

3/23/2023

Class Agenda 19 - Graphing
Quadratic Functions - using
The vertex formula and transformations
of functions

So far: Quadratic Equations

$$ax^2 + bx + c = 0$$

When we solve this equation,
looking for the value(s) of
 x which make the statement
above true

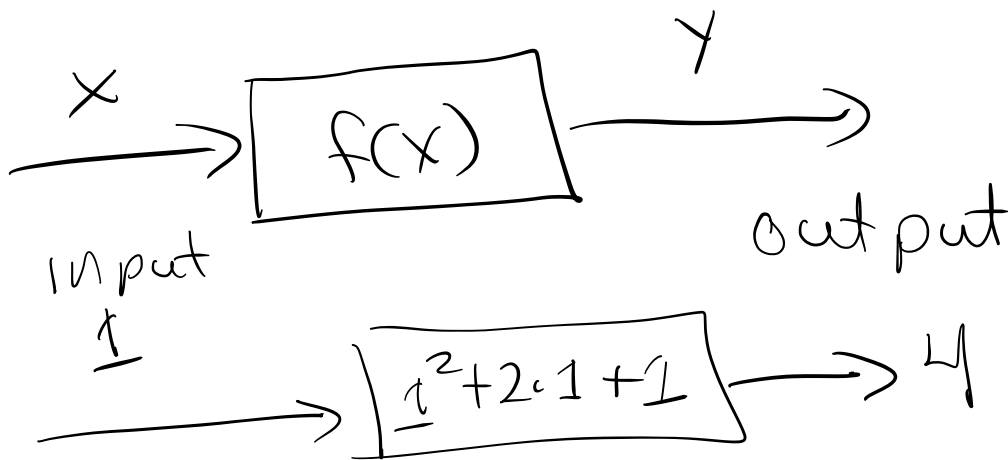
Today: Quadratic Functions

$$f(x) = ax^2 + bx + c$$

Ex Given
 $f(x) = x^2 + 2x + 1$

find $f(1) = 1^2 + 2 \cdot 1 + 1 = 4$

plot the point $(1, 4) \rightarrow$ on the graph of
 $f(x) = x^2 + 2x + 1$



Note: Graphs of quadratic functions are parabolas.

Ex $f(x) = x^2$
↑

Experimented in Desmos!

Note: Where the quadratic function $f(x) = 0 \rightarrow$ cross the x-axis, these are called the "roots"

Transformations

1) $f(x) = x^2 + k$ if k is positive
The graph of x^2 is shifted up
 k units

2) $f(x) = x^2 + k$ if k is negative
" " " " " "down
 k units

3) $f(x) = (x - k)^2$ if k is positive
The graph of x^2 is shifted
right k units

3) $f(x) = (x + k)^2$ if k is positive
" " " " " "
left k units

4) Graphing $f(x) = ax^2$
If a is positive the parabola

opens up and if a is negative
the parabola opens down.

a) If $|a| > 1$ the graph of the
parabola is narrower than
the graph of x^2

b) If $|a| < 1$ " " " "
" " is wider than x^2

" Shift right ^{1 unit}, make it narrow
by a factor of 2 then shift
up 4 units" | $f(x) = 2(x-1)^2 + 4$

What if we are given
a quadratic function.....
but it is not represented
in the form $f(x) = a(x-h)^2 + k$?

$$\text{vertex} = (h, k)$$

Consider the quadratic function

$$f(x) = 3x^2 + 3x + 1$$

$$= 3(x^2 + x) + 1$$

Next complete the square!

$$\text{take } \frac{1}{2} \text{ of } 1 = \frac{1}{2}$$

$$\text{Square it! } \left(\frac{1}{2}\right)^2 = \frac{1^2}{2^2} = \frac{1}{4}$$

Now add this inside the parentheses

$$\rightarrow = 3\left(x^2 + x + \frac{1}{4}\right) + 1 - \frac{3}{4}$$

we did all of this because
now $\left(x^2 + x + \frac{1}{4}\right) = \left(x + \frac{1}{2}\right)^2$

$$f(x) = 3\left(x + \frac{1}{2}\right)^2 + \frac{1}{4}$$

x^2 shifted left $\frac{1}{2}$ unit
up $\frac{1}{4}$ unit and narrowed by
a factor of 3. $\left(-\frac{1}{2}, \frac{1}{4}\right)$

What if we wished to
just graph

$$f(x) = 3x^2 + 3x + 1$$

without completing the
square?

We can use The vertex
formula: Given $f(x) =$
 $ax^2 + bx + c$ Then the

Vertex is $(x = -\frac{b}{2a}, y = f(x))$

$$\text{Ex } f(x) = 3x^2 + 3x + 1$$

$$a = 3, b = 3, c = 1$$

$$x = \frac{-3}{2(3)} = \frac{-3}{6} = \boxed{-\frac{1}{2}}$$

$$y = f\left(-\frac{1}{2}\right) = 3\left(-\frac{1}{2}\right)^2 + 3\left(-\frac{1}{2}\right) + 1$$

$$= 3 \cdot \frac{1}{4} - \frac{3}{2} + 1$$

$$= \frac{3}{4} - \frac{6}{4} + \frac{4}{4}$$

$$= -\frac{3}{4} + \frac{4}{4} = \boxed{\frac{1}{4}}$$

$$\text{Vertex} = \left(-\frac{1}{2}, \frac{1}{4}\right)$$