabola

opens up and if a is negative The parabola opens down.

a) If $|a| > 1$ the graph of the parabola is narrower than the graph of x^2

b) If $|a| < 1$ " " " " " " is wider than x^2

"Shift right 1 unit, make it narrow by a factor of 2 then shift up 4 units" $\boxed{f(x) = 2(x-1)^2 + 4}$

What if we are given a quadratic function...... but it is not represented in the form $f(x) = a(x-h)^2 + k$?

vertex = (h, k)

Consider the quadratic function

$f(x) = 3x^2 + 3x + 1$

$= 3(x^2 + x \quad) + 1$

Next complete the square!

take $\frac{1}{2}$ of $1 = \frac{1}{2}$

square it! $\left(\frac{1}{2}\right)^2 = \frac{1^2}{2^2} = \frac{1}{4}$

Now add this inside the parenthesis

$= 3(x^2 + x + \frac{1}{4}) + 1 - \frac{3}{4}$

we did all of this because now $(x^2 + x + \frac{1}{4}) = (x + \frac{1}{2})^2$

$f(x) = 3(x+\frac{1}{2})^2 + \frac{1}{4}$

x^2 shifted left $\frac{1}{2}$ unit up $\frac{1}{4}$ unit and narrowed by a factor of 3. $(-\frac{1}{2}, \frac{1}{4})$

What if we wished to just graph

$$f(x) = 3x^2 + 3x + 1$$

without completing the square?

We can use the vertex formula: Given $f(x) = ax^2 + bx + c$ then the vertex is $\left(x = \frac{-b}{2a}, y = f(x) \right)$

Ex $f(x) = 3x^2 + 3x + 1$

$a = 3, b = 3, c = 1$

$$x = \frac{-3}{2(3)} = \frac{-3}{6} = \boxed{-\frac{1}{2}}$$

$$y = f\left(-\frac{1}{2}\right) = 3\left(-\frac{1}{2}\right)^2 + 3\left(-\frac{1}{2}\right) + 1$$

$$= 3 \cdot \frac{1}{4} - \frac{3}{2} + \frac{1}{1}$$

$$= \frac{3}{4} - \frac{6}{4} + \frac{4}{4}$$

$$= -\frac{3}{4} + \frac{4}{4} = \boxed{\frac{1}{4}}$$

vertex $= \left(-\frac{1}{2}, \frac{1}{4}\right)$