Absolute Value

The **absolute value** (or **modulus**) of the complex number $a + bi$ is

$$|a + bi| = \sqrt{a^2 + b^2}.$$

Polar Multiplication and Division Rules

If $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$ are any two complex numbers, then

$$z_1 z_2 = r_1 r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$$

and

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)] \quad (z_2 \neq 0).$$

Section 9.1

Section 9.2

DeMoivre's Theorem

For any complex number $z = r(\cos\theta + i\sin\theta)$ and any positive integer n,

$$z^n = r^n(\cos n\theta + i\sin n\theta).$$

Formula for nth Roots

For each positive integer n, nonzero complex number

$$r(\cos\theta + i\sin\theta)$$

has exactly n distinct nth roots. They are given by

$$\sqrt[n]{r}\left[\cos\left(\frac{\theta + 2k\pi}{n}\right) + i\sin\left(\frac{\theta + 2k\pi}{n}\right)\right],$$

where $k = 0, 1, 2, 3, \ldots n - 1$.

Roots of Unity

For each positive integer n, there are n distinct nth roots of unity:

$$\cos\frac{2k\pi}{n} + i\sin\frac{2k\pi}{n} \quad (k = 0, 1, 2, \ldots, n - 1).$$