11/1/2021

WeBWork Problem Set: Rational Inequalities

#3) $\frac{5x-9}{x^2+x-42} > 0$

Find root, need to find $\frac{5x-9}{x^2+x-42} = 0$

Recall $\frac{a}{b} = 0 \Rightarrow$ if $a = 0$

$5x - 9 = 0$

$+9 \quad +9$

$5x = 9 \Rightarrow \boxed{x = 9/5}$

To find vertical asymptotes, set denominator $= 0$

$x^2 + x - 42 = 0$

$(x+7)(x-6) = 0$

$\boxed{x = -7}$ $\boxed{x = 6}$ check against graph

(Note: sometimes there is a hole, not an asymptote! This happens when a factor in the denominator gets cancelled by a factor in the numerator.)

Aside:

$$\frac{x^2+1}{(x+7)(x-6)}$$

here $x=-7$ and $x=6$ are both vertical asymptotes

vs.

$$\frac{\cancel{(x+7)}(x^2+1)}{\cancel{(x+7)}(x-6)}$$

here $x=-7$ is a HOLE and $x=6$ is a vertical asymptote

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where is our function non-zero?

$$(-\infty,-7)\cup(-7,9/5)\cup(9/5,6)\cup(6,+\infty)$$

vertical asymptote

root

vertical asymptote

Test whether the function is "+" or "−" on each interval.

$$\frac{5x-9}{x^2+x-42} > 0 \quad \text{on} \quad \boxed{(-7, 9/5) \cup (6, +\infty)}$$

positive

WeBWork:

Exponential functions graphs

#8) $f(x) = A \cdot B^x$ $(0,-6)$ $(2,-3)$

$f(0) = -6$

$A \cdot B^0 = -6$

Recall: $\boxed{B^0 = 1}$

$A \cdot 1 = -6$

$A = -6$

$f(2) = -3$

$f(2) = \underset{-6}{A} \cdot B^2 = -3$

$\frac{-6B^2}{-6} = \frac{-3}{-6}$

$B^2 = \frac{1}{2}$

$B = \sqrt{\frac{1}{2}}$

$f(x) = A \cdot B^x$

$f(x) = (-6) \cdot \left(\frac{1}{\sqrt{2}}\right)^x$

Lesson 15: Applications of Exponential functions

Note: The exponential function is $y = c \cdot b^x$

Ex: Let $f(x) = c \cdot b^x$. Determine the constant c and base b under the given conditions.

$f(2) = 160$ and $f(7) = 5$

$f(2) = c \cdot b^2 = 160$ $\qquad f(7) = c \cdot b^7 = 5$

First solve for b. Trick: Form a quotient!

$$\frac{f(2)}{f(7)} = \frac{\cancel{c} \cdot b^2}{\cancel{c} \cdot b^7} = \frac{160}{5}$$

Recall:

$$\frac{b^m}{b^n} = b^{m-n}$$

$$b^{-5} = \frac{1}{b^5}$$

$$\frac{\cancel{b^2}}{b^{\cancel{7}\,5}} = \frac{160}{5}$$

$$b^{-5} = \boxed{\frac{1}{b^5} = \frac{160}{5}}$$

take the reciprocal of both sides

$$b^5 = \frac{5}{160} \Rightarrow$$

solve for b, take 5th root of both sides

$$b = \sqrt[5]{\frac{5}{160}} = \left(\frac{5}{160}\right)^{1/5}$$

$\boxed{b = \frac{1}{2}}$ So now let's figure out c!

Now $f(7) = c \cdot \left(\frac{1}{2}\right)^7 = 5$ (based on given info)

now solve for c

$$\boxed{c = \frac{5}{(1/2)^7} = 640}$$

Ex The population size of a country was 12.7 million in the year 2000 and 14.3 in the year 2010

$t=0$ (2000)

"$t=10$" (2010)

a) Assuming an exponential growth for the population size, find the formula for the population depending on the year t.

b) What will the population size be in 2015?

c) When will the population reach 18?

$f(t) = c \cdot b^t$

$t=0 \rightarrow 12.7m$

$t=10 \rightarrow 14.3m$

Need to find the exponential model
$\Rightarrow$ need to figure out c and b

$t=0$ $\quad f(0) = c \cdot \underset{\uparrow \atop 1}{b^0} = 12.7$

$\boxed{c = 12.7}$

Now let's find $\underline{\underline{b}}$.

$t=10$ $\quad f(10) = 12.7 \cdot b^{10} = 14.3$

$\dfrac{\cancel{12.7} \cdot b^{10}}{\cancel{12.7}} = \dfrac{14.3}{12.7}$ Solve for b.

$b^{10} = \dfrac{14.3}{12.7}$

$b = \left(\dfrac{14.3}{12.7}\right)^{1/10} \approx \boxed{1.012}$

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a) $f(t) = 12.7 \cdot (1.012)^t$ model $x =$ time

b) let $t = \overset{2015}{15}$ $f(t) = ?$

$f(15) = 12.7 (1.012)^{15} \approx \boxed{15.19}$ million

c) When will $f(t) = 18m$? $t = ?$

$\cancel{12.7} (1.012)^t = 18$ solve for t

$\cancel{12.7}$ $\quad$ 12.7

$$1.012^t = \left(18/12.7\right)$$

take the log of both sides

$$\log(1.012)^t = \log\left(18/12.7\right)$$

use property of log s

$$\frac{t \log\cancel{(1.012)}}{\cancel{\log(1.012)}} = \frac{\log\left(18/12.7\right)}{\log(1.012)}$$

$$t = \frac{\log\left(18/12.7\right)}{\log(1.012)} \approx \boxed{29.24 \text{ years}}$$

$t = 0 \Rightarrow 2000$

$t = 29.24 \longrightarrow \boxed{2029}$