11/24/2021 Lesson 20: Trig Equations

Start by considering some basic trig equations:

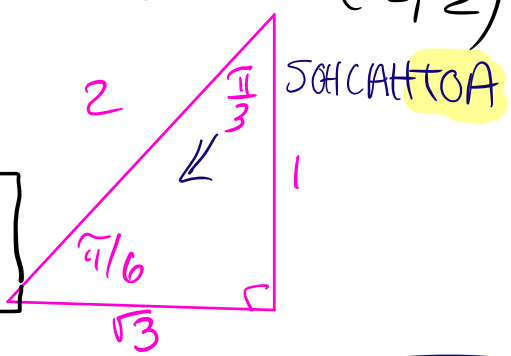
$\sin x = c$, $\cos x = c$, $\tan x = c$

Ex Solve for x: $\tan(x) = \sqrt{3}$

$\tan^{-1}(\sqrt{3}) = x = \frac{\pi}{3}$

Be careful this only gives one solution coming from the restricted domain $(-\frac{\pi}{2}, \frac{\pi}{2})$

Recall the special $\triangle$



Solution:

$$x = \frac{\pi}{3} + n \cdot \pi \quad n = 0, \pm 1, \pm 2, \pm 3, \ldots$$

Means

$n=0$ $\frac{\pi}{3} + 0\pi = \frac{\pi}{3}$

$n=1$ $\frac{\pi}{3} + \pi = \frac{4\pi}{3}$

$n=-1$ $\frac{\pi}{3} - \pi = -\frac{2\pi}{3}$

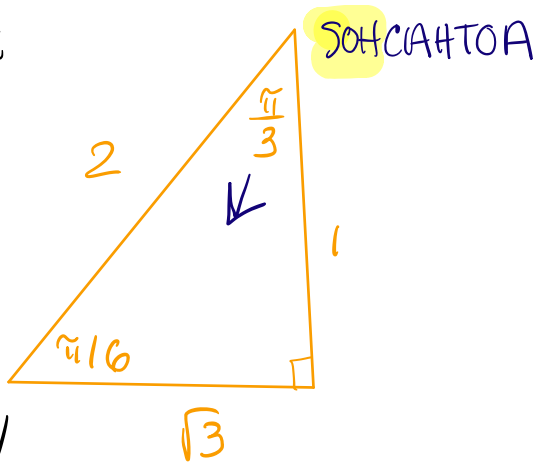
$\vdots$

Question:

$\cos x = \frac{\sqrt{3}}{2}$

Ex $\sin x = \frac{\sqrt{3}}{2}$ Solve for x

SOHCAHTOA

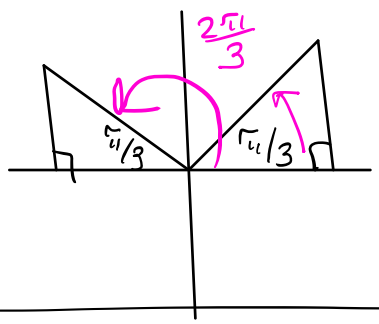


$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = x$

$\frac{\pi}{3}$ works but

There are ∞-many more!

π S	A I
$\sin x$ $\csc x > 0$	all positive
$\tan x$ $\cot x > 0$ III	$\cos x$ $\sec x > 0$ C IV

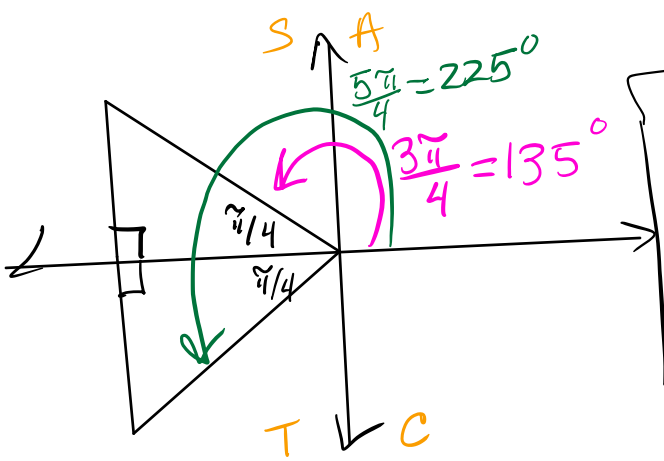


$x = \frac{2\pi}{3} + 2\pi \cdot n$ $n = 0, \pm1, \pm2, \pm3, \ldots$

$x = \frac{\pi}{3} + 2\pi \cdot n$

Solve for x: $\cos x = -\frac{\sqrt{2}}{2}$

$\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = x = 135^\circ = \frac{3\pi}{4}$



$x = \frac{3\pi}{4} + 2\pi \cdot n$ $n = 0, \pm1, \pm2, \ldots$

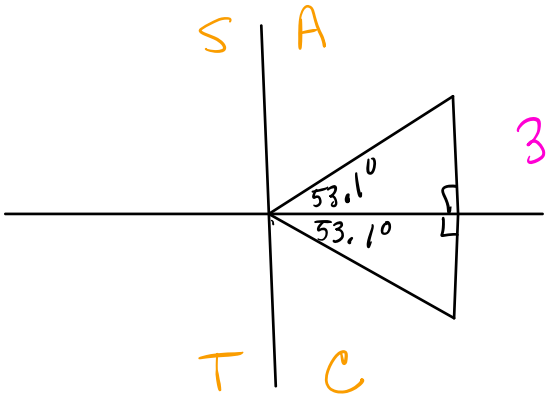
$x = \frac{5\pi}{4} + 2\pi \cdot n$

Ex Solve for x

$\cos x = 0.6$

$\cos^{-1}(0.6) = x \approx 51.1^\circ$ ← our ref ∠

↑ use a calculator



$360^\circ - 53.1^\circ = 306.9^\circ$

$x = 53.1^\circ + 360^\circ \cdot n$

$x = 306.9^\circ + 360^\circ \cdot n$

$n = 0, \pm 1, \pm 2, \ldots$

Ex Solve for x: $2\sin x - 1 = 0$

$+1 \quad +1$

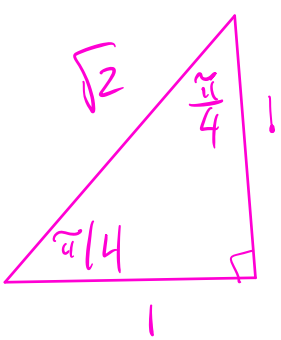
$2\sin x = 1$

$\frac{2\sin x}{2} = \frac{1}{2}$

$\sin x = \frac{1}{2}$

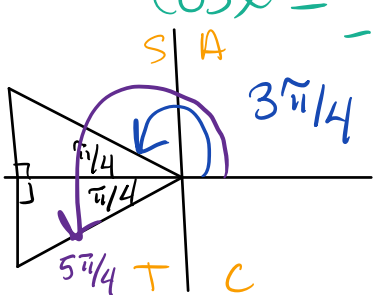
(solve from here) you finish!

Ex Solve for x: $\sec x = -\sqrt{2}$



so

$\cos x = \frac{1}{-\sqrt{2}}$



Reciprocal Identities

$\sin x \leftrightarrow \csc x$

$\cos x \leftrightarrow \sec x$

$\tan x \leftrightarrow \cot x$

$$X = \frac{3\pi}{4} + 2\pi \cdot n \qquad n = 0, \pm 1, \pm 2, \ldots$$

$$X = \frac{5\pi}{4} + 2\pi \cdot n$$

Ex: $\tan^2(x) + 2\tan(x) + 1 = 0$

Use a "dummy variable"

let $u = \tan(x)$

Rewrite the equation using the dummy variable

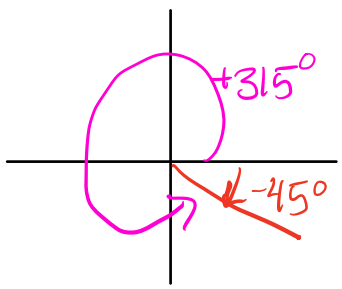
$$u^2 + 2u + 1 = 0$$

This is just a quadratic equation!

$$(u+1)(u+1) = 0$$

$u + 1 = 0$ $\quad$ $u + 1 = 0$

$u = -1$ $\quad$ $u = -1$



Don't forget $u = \tan(x)$

$\tan(x) = -1$

$\tan^{-1}(-1) = X = -45^\circ = 315^\circ$

$$X = 315^\circ + 180^\circ \cdot n \qquad n = 0, \pm 1, \pm 2, \ldots$$

WeBWork: COVID Project

#1: a) Recall $P(t) = C \cdot b^t$ ← t = time

C = initial value = 684, b = base

$P(t) = 684 \cdot b^t$ ← exponential model.

↑ solve for b!

Given: $t = 76$ $P(t) = 16{,}003$

$684 \cdot b^{76} = 16{,}003$

exponential equation!

Solve for b.

---

b) Evaluate $P(39) = \#$ → into WeBWork

---

c) Start 684 → double = $2 \cdot 684$

$2 \cdot 684 = 684 \cdot b^t$ ↖ solve for t!

$2 = b^t$ ↖ solve

d) $86985 = 684 \cdot b^{t}$ solving for t