Exam 4 Review (partial) solutions Fall 2011

1. $x^2=4x$ has $\{0,4\}$ as its solution set
   $x=4$ has $\{4\}$ as its solution set.

   We have lost a solution as a result of dividing by 0.
   (we can divide by x, but then must look at $x=0$ separately).

2. Squaring both sides may introduce extraneous solutions
   $x=-1$ has $\{-1\}$ as its solution set
   $(x)^2=(-1)^2$ or $x^2=1$ has $\{-1,1\}$ as its solution set
   You can finish with the correct solution set by requiring that you (must) check the solutions in the original equation.

3.

	advantage	disadvantage
Factor	can be fast	only works for rational solutions
square rt principal or property	fast	only works in special form
quadratic formula	always works	more work to simplify

factor: $2x^2+5x-3=0$ $(3x-1)^2=5$ sqrt prop $2x^2-6x+3$ quad. form

4. a) $7x^5$ b) $2p\sqrt{9p^4}\sqrt{7p}=6p^3\sqrt{7p}$ c) $-3pq^2\sqrt{25k^6q^8}\sqrt{5k}$
   $=-3pq^2\,5k^3q^4\sqrt{5k}$
   $=-15pk^3q^6\sqrt{5k}$

5. a) $(2\sqrt{3})^2-2(2\sqrt{3})(\sqrt{5})+(\sqrt{5})^2=4\cdot 3-4\sqrt{15}+5$
   $=17-4\sqrt{15}$

   b) $3\sqrt{6x}-12x\sqrt{45}=3\sqrt{6x}-12x\sqrt{9}\sqrt{5}=3\sqrt{6x}-36x\sqrt{5}$

   c) $6\sqrt{100}-9\sqrt{24}+4\sqrt{150}-6\sqrt{36}=6\cdot 10-9\sqrt{4}\sqrt{6}+\sqrt{25}\sqrt{6}-6\cdot 6$
   $=60-18\sqrt{6}+5\sqrt{6}-36=24-13\sqrt{6}$

6. a) $\sqrt{\dfrac{8x^3}{y}}=\dfrac{\sqrt{4x^2}\sqrt{x}}{\sqrt{y}}\cdot\dfrac{\sqrt{y}}{\sqrt{y}}=\dfrac{2x\sqrt{xy}}{y}$ b) $\dfrac{\sqrt{4x}}{2\sqrt{3}}\cdot\dfrac{\sqrt{3}}{\sqrt{3}}=\dfrac{\sqrt{4}\sqrt{3x}}{2\cdot 3}=\dfrac{\sqrt{3x}}{3}$

   c) $\dfrac{\sqrt{4}\sqrt{6}}{3-\sqrt{3}}\cdot\dfrac{3+\sqrt{3}}{3+\sqrt{3}}=\dfrac{2\sqrt{6}(3+\sqrt{3})}{9-3}=\dfrac{\cancel{2}\sqrt{6}(3+\sqrt{3})}{\cancel{6}_{3}}=\dfrac{3\sqrt{6}+\sqrt{18}}{3}$
   $=\dfrac{3\sqrt{6}+\sqrt{9}\sqrt{2}}{3}=\dfrac{3\sqrt{6}+3\sqrt{2}}{3}=\dfrac{\cancel{3}(\sqrt{6}+\sqrt{2})}{\cancel{3}}=\sqrt{6}+\sqrt{2}$

6 d) $\frac{2\sqrt{3}-3\sqrt{2}}{\sqrt{3}+2\sqrt{2}} \cdot \frac{\sqrt{3}-2\sqrt{2}}{\sqrt{3}-2\sqrt{2}} = \frac{2\cdot 3-4\sqrt{6}-3\sqrt{6}+6\cdot 2}{3-4\cdot 2} = \frac{18-7\sqrt{6}}{-5}$

$= \frac{-18+7\sqrt{6}}{5}$

e) $\frac{x\sqrt{2}}{x\sqrt{2}+3\sqrt{5}} \cdot \frac{x\sqrt{2}-3\sqrt{5}}{x\sqrt{2}-3\sqrt{5}} = \frac{2x^2-3x\sqrt{10}}{2x^2-45}$

7. a) $x-4=\sqrt{x-2}$

$x^2-8x+16=x-2$

$x^2-9x+18=0$

$(x-6)(x-3)=0$

$x=6$ or $x=3$

$6-4=\sqrt{6-2}$

$2=\sqrt{4}$ ✓

$3-4=\sqrt{3-2}$

$-1=1$ false.

$\{6\}$.

b) $3\sqrt{x-4}=x-4$

$9(x-4)=x^2-8x+16$

$9x-36=x^2-8x+16$

$0=x^2-17x+52$

$0=(x-13)(x-4)$

$x=13$ or $x=4$

$3\sqrt{13-4}=13-4$

$3\sqrt{9}=9$

$3\cdot 3=9$ ✓

$3\sqrt{4-4}=4-4$

$3\cdot 0=0$ ✓

$\{4, 13\}$

c) $\sqrt{5-x}=x+1$

$5-x=x^2+2x+1$

$0=x^2+3x-4$

$0=(x+4)(x-1)$

$x=-4$ or $x=1$

$\sqrt{5-(-4)}=-4+1$

$3=-3$ false

$\sqrt{5-1}=1+1$

$2=2$ ✓

$\{1\}$

8. a) $y^2=135$

$y=\pm\sqrt{135}$

$=\pm\sqrt{9}\sqrt{15}$

$=\pm 3\sqrt{15}$

b) $y+4=\pm 13$

$y=-4\pm 13$

$y=-4+13$

$=9$

$y=-4-13$

$=-17$

$\{-17, 9\}$

9. a) $x=\frac{-10\pm\sqrt{10^2-4\cdot 7}}{2}$

$=\frac{-10\pm\sqrt{72}}{2}=\frac{-10\pm 6\sqrt{2}}{2}=-5\pm 3\sqrt{2}$

b) $3x^2-8x+3=0$

$x=\frac{8\pm\sqrt{(-8)^2-4\cdot 3\cdot 3}}{2\cdot 3}=\frac{8\pm\sqrt{28}}{2\cdot 3}=\frac{\cancel{2}(4\pm\sqrt{7})}{\cancel{2}\cdot 3}=\frac{4\pm\sqrt{7}}{3}$

c) $x^2-6x-4=0$ $\quad x=\frac{6\pm\sqrt{(-6)^2-4(-4)}}{2}=\frac{6\pm\sqrt{54}}{2}=\frac{6\pm 3\sqrt{6}}{2}$

9 d) $x^2+6x+6=0 \quad x=\dfrac{-6\pm\sqrt{6^2-4\cdot 6}}{2}=\dfrac{-6\pm\sqrt{12}}{2}$

$=\dfrac{-6\pm 2\sqrt{3}}{2}=\dfrac{\cancel{2}(-3\pm\sqrt{3})}{\cancel{2}}=3\pm\sqrt{3}$

10. $(x-1)^2+(2x)^2=(2x+1)^2$

$x^2-2x+1+4x^2=4x^2+4x+1$

$x^2-6x=0$

$x(x-6)=0$

$\cancel{x=0}$ or $x=6$

a) $x=6$

b) $AC=5=6-1$

$BC=12=2(6)$

$AB=13=2(6)+1$

c) $A_\triangle=\frac{1}{2}bh$

$=\frac{1}{\cancel{2}}5(\cancel{12}^{6})$

$=30$

11. $x^2+3^2=7^2$

$x^2+9=49$

$x^2=40$

$x=\pm 2\sqrt{10}$

a) $x=2\sqrt{10}\approx 6.3$

b) $\text{perim}_\triangle=3+7+2\sqrt{10}=10+2\sqrt{10}\text{ cm}$

$\approx 16.3\text{ cm}$

c) $A_\triangle=\frac{1}{2}bh=\frac{1}{2}(2\sqrt{10})\cdot 3=3\sqrt{10}\text{ cm}^2$

$=9.5\text{ cm}^2$

12. $\text{Perim}_\triangle=\sqrt{12}+\sqrt{48}+\sqrt{75}=2\sqrt{3}+4\sqrt{3}+5\sqrt{3}=11\sqrt{3}\text{ ft}\approx 19.1\text{ ft}^2$

13. a) $12\sqrt{7}=\sqrt{112}+x+\sqrt{112}+x=2\sqrt{112}+2x=2\sqrt{16}\sqrt{7}+2x=8\sqrt{7}+2x$

$4\sqrt{7}=2x \quad x=2\sqrt{7}\text{ cm}$ b) $A_\square=lw=4\sqrt{7}\cdot 2\sqrt{7}=8\cdot 7=56\text{ cm}^2$

(It was a mistake to ask but) Perimeter is $12\sqrt{7}$ cm

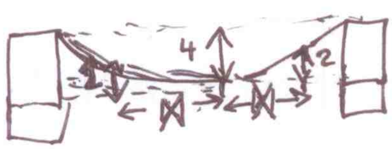
14. a)

let H be vertical height

$x^2=16h$

$8^2=16\cdot H \quad H=4$ inches

b)



$x^2=16h$ let X be where $h=2$

$X^2=16(2) \quad X^2=32 \quad X=\pm 4\sqrt{2}$

The locations where the cable is 2 inches above its lowest point are $4\sqrt{2}\approx 5.66$ inches on either side of the lowest point.

If we let the lowest point be the origin $(0,0)$ then the locations are $(-4\sqrt{2},2)$ and $(4\sqrt{2},2)$ or $(-5.66,2)$ and $(5.66,2)$.