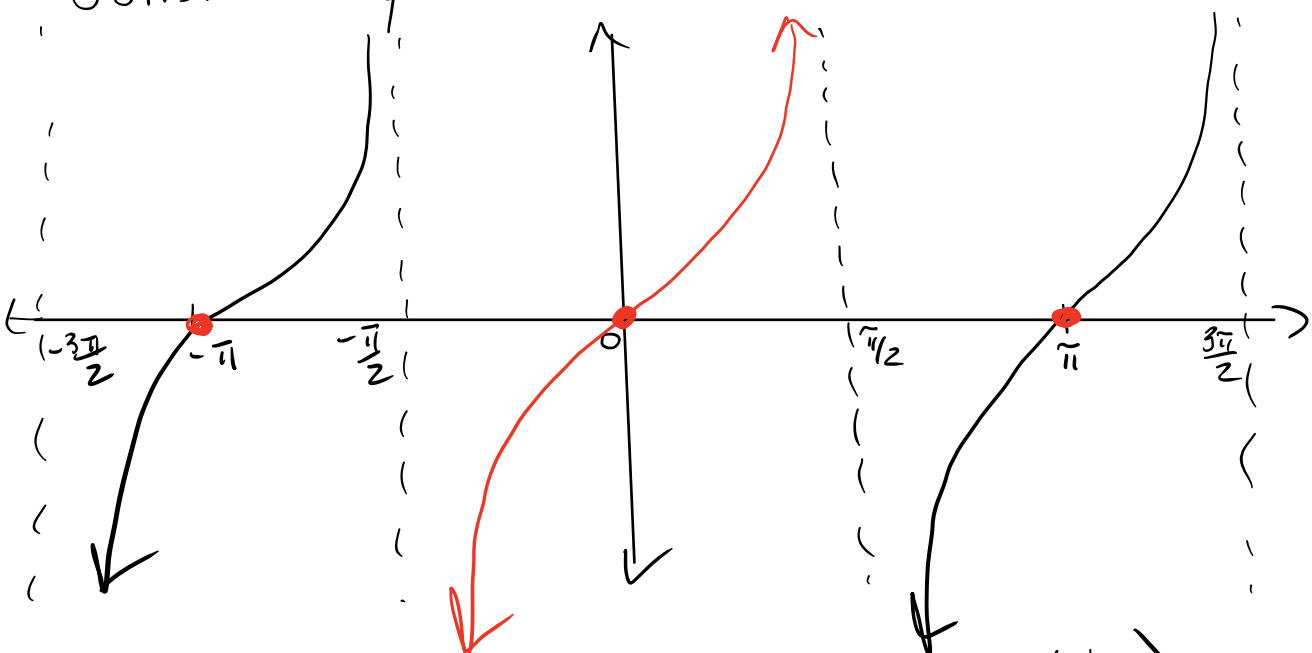
Monday	Wednesday
11/22 (19)	11/24 (20)
11/29 (21)	12/1 (22)
12/6 (23)	12/8 (24)
12/13 (25)	12/15 Review

Final Exam 12/20

Lesson 19: Inverse Trig Functions

Consider $y = \tan x$

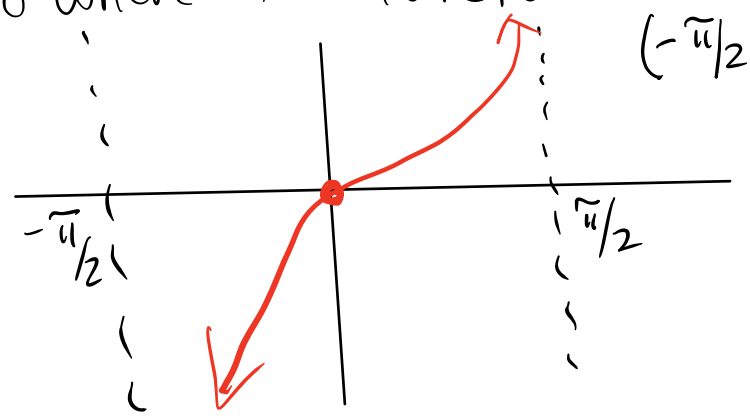


$y = \tan x$
output # ↑ input ✗

$x = \tan^{-1}(y)$
output ✗ ↑ input #

Question: $\tan^{-1}(0) = ?$ Trouble is that $\tan(\pi)$, $\tan(0)$, $\tan(-\pi)$ all give zero value. There are many answers.

Trick: Restrict the domain of $y = \tan x$ to where the function is one-to-one $(-\pi/2, \pi/2)$



Now if I ask the same question: $\tan^{-1}(0) = 0$ only 1 solution

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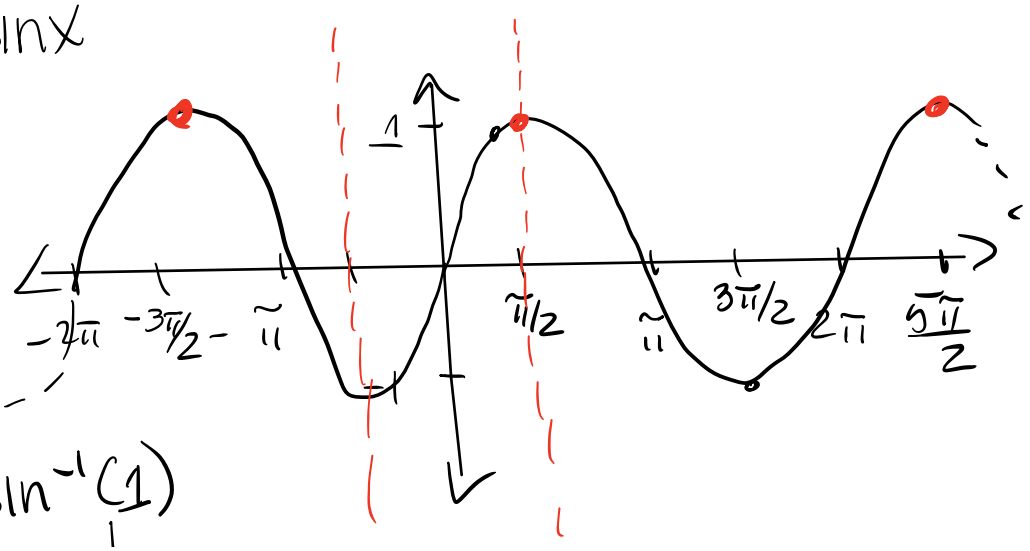
$y = \sin x$

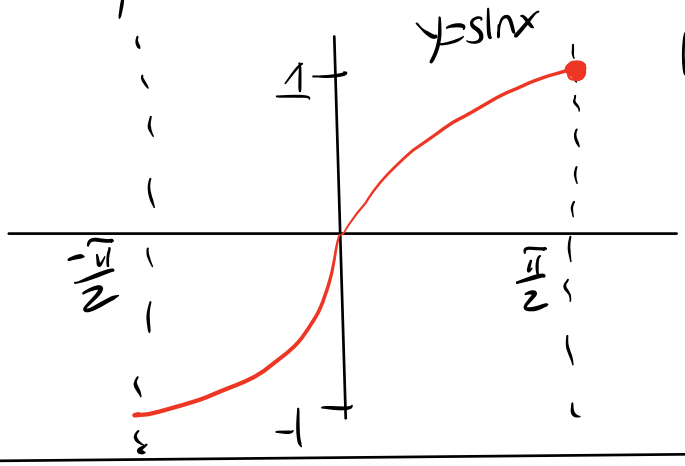


$x = \sin^{-1}(1)$

$\pi/2, \frac{5\pi}{2}, -\frac{3\pi}{2}, \ldots$

infinitely many choices!

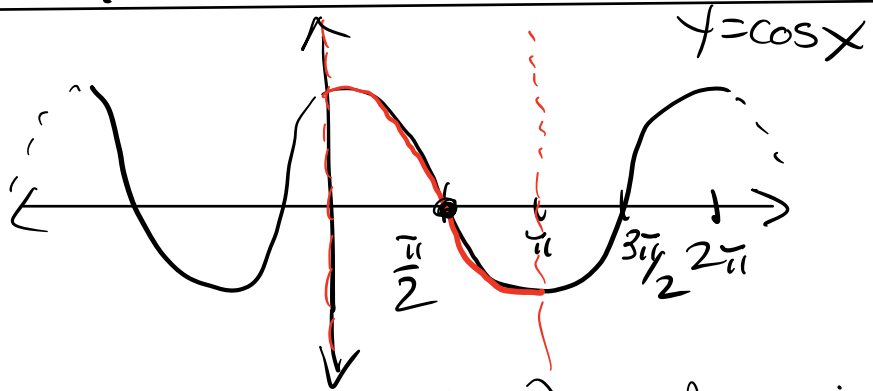
Like before, we restrict the domain to $[-\frac{\pi}{2}, \frac{\pi}{2}]$



Ask same question

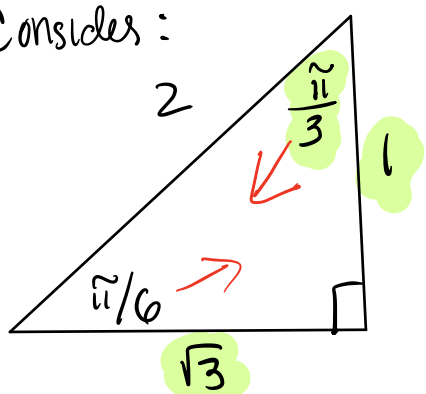
$\sin^{-1}(1) = \frac{\pi}{2}$

Consider



For cosine we restrict the domain to $[0, \pi]$ $\cos^{-1}(0) = \frac{\pi}{2}$

Consider:



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$\cos^{-1}(\frac{\sqrt{3}}{2}) = \frac{\pi}{6}$ ✗

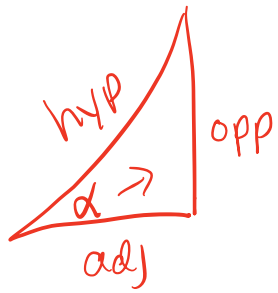
$\sin^{-1}(\frac{1}{2}) = \pi/6$ ✓

$\tan^{-1}(\frac{\sqrt{3}}{1}) = \frac{\pi}{3}$

$\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$ ✗

$\tan\frac{\pi}{3} = \frac{\sqrt{3}}{1}$ ✓

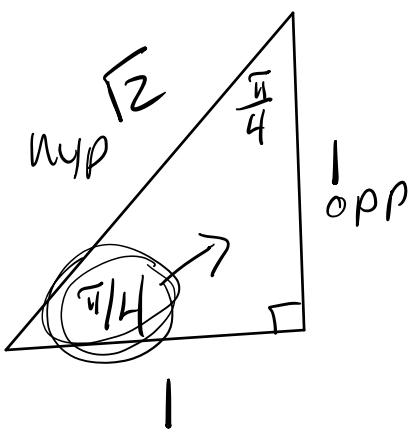
$\cos\alpha = \frac{adj}{hyp}$ $\frac{\sqrt{3}}{2}$



α $\frac{\pi}{6}$

Work backwards $\cos^{-1}\left(\frac{adj}{hyp}\right) = \alpha$

$\frac{adj}{hyp} = \frac{\sqrt{3}}{2}$ $\alpha = \frac{\pi}{6}$



$\tan^{-1}(1) = \frac{\pi}{4}$

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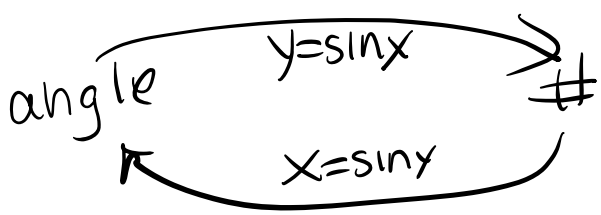
$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$

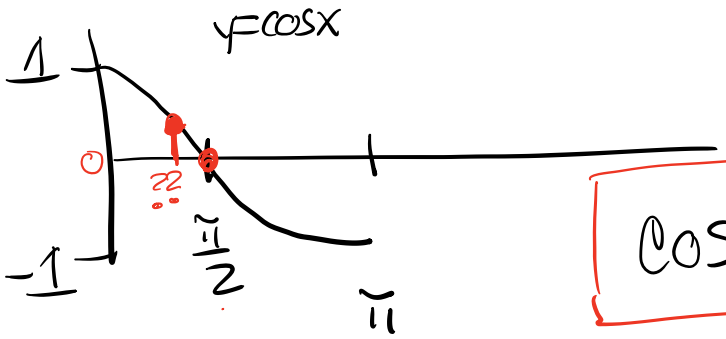
$y = \sin(x)$

output is a # — input is an $\angle$

$x = \sin^{-1}(y)$

an angle — a number





$\cos^{-1}(0) = \frac{\pi}{2}$

∠

$\cos^{-1}(.27895) = ??$ $\approx 73.8°$

not from a special Δ or easy to read off graph → Then use calculator

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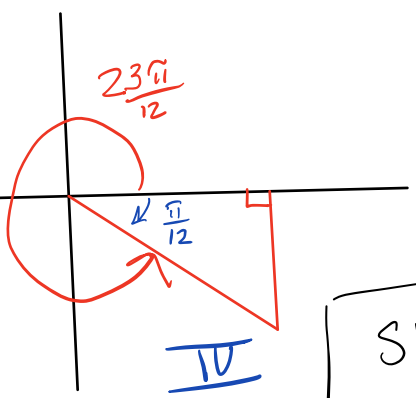
WeBWork set: Trig: Sum + Difference

Problem #1: $\sin\left(\frac{23\pi}{12}\right)$

$\frac{23\pi}{12}$

S | A
T | C
IV



$2\pi = \frac{24\pi}{12}$

Use $\frac{\pi}{12}$ as the ref ∠

$\sin\left(\frac{\pi}{12}\right)$

$\frac{\pi}{12}$ as a sum or difference of ∠'s? $\frac{\pi}{3}, \frac{\pi}{6}, \frac{\pi}{4}$

$\frac{4\pi}{12}, \frac{2\pi}{12}, \frac{3\pi}{12}$

What works?

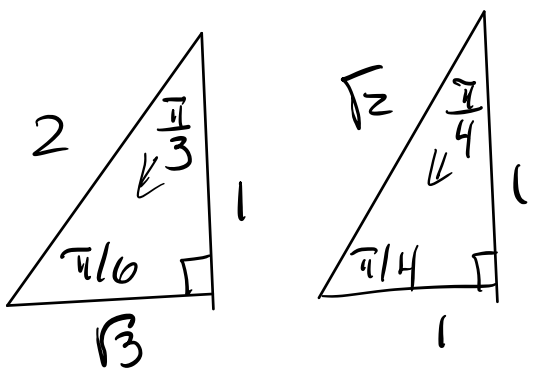
$$\frac{4\pi}{12} - \frac{3\pi}{12} = \frac{\pi}{12}$$

$$\frac{\pi}{3} \quad \frac{\pi}{4}$$

$$60° - 45° = 15°$$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \sin\frac{\pi}{3}\cos\frac{\pi}{4} - \cos\frac{\pi}{3}\sin\frac{\pi}{4}$



$= \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}} - \frac{1}{2}\cdot\frac{1}{\sqrt{2}}$

$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \boxed{\frac{(\sqrt{3}-1)}{2\sqrt{2}}}$

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WeBWork: Trig Inverse Functions.

#3. Keep in mind:

$\arccos(x) = \cos^{-1}(x)$

$\arcsin(x) = \sin^{-1}(x)$

$\arctan(x) = \tan^{-1}(x)$

domain: $\cos^{-1}(x)$: $[0, \pi]$

$\sin^{-1}(x)$: $[-\pi/2, \pi/2]$

$\tan^{-1}(x)$: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

a) $\cos^{-1}(.37)$

a) rad 1.1979

b) deg 68.2844

WeBWorK Set: Trig: Double +1/2 ∡ Formula

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#1) Given α in Q1 and $\tan(\alpha) = \frac{7}{8}$

a) $\sin(2\alpha)$

b) $\cos(2\alpha)$

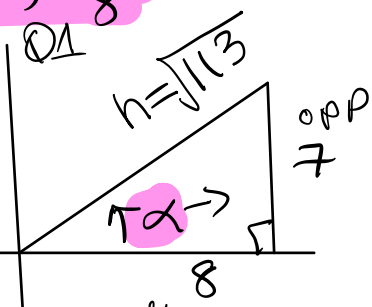
c) $\tan(2\alpha)$

$h^2 = 8^2 + 7^2$

$h = \sqrt{8^2+7^2}$

$= \sqrt{64+49}$

$= \sqrt{113}$



$\sin\alpha = \frac{7}{\sqrt{113}}$

$\cos\alpha = \frac{8}{\sqrt{113}}$

$\tan\alpha = \frac{7}{8}$

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a) $\sin(2\alpha) = 2\sin\alpha\cos\alpha$

$= 2\frac{7}{\sqrt{113}} \cdot \frac{8}{\sqrt{113}} =$

b) $\cos(2\alpha) = 1 - 2\sin^2\alpha$

$= 1 - 2\cdot\left(\frac{7}{\sqrt{113}}\right)^2 =$

you finish!

c) $\tan(2\alpha) = \frac{2\tan\alpha}{1-\tan^2(\alpha)} = \frac{2\left(\frac{7}{8}\right)}{1-\left(\frac{7}{8}\right)^2} =$