

3/21/2022

WebWork set: Quadratic Formula

#2) List of The "roots" of

$$x^2 - 16x + 100$$

i.e. The solutions to The quadratic equation $x^2 - 16x + 100 = 0$

First check if

it is possible to factor:

Try to find 2 values which multiply to 100 and sum to -16.

Try: 10×10

2×50

4×25

5×20

\vdots

If factoring

fails then

Solve for x

using The quadratic
formula

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$ax^2 + bx + c = 0$$

$$x^2 - 16x + 100 = 0$$

$$a = 1$$

$$b = -16$$

$$c = 100$$

$$= \frac{-(-16) \pm \sqrt{(-16)^2 - 4(1)(100)}}{2(1)}$$

$$= \frac{16 \pm \sqrt{-144}}{2} = \frac{16 \pm \sqrt{144} \sqrt{-1}}{2} = i$$

$$= \frac{16 \pm 12i}{2} = \frac{2 \cdot (8 \pm 6i)}{2} = 8 \pm 6i$$

Listed: $8+6i, 8-6i$

Quadratic Equation Applications

Ex Need \$! Go to The loan shark, borrow \$100 at 30% simple interest rate monthly

How much do we owe after 1 month?

Month	Owe
0	\$ 100
1	\$ 130 → $(1.3 \cdot 100)$
2	\$ 160
3	\$ 190
4	\$ 220

← principal

$$(1+r)P$$

↑

interest rate as a decimal

Compound the interest

every month
we can just add
on \$ 30 to what
we owe → "simple"

↓
new
table

Month	interest	Total owed
0	0	\$100
1	\$30 = (.3 × 100)	\$130
2	\$39 = (.3 × 130)	\$169
3	\$50.7 = (.3 × 169)	\$219.70
4	\$65.91 = (.3 × 219.7)	\$285.61
	⋮	

“

Amortization Schedule

Formula

$$A = P(1+r)^t$$

total amount \rightarrow A
 principal \uparrow P
 rate in decimal \uparrow r
 # time periods elapsed \uparrow t

Ex At the end of 3 years if
\$100 is invested at 5%
compounded annually

$$A = 100(1 + .05)^3 = \$$$

$$100(1.05)^3 = \$115.76$$

Ex Find the interest rate r
if \$2000 compounded annually
grows to \$2420 in 2 years.

$$A = P(1 + r)^t$$

$$t = 2$$

$$A = \$2420$$

$$\frac{2420}{2000} = \frac{2000(1+r)^2}{2000}$$

$$P = \$2000$$

Solve for r !

$$\frac{2420}{2000} = (1+r)^2$$

$$\pm \sqrt{\frac{2420}{2000}} = 1+r$$

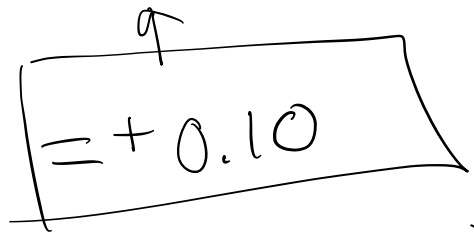
This is a quadratic
equation \rightarrow take
the square root of
both sides

-1

$$-1 \pm \sqrt{\frac{2420}{2000}} = r$$



$$r = -1 + \sqrt{\frac{2420}{2000}}$$



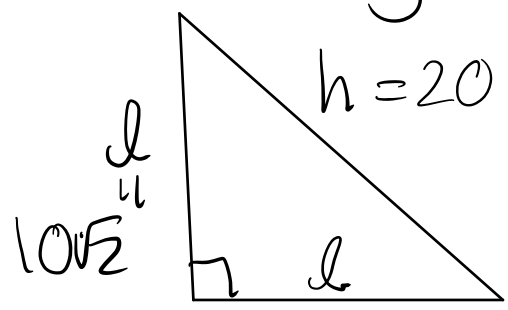
10% interest rate

$$r = -1 - \sqrt{\frac{2420}{2000}}$$

$$r = -2$$

reject we don't use this

Ex An isosceles ^{right} triangle has legs of equal length. If the hypotenuse is 20 cm long, find the length of each leg.



Check:

$$(10\sqrt{2})^2 + (10\sqrt{2})^2 \stackrel{?}{=} 20^2$$

$$100 \cdot 2 + 100 \cdot 2 \stackrel{?}{=} 400$$

$$200 + 200 = 400$$

Challenge: The sum of two consecutive odd numbers is -100 . Find the numbers!

odd # can be written as

$$2n+1 \text{ where } n=0, \pm 1, \pm 2, \dots$$

→ two consecutive odds

$$2n+1, 2n+3$$

$$\rightarrow (2n+1) + (2n+3) = -100$$

$$\rightarrow \text{solve for } \boxed{n = -26}$$

$$\text{Then } 2n+1 = -51$$

$$2n+3 = -49$$

Solution: $-49, -51$

Ex Find the rate r at which
\$3000 grows to 4320 in
2 years.

$$r = 0.20$$

20%

$$A = P(1+r)^t$$

$$\frac{4320}{3000} = \frac{3000}{3000} (1+r)^2$$

$$\frac{4320}{3000} = (1+r)^2$$

$$\pm \sqrt{\frac{4320}{3000}} = 1+r$$

$$-1 \pm \sqrt{\frac{4320}{3000}} = r$$

$$\begin{aligned} -1 + \sqrt{\frac{4320}{3000}} & \text{ Keep!} \\ & = 0.20 \\ & \rightarrow 20\% = r \end{aligned}$$

$$-1 - \sqrt{\frac{4320}{3000}}$$

||
~~-2.2~~ reject