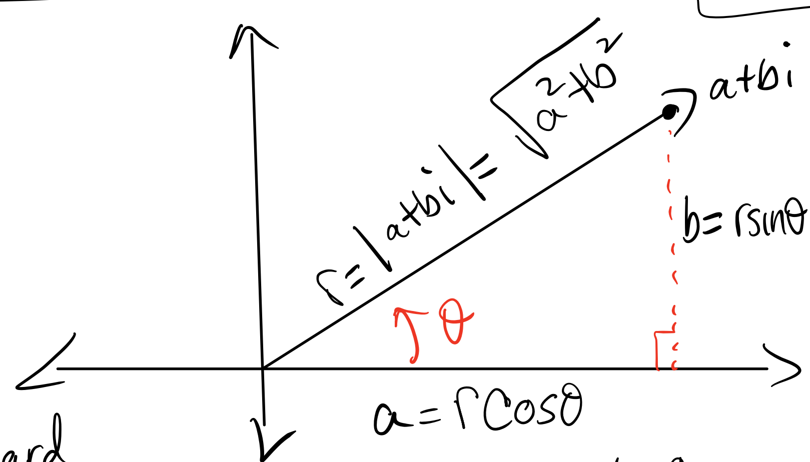
12/1/2021 Continue: Lesson 21: Complex Numbers

Standard form $\longrightarrow$ polar form

$a+bi$ $\longleftarrow$ $re^{i\theta}$

alternative



SOHCAHTOA

$\sin\theta=\frac{b}{r}$

$\cos\theta=\frac{a}{r}$

standard form

$a+bi = r\cos\theta + i\,r\sin\theta$

$= r(\cos\theta + i\sin\theta)$

$\parallel$ > both polar form

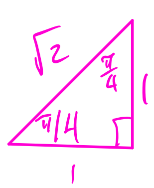
$re^{i\theta}$

can solve for a+b!

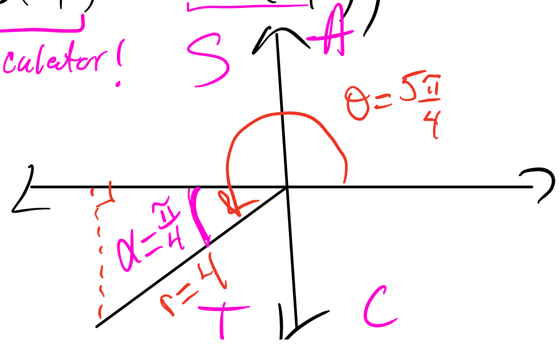
$b = r\sin\theta$

$a = r\cos\theta$

Practice: Convert $4\left(\underbrace{\cos\left(\frac{5\pi}{4}\right)}_{\text{Calculator!}}+i\sin\left(\frac{5\pi}{4}\right)\right)$ to standard form.



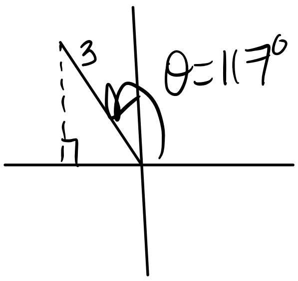
$4\left(-\frac{1}{\sqrt{2}}-i\frac{1}{\sqrt{2}}\right)$



$$= -\frac{4}{\sqrt{2}} - i\frac{4}{\sqrt{2}} = -4\cdot\frac{\sqrt{2}}{2} - i\,4\cdot\frac{\sqrt{2}}{2} = \boxed{-2\sqrt{2} - i\,2\sqrt{2}}$$

rationalize

Convert to standard form:



$$3(\cos(117^\circ) + i\sin(117^\circ))$$

$$= 3(-0.454 + i(0.891))$$

$$= \boxed{-1.312 + 2.673\,i}$$

WeBWork: Precalc Sample Final

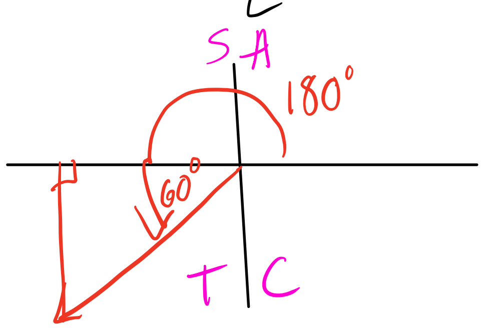
Problem #5: Find the product and write the result in standard form:

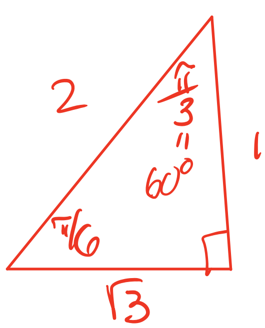
$$[2\cdot(\cos(190^\circ) + i\sin(190^\circ))]\cdot[7(\cos 50^\circ + i\sin 50^\circ)]$$

$$= 2\cdot 7[\cos(190^\circ + 50^\circ) + i\sin(190^\circ + 50^\circ)]$$

$$= 14[\cos(240^\circ) + i\sin(240^\circ)]$$

$$= 14\left[-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right]$$

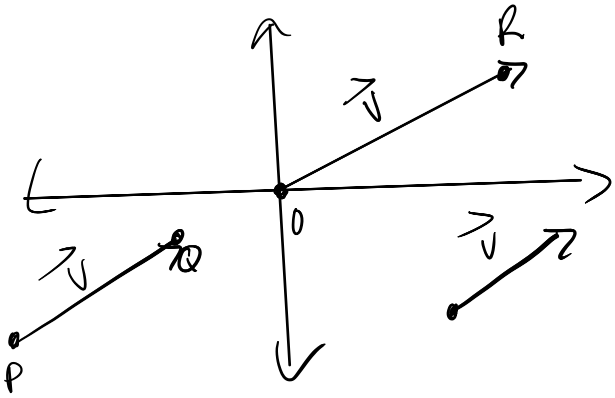




$$\boxed{= -7 - 7\sqrt{3}\,i}$$

Lesson 22: Vectors in the plane

Def: A geometric vector in the plane is a geometric object in $\mathbb{R}^2$ that is given by a magnitude and a direction. We denote a vector by $\vec{v}$ (sometimes $\hat{v}$, $\dot{v}$, $\mathbf{v}$, $\vec{v}$), and its magnitude is denoted by $\|\vec{v}\|$ and its directional angle θ.

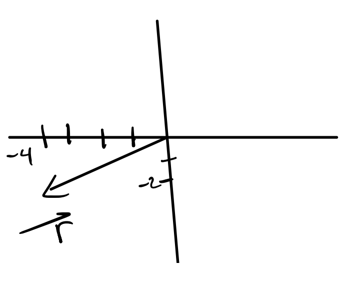


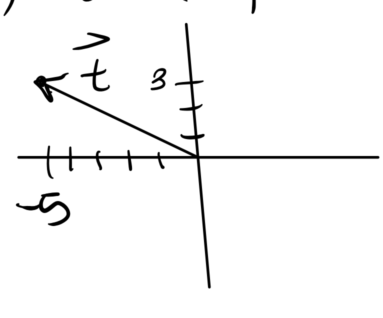
In particular, we can always represent a vector $\vec{v}$ by $\vec{OR}$ by arranging the starting point of $\vec{v}$ at the origin $(0,0) = O$. If R is given by the coordinates $R(a,b)$ then we can also write for $\vec{v}$

$$\vec{v} = \langle a, b \rangle \text{ or alternatively } \vec{v} = \begin{bmatrix} a \\ b \end{bmatrix}$$

Ex. Graph the vectors

a) $\vec{r} = \langle -4, -2 \rangle$ b) $\vec{t} = \langle -5, 3 \rangle$

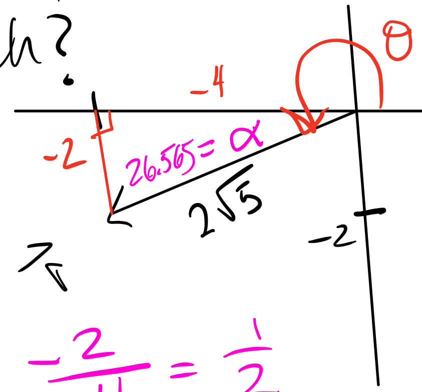




Go further, what are the magnitude + direction for each?

Find the length of the hypotenuse!

a)



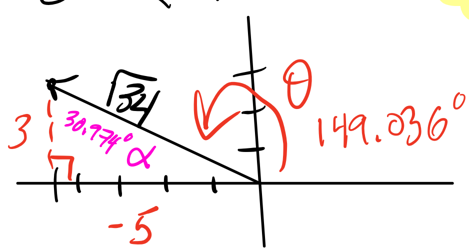
$$\|\vec{r}\| = \sqrt{(-2)^2 + (-4)^2} = \sqrt{20} = \sqrt{4 \cdot 5} = 2\sqrt{5}$$

$$\tan\alpha = \frac{-2}{-4} = \frac{1}{2}$$

$$\tan^{-1}\left(\tfrac{1}{2}\right) = 26.565^\circ$$

$$\theta = 180^\circ + 26.565^\circ = 206.565^\circ$$

b) $\vec{t} = \langle -5, 3 \rangle$



$$\|\vec{t}\| = \sqrt{(-5)^2 + 3^2} = \sqrt{25 + 9} = \sqrt{34}$$

$$\tan\alpha = \frac{3}{5} \rightarrow \tan^{-1}\left(\frac{3}{5}\right) = \alpha = 30.974^\circ$$

$$\theta = 180^\circ - 30.974^\circ = 149.036^\circ$$

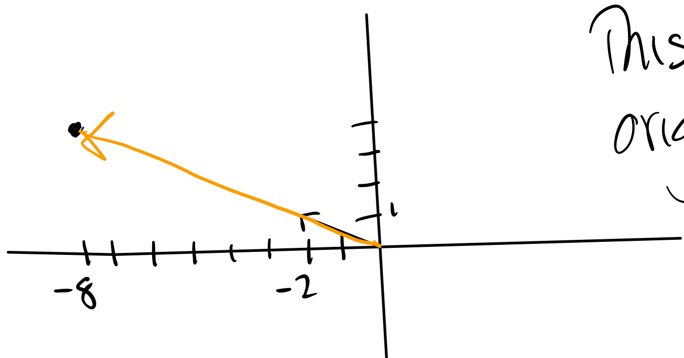
Operations on Vectors: Scalar multiplication and vector addition

Def: The scalar multiplication of a real # r with a vector $\vec{v} = \langle a, b \rangle$ is defined to be the vector given by multiplying r to each coordinate

$$r \cdot \langle a, b \rangle = \langle r \cdot a, r \cdot b \rangle$$

Ex Multiply + graph

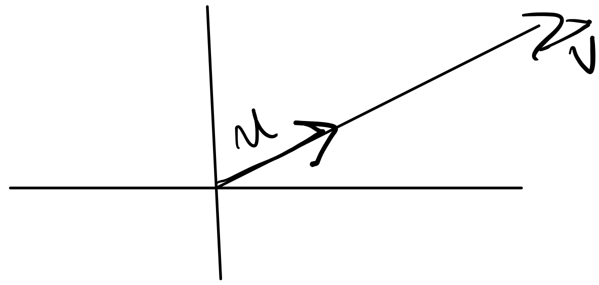
$$4 \cdot \langle -2, 1 \rangle = \langle -8, 4 \rangle$$



This stretches the original vector by a power of 4!

Well, if we stretch we could contract.

Observation: Let $\vec{v}$ be a vector with magnitude $\|\vec{v}\|$ and angle θ. Then if we divide $\vec{v}$ by $\|\vec{v}\|$ then the resulting vector will have magnitude 1 and the same direction.



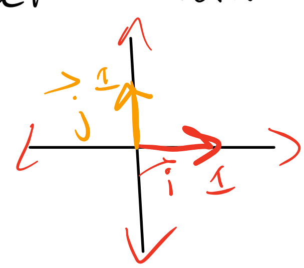
$\|\vec{v}\| = 4$

take $\dfrac{\vec{v}}{\|\vec{v}\|} = \dfrac{\vec{v}}{4} = u$

Then u has length 1 and the same direction.

Def: A vector $\vec{u}$ is called a unit vector if it has magnitude 1 i.e. $\|\vec{u}\| = 1$.

Special: unit vectors $\vec{i} = \langle 1,0 \rangle$, $\vec{j} = \langle 0,1 \rangle$



Ex Find a unit vector in the direction of $\vec{v} = \langle 8,6 \rangle$

First find $\|\vec{v}\| = \sqrt{8^2+6^2} = \sqrt{64+36}$ $\sqrt{100}$ $= 10$

Now divide $\vec{v}$ by 10 and the result should be a unit vector.

$\frac{1}{10} \cdot \vec{v} = \frac{1}{10}\langle 8,6 \rangle = \langle \frac{8}{10}, \frac{6}{10} \rangle = \vec{u}$

This is a unit vector with length 1

Check: $\|\vec{u}\| = \sqrt{\left(\frac{8}{10}\right)^2 + \left(\frac{6}{10}\right)^2} = \sqrt{\frac{64}{100} + \frac{36}{100}}$

$= \sqrt{\frac{100}{100}} = \frac{10}{10} = 1$ ✓

WeBWork Set: Complex Polar Form

#5) $\theta_1 = 255°$ $\theta_2 = 60°$

Come back to this Monday

argument $\theta_1 + \theta_2 = 315°$

$r_1 = 3$, $r_2 = 3$, $r_1 r_2 = 9$
