11/17/2021

# WebWork Problem Set: Trig: Unit Circle

## Problem 2:

a) $\cos\left(\frac{\pi}{6}\right)$

$= +\frac{\sqrt{3}}{2}$

SOHCAHTOA

| II | I |
|---|---|
| sinx csc x + S | A + |
| tanx cot + T | C cosx sec x + |
| III | IV |

hypotenuse, opposite, adjacent, $\alpha$

$180° = \pi$, $0° = 2\pi$, $360°$

$\pi/6$, $\pi/4 = 45°$, $\pi/3 = 60°$, $\pi/6 = 30°$

$\sqrt{2}$, $\sqrt{3}$, 1, 2

Special $\Delta$'s

b) $\cos\left(\frac{-5\pi}{6}\right)$

$= -\frac{\sqrt{3}}{2}$

$\frac{\pi}{6}$ is the "reference"

$\frac{6\pi}{6} = \pi$

| S | A |
|---|---|
| T | C |

III

$\frac{-5\pi}{6}$

SOHCAHTOA

Cosine = adj / hyp

c) $\cos\left(\frac{11\pi}{6}\right)$

$= +\frac{\sqrt{3}}{2}$

$\frac{11\pi}{6}$, $2\pi = \frac{12\pi}{6}$

IV

Given $f(x) = a\sin(bx+c)$ or $f(x) = a\cos(bx+c)$

$|a|$ = amplitude

$\left|\frac{2\pi}{b}\right|$ = period

$-\frac{c}{b}$ = phase shift

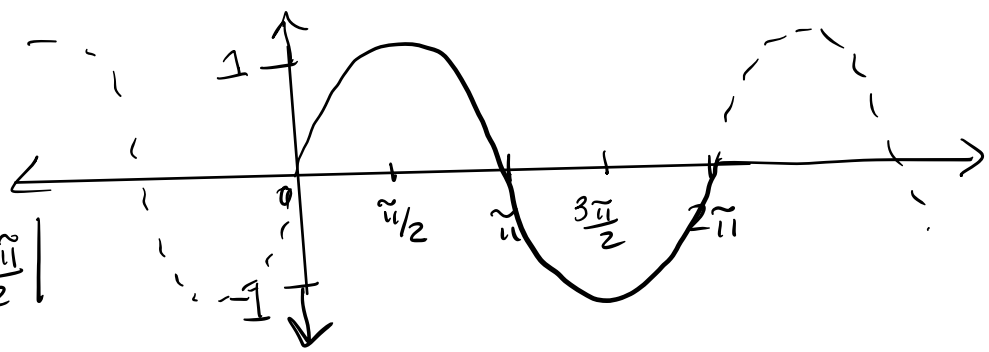
WeBWork Problem Set: Trig-Graphing Comprehensive

Problem 3: $f(x) = \sin(2x - \pi)$ ($b$, $c$)

$y = \sin x$

$|a| = |1| = 1$

period $= \left|\frac{2\pi}{b}\right| = \left|\frac{2\pi}{2}\right|$



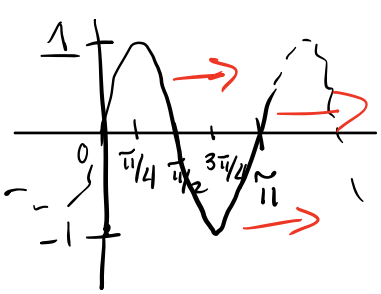

$\pi$ = period

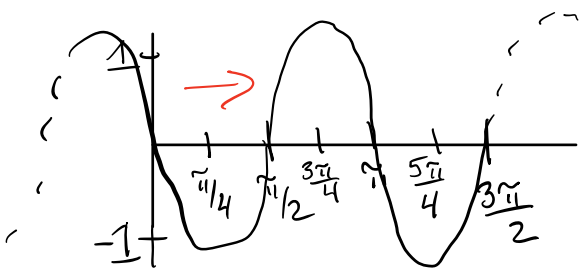
Compress one wave

phase shift: $\frac{-(-\pi)}{2} = +\frac{\pi}{2}$ units right!

First compress: → Next shift

$y = \sin(2x)$ $\qquad$ $y = \sin(2x - \pi)$



Session 18: Sum/Difference, Double/Half angle formulas

Ex Find the exact values of the trig functions

a) $\cos\left(\frac{\pi}{12}\right)$ b) $\tan\left(\frac{5\pi}{12}\right)$

Want to use the $\theta$'s for which we already know the values of the trig functions.

Goal write $\frac{\pi}{12}$ ($15^\circ$) as a sum or difference of $\frac{\pi}{6}$, $\frac{\pi}{3}$, $\frac{\pi}{4}$

$\frac{\pi}{6} = 30^\circ$, $\frac{\pi}{3} = 60^\circ$, $\frac{\pi}{4} = 45^\circ$

$60^\circ - 45^\circ = 15^\circ$

or

$45^\circ - 30^\circ = 15^\circ$

$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\cos\left(\frac{\pi}{12}\right) = \cos\left(\underbrace{\frac{\pi}{3}}_{\alpha} - \underbrace{\frac{\pi}{4}}_{\beta}\right) =$

$= \cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4}$

$= \frac{1}{2}\cdot\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}}$

$= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} = \boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$

b) $\tan\left(\frac{5\pi}{12}\right)$

Goal: Write $\frac{5\pi}{12}$ as a sum or difference of $\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{4}$

$\frac{2\pi}{12}, \frac{4\pi}{12}, \frac{3\pi}{12}$

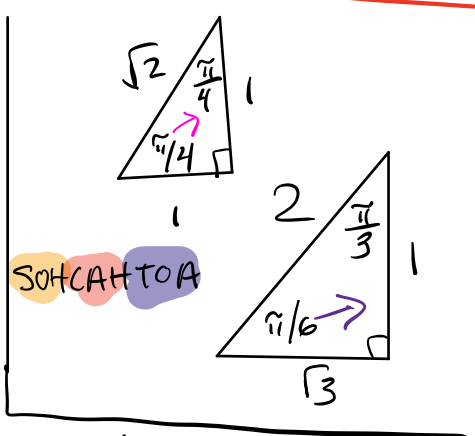
$$\frac{5\pi}{12} = \frac{2\pi}{12} + \frac{3\pi}{12}$$

$$\tan\left(\frac{5\pi}{12}\right) = \tan\left(\underbrace{\frac{\pi}{6}}_{\alpha} + \underbrace{\frac{\pi}{4}}_{\beta}\right)$$

$$= \frac{\tan\left(\frac{\pi}{6}\right) + \tan\left(\frac{\pi}{4}\right)}{1 - \tan\left(\frac{\pi}{6}\right)\tan\left(\frac{\pi}{4}\right)}$$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$= \frac{\frac{1}{\sqrt{3}} + 1}{1 - \frac{1}{\sqrt{3}} \cdot 1} = \frac{\frac{1}{\sqrt{3}} + 1}{1 - \frac{1}{\sqrt{3}}}$$




To rationalize multiply all by $\sqrt{3}$

$$\frac{\left(\frac{1}{\sqrt{3}} + 1\right) \cdot \sqrt{3}}{\left(1 - \frac{1}{\sqrt{3}}\right) \cdot \sqrt{3}} = \frac{1 + \sqrt{3}}{\sqrt{3} - 1}$$

go further
Multiply top + bottom by $\sqrt{3}+1$

$$\frac{(1+\sqrt{3})}{(\sqrt{3}-1)} = \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)} = \frac{\sqrt{3}+1+\sqrt{3}}{3-1} = \frac{4+2\sqrt{3}}{2} = 2+\sqrt{3}$$

Ex Find the trig function using the 1/2 $\alpha$ formula.

$\sin\left(\frac{\pi}{8}\right)$

$= \pm\sqrt{\frac{1-\cos\frac{\pi}{4}}{2}}$

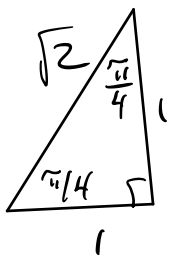
$= \pm\sqrt{\frac{1-\frac{1}{\sqrt{2}}}{2}}$

$= \pm\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}} = \pm\sqrt{\frac{1}{2}-\frac{\sqrt{2}}{4}}$

Notice: $\frac{\pi}{8} = \left(\frac{\pi}{4}\right)/2$ ($\alpha$)

$\sin\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1-\cos\alpha}{2}}$

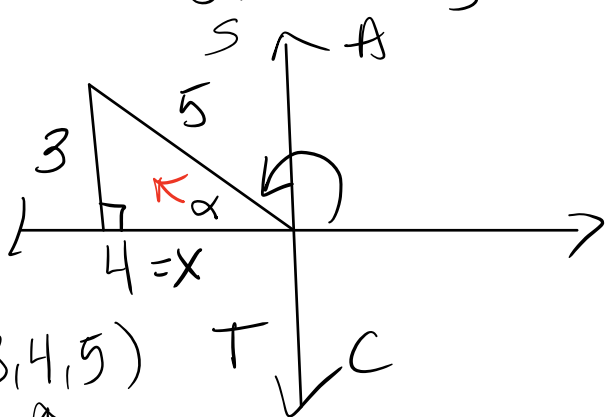
SOHCAHTOA




Ex Find the trig functions of $2\alpha$ when $\alpha$ has the property

$\sin\alpha = \frac{3}{5}$ and $\alpha$ is in quadrant II

SOHCAHTOA




(3,4,5) ↑ pythagorean triplet

By the pythagorean theorem:

$x^2 + 3^2 = 5^2$

$x^2 = 5^2 - 3^2 = 25 - 9$

$= 16$

$x = 4$

$\sin 2\alpha = 2\sin\alpha\cos\alpha \rightarrow 2\cdot\frac{3}{5}\cdot\frac{4}{5} = \frac{24}{25}$

$\cos 2\alpha = 2\cos^2\alpha - 1 \rightarrow 2\cdot\left(\frac{4}{5}\right)^2 - 1 = \frac{32}{25} - 1 = \frac{7}{25}$

$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} \rightarrow \frac{2\cdot\frac{3}{4}}{1-\left(\frac{3}{4}\right)^2} = \frac{-6/4}{1-9/16} = \frac{-3/2}{7/16} = \frac{-3}{7/8}$

$= \frac{-24}{7}$