11/29/2021

WebWork Set: Trig Equations

Problem #5: Give all solutions

$$4\cos^2 x - 1 = 0$$

let $u = \cos x$ $\qquad 4u^2 - 1 = 0$

$$(2u)^2 - 1^2 = 0$$

looks like the difference of 2 squares so factor it! $a^2 - b^2 = (a+b)(a-b)$

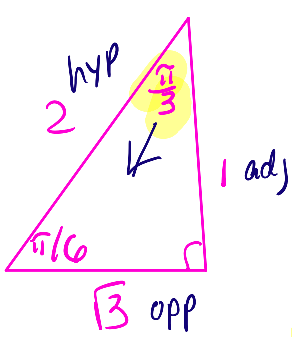
$$(2u+1)(2u-1) = 0$$

$$(2\cos x + 1)(2\cos x - 1) = 0$$

"Zero product" property:
$A \cdot B = 0$
$\Leftrightarrow A = 0$ or $B = 0$
or both

$$2\cos x + 1 = 0$$
$$-1 \quad -1$$
$$2\cos x = -1$$
$$\cos x = -\frac{1}{2} \quad \frac{\text{adj}}{\text{hyp}}$$
$$\cos^{-1}\left(-\frac{1}{2}\right) = x$$

$$2\cos x - 1 = 0$$
$$+1 \quad +1$$
$$2\cos x = 1$$
$$\cos x = \frac{1}{2}$$
$$\cos^{-1}\left(\frac{1}{2}\right) = x$$



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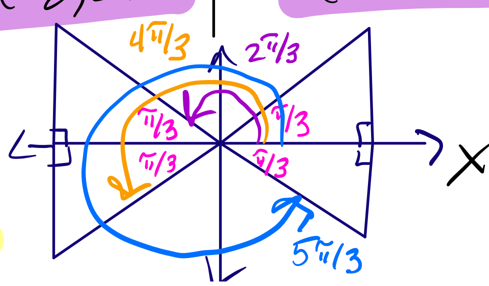
$$x = \frac{\pi}{3} + 2\pi \cdot n$$
$$\frac{2\pi}{3} + 2\pi \cdot n$$
$$4\pi/3 + 2\pi \cdot n$$

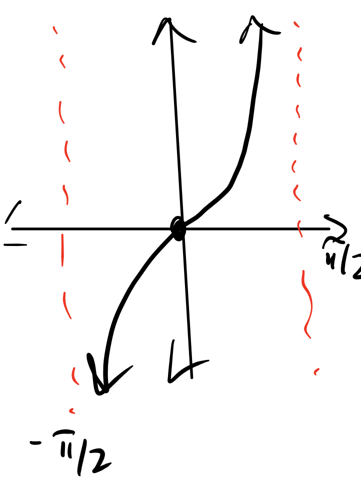


$5\pi/3 + 2\pi \cdot n$

$n = 0, \pm 1, \pm 2, \pm 3, \ldots$

Problem #6: $2\sin x \tan x - \tan x = 0$

$$\tan x(2\sin x - 1) = 0$$



$\tan x = 0$

$\tan^{-1}(0) = x$

$x = 0 + \pi \cdot n$

$n = 0, \pm 1, \pm 2, \ldots$

(recall π is the period for $\tan x$)

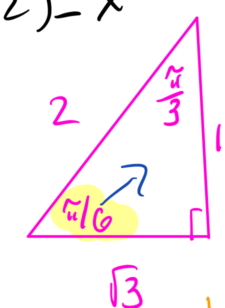
$2\sin x - 1 = 0$

$\quad +1 \quad +1$

$2\sin x = 1$

$\sin x = \frac{1^{\text{opp}}}{2_{\text{hyp}}}$ SOHCAHTOA

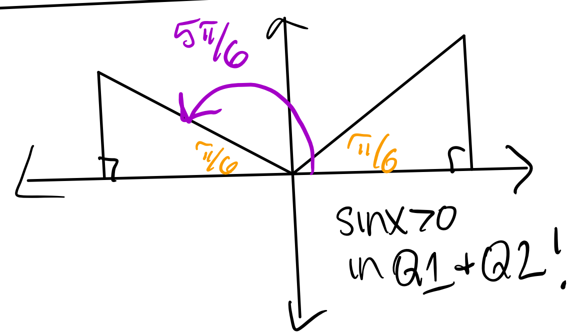
$\sin^{-1}\left(\frac{1}{2}\right) = x$



$x = \frac{\pi}{6} + 2\pi n$

$n = 0, \pm 1, \pm 2, \ldots$

$\frac{5\pi}{6} + 2\pi n$



$\sin x > 0$ in Q1 + Q2!

Problem #7: $\tan^2 x - \sqrt{3}\tan x = 0$

let $u = \tan x$ $\qquad u^2 - \sqrt{3}u = 0$

$u(u - \sqrt{3}) = 0$

$$\tan x(\tan x - \sqrt{3}) = 0$$

$\tan x = 0$

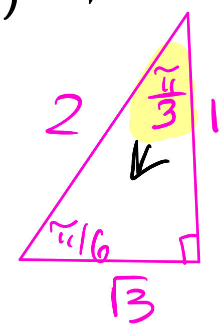
$\tan^{-1}(0) = x$

$x = 0 + n\pi$
$n = 0, \pm 1, \pm 2, \ldots$

$\tan x - \sqrt{3} = 0$
$\quad + \sqrt{3} \quad + \sqrt{3}$

$\tan x = \sqrt{3}$ SOHCAHTOA

$\tan^{-1}(\sqrt{3}) = x$



$x = \frac{\pi}{3} + n\pi$
$n = 0, \pm 1, \pm 2, \ldots$

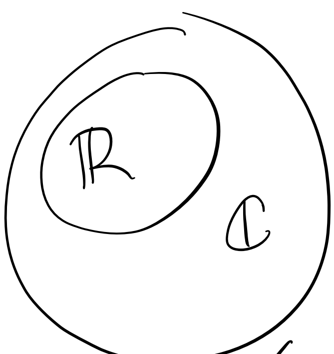
Lesson 21: Complex Numbers

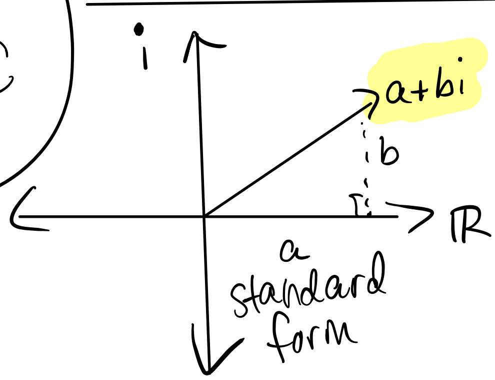
Standard Form: $a + bi$ where $a, b \in \mathbb{R}$

(a, b in the set of real numbers)

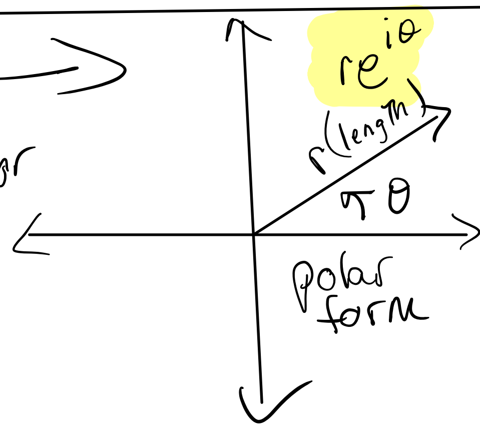
and $i = \sqrt{-1}$

so $i^2 = -1$





or



First some basic arithmetic w/ complex #'s:

a) $(2-3i)+(-6+4i)$

$= (2-6)+(-3+4)i$

$= \boxed{-4+i}$

add real & imaginary parts

b) $(3+5i)(-7+i) = -21+3i-35i+5i^2$ $(i^2=-1)$

"FOIL"

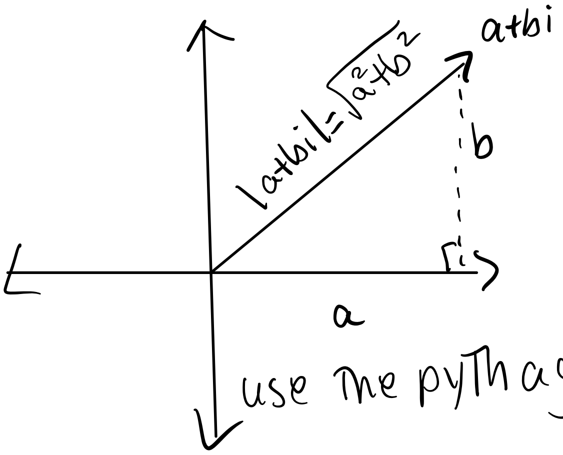
$= -21-32i-5$

$= \boxed{-26-32i}$

c) $\dfrac{5+4i}{3+2i}$ Can simplify the quotient of complex #'s by multiplying the num + den by the complex conjugate of the denominator.

$3+2i \longrightarrow 3-2i$

$$\frac{(5+4i)}{(3+2i)}\cdot\frac{(3-2i)}{(3-2i)} = \frac{15-10i+12i-8i^2}{9+\cancel{6i}-\cancel{6i}-4i^2} = \frac{23+2i}{13} = \boxed{\frac{23}{13}+\frac{2}{13}i}$$

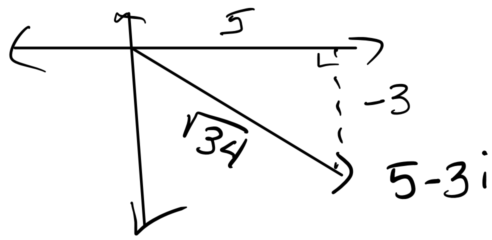


absolute value

$|a+bi| = \sqrt{a^2+b^2}$

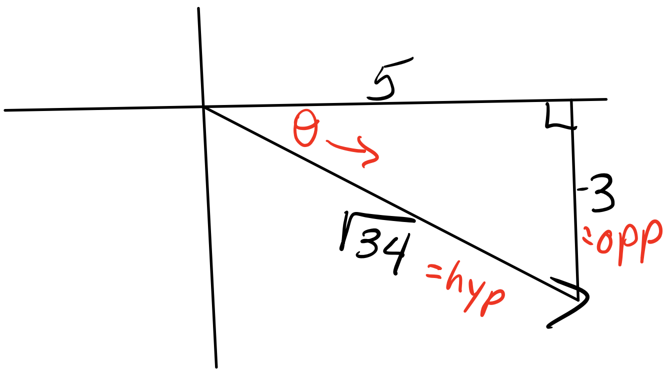
use the pythagorean theorem!

Consider: $5-3i$



$$|5-3i| = \sqrt{5^2+(-3)^2}$$
$$= \sqrt{25+9}$$
$$= \sqrt{34}$$

Suppose we wish to write this in <u>polar</u> <u>form</u>. We already have r = length, we need the ∡! SOHCAHTOA



$$\sin\theta = \frac{-3}{\sqrt{34}}$$

$$\sin^{-1}\left(\frac{-3}{\sqrt{34}}\right) = \theta$$
$$\approx -30.96^\circ$$
$$\text{II}$$
$$329.04^\circ$$

So $5-3i$ in polar form $\sqrt{34}\, e^{i\,329.04^\circ}$

In the next class:

Given Polar form ⟶ <u>standard form</u>!