Exam 3 Review

Math 1175, Fundamentals of Mathematics, Fall 2011
Mathematics and The Brooklyn Bridge

1. Factoring is often taught directly after multiplying polynomials. Explain the reason for this order of topics.
2. In the context of polynomials, what is a prime factor? Give an example of a prime factor of degree 2.
3. Multiply: a) $7x^2y^3(4x-3xy+2y)$ b) $(2x-3)(x+5)$ c) $(3y^3-y)(3y^3-6y+1)$
4. Divide:
   a) $\frac{27a^2x^3y^5-15x^2y^4+3xy^3}{3xy^3}$ b) long division: $(2x^3-5x^2+5x+2)\div(2x-3)$
5. Factor completely: a) $x^2y^2+4xy^2+3y^2$ b) $3x^2+8x+4$
   c) $4x^2z+4xz-24z$ d) $4x^2-25$ e) $x^2+10x+25$ (special product)
6. Imagine your younger brother or sister is learning factoring for the first time. Explain to them the process of factoring using the AC method (factoring by grouping), using problem #5b (above) as an example. Use complete sentences.
7. Solve and check your answer(s):
   a) $x(2x-3)=2$ b) $y(3+2y)=y^2+4y$ c) $9x^2+30x+25=0$
8. A rocket is launched from the pedestrian pathway of the Brooklyn Bridge, near the Brooklyn side of the bridge, at a point where the pathway is 192 feet above the river. It flies into the air and then falls into the river. The height *h* in feet of the rocket (above the river) at time *t* seconds is given by $h=-16t^2+64t+192$. How many seconds will it take for the rocket to complete its flight?
9. Daisy is making a model of the bridge and knows that her tower is 8 inches tall and that the distance from the base of one tower to the top of other tower is 2 inches less than twice the distance between the 2 towers. Find the distance between the towers in Daisy's model.
10. Simplify: a) $\frac{x^2-8x+16}{x-4}$ b) $\frac{2x^2-2}{10x+30}\cdot\frac{12x+36}{3x-3}$ c) $\frac{3x}{5}\div\frac{x^2-10x+25}{5x-25}$
11. Add or subtract, and simplify:
    a) $\frac{6}{x^2}+\frac{11}{3x}$ b) $\frac{4x-2}{x^2-x-20}-\frac{2}{x+4}$ c) $\frac{x+2}{x^2-36}-\frac{x}{x^2+9x+18}$

Expanded Answer Key

1. (Answers will vary.) Factoring is the inverse of multiplying. More precisely, if the original 2 factors of the product are prime, then factoring will return you to what you started with. Also note that multiplying can serve as a check on factoring. If you multiply your factors, the result should be the polynomial that you started with.
2. (Answers will vary.) A prime factor is something that can not be factored further. x^2+1 is an example of prime factor of degree 2 (over the real numbers).

3a. $28x^3y^3-21x^3y^4+14x^2y^4$ b. $2x^2+7x-15$

c.

$$\begin{array}{r} 3y^3+0y^2-6y+1 \\ 3y^3-y \\ \hline -3y^4+0y^3+6y^2-y \\ 9y^6+0y^5-18y^4+3y^3 \\ \hline 9y^6+0y^5-21y^4+3y^3+6y^2-y \end{array}$$

Answer simplified: $9y^6-21y^4+3y^3+6y^2-y$

4a. $\frac{27a^2x^3y^5}{3xy^3}-\frac{15x^2y^4}{3xy^3}+\frac{3xy^3}{3xy^3}=9a^2x^2y^2-5xy+1$

b.

$$\begin{array}{r} x^2-x+1 \\ 2x-3\,|\,\overline{2x^3-5x^2+5x+2} \\ 2x^3-3x^2 \\ \hline -2x^2+5x \\ -2x^2+3x \\ \hline 2x+2 \\ 2x-3 \\ \hline 5 \end{array}$$

Answer is:

$$x^2-x+1+\frac{5}{2x-3}$$

5 a. $y^2(x^2+4x+3)=y^2(x+3)(x+1)$

b. Using the AC method, the product of the first and last terms is $12x^2$. The factors which multiply to this product but add to the middle term are $2x$and $6x$. Replace the middle term with the sum of these 2 expressions:$3x^2+8x+4=3x^2+2x+6x+4$. Now take out the common factors from the first 2 and then the last 2 terms:$3x^2+2x+6x+4=x(3x+2)+2(3x+2)$ Finally, pull out the common $(3x+2)$to get $(3x+2)(x+2)$

c. $4z(x^2+x-6)=4z(x+3)(x-2)$

d. This is a difference of perfect squares. Take the square roots of the terms and then write the sum and difference down as factors: $(2x+5)(2x-5)$

e. This is a perfect square trinomial. The middle term is twice the product of the square roots of the first and last terms. To factor, take the square roots of the first and last terms and the sign of the middle term and square the resulting binomial: $(x+5)^2$

6. Answers will vary. See the answer to 5b) above.

7a.

$$2x^2-3x=2;\ 2x^2-3x-2=0;(2x+1)(x-2)=0;x=-\frac{1}{2},\ x=2;\left\{-\frac{1}{2},2\right\}$$

b. $y(3+2y)=y^2+4y;3y+2y^2=y^2+4yy^2-y=0;y(y-1)=0;y=0,\ y=1;\{0,1\}$

c. $(3x+5)^2=0;\ x=-\frac{5}{3}$

8. The rocket will end its flight when its height is 0:

$$h=-16t^2+64t+192=0.$$

$$-16(t^2-4t-12)=-16(t-6)(t+2)=0\ or\ t=-2,t=6.$$

We reject the negative answer since the equation is only valid for nonnegative time.

Answer: The rocket takes 6 seconds to complete its flight.

9. Let x be the distance between the towers (which is one of the legs).

The hypotenuse is $2x-2$. Using the Pythagorean theorem:

$$x^2+8^2=(2x-2)^2$$

$$x^2+64=4x^2-8x+4$$

$$0=3x^2-8x-60$$

$$3x^2-18x+10x-60=3x(x-6)+10(x-6)=\ (x-6)(3x+10)=0$$

$$\left\{-\frac{10}{3},6\right\}$$

Reject the negative answer since x is a distance.

Answer: the distance between the towers is 6 inches.

10 a.

$$\frac{x^2-8x+16}{x-4}=\frac{(x-4)^2}{x-4}=x-4$$

b.

$$\frac{2x^2-2}{10x+30}\cdot\frac{12x+36}{3x-3}\quad=\frac{2(x-1)(x+1)}{10(x+3)}\cdot\frac{12(x+3)}{3(x-1)}=\frac{4(x+1)}{5}$$

c.

$$\frac{3x}{5}\cdot\frac{5x-25}{x^2-10x+25}=\frac{3x}{5}\cdot\frac{5(x-5)}{(x-5)^2}=\frac{3x}{x-5}$$

11a.

$$\frac{6}{x^2}+\frac{11}{3x}=\frac{3}{3}\frac{6}{x^2}+\frac{11}{3x}\frac{x}{x}=\frac{18+11x}{3x^2}$$

b.

$$\frac{4x-2}{x^2-x-20}-\frac{2}{x+4}=\frac{4x-2}{(x-5)(x+4)}-\frac{2}{x+4}=\frac{4x-2}{(x-5)(x+4)}-\frac{2}{x+4}\cdot\frac{x-5}{x-5}$$

$$=\frac{4x-2-2(x-5)}{(x-5)(x+4)}=\frac{4x-2-2x+10)}{(x-5)(x+4)}=\frac{2x+8}{(x-5)(x+4)}$$

c.

$$\frac{x+2}{x^2-36}-\frac{x}{x^2+9x+18}=\frac{x+2}{(x+6)(x-6)}-\frac{x}{(x+6)(x+3)}$$

$$=\frac{x+2}{(x+6)(x-6)}\cdot\frac{x+3}{x+3}-\frac{x}{(x+6)(x+3)}\cdot\frac{x-6}{x-6}=\frac{x^2+5x+6-x(x-6)}{(x+6)(x-6)(x+3)}$$

$$=\frac{x^2+5x+6-x^2+6x}{(x+6)(x-6)(x+3)}=\frac{11x+6}{(x+6)(x-6)(x+3)}$$