10/6/2021

Recall from last lecture: The Remainder Theorem:

The remainder when dividing $f(x)$ by $\boxed{(x-c)}$ is $\boxed{r = f(c).}$

In particular: The Factor Theorem

$\boxed{f(c) = 0}$ if and only if $\boxed{x-c}$ is a factor of $f(x)$

i.e. c is a root!

Ex Show that 5 is a root of $f(x) = x^3 - 19x - 30$ and use this to factor $f(x)$ completely.

Check: $f(5) = 5^3 - 19x - 30$

$= 125 - 19 \cdot 5 - 30 = 0$

$= 125 - 95 - 30 = 0$

$\Rightarrow$ $x-5$ is a factor

use long division

$x - 5 \overline{\smash{)}\, x^3 - 19x' - 30}$

notice we are missing the x^2 term

so insert "$0x^2$"

$$
\begin{array}{r}
\boxed{x^2+5x+6} \\
x-5\,\overline{)\,x^3+0x^2-19x-30} \\
-(x^3-5x^2) \\ \hline
5x^2-19x \\
-(5x^2-25x) \\ \hline
6x-30 \\
-(6x-30) \\ \hline
0
\end{array}
$$

Recall that we are trying to factor

$$\underline{x^3-19x-30} = (x-5)(x^2+5x+6)$$

prime

try factoring further

$$= \underline{(x-5)(x+2)(x+3)}$$

all primes!

$x-c$ are factors

$$= (x-5)(x-(-2))(x-(-3))$$

$5, -2$ and -3 are roots of $f(x)$

Graphs of polynomial functions

$$f(x) = a_n x^n + a_1 x^{n-1} + \ldots + a_1 x + a_0$$

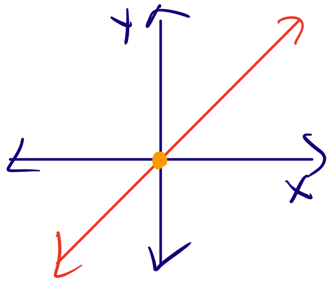
a's are real (or complex values)

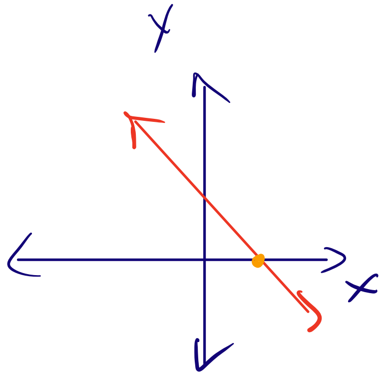
n's are positive integers

Degree 1

Lines

Ex $f(x)=x$

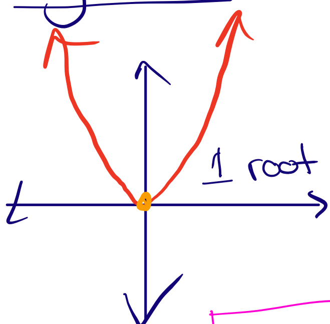


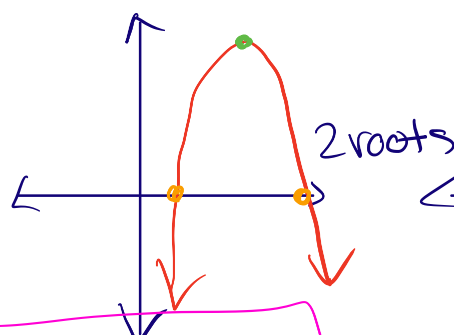


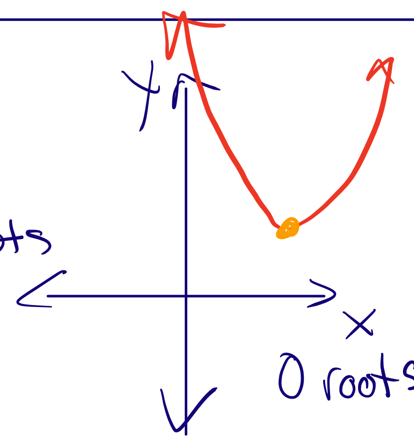
Observe: $\boxed{\text{1 root}}$

0 extreme points

Degree 2:



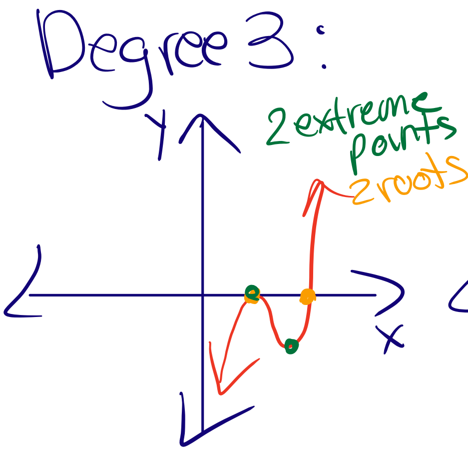


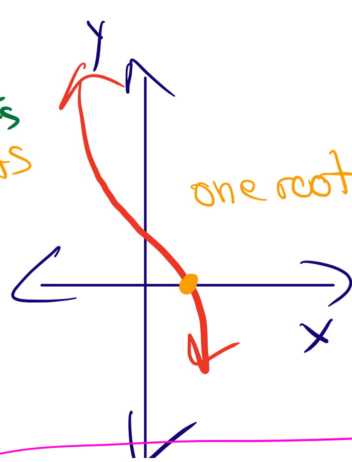


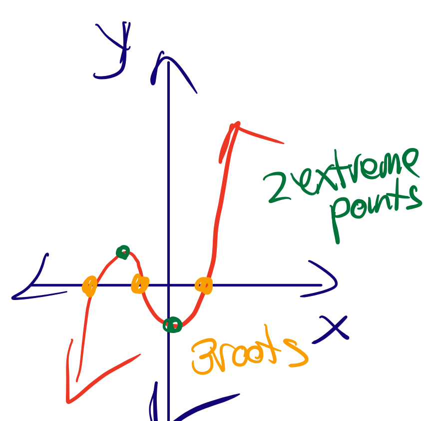
Observe: $\boxed{\text{0, 1 or 2 roots}}$

1 extreme point

Degree 3:







Observe : $\boxed{1, 2 \text{ or } 3 \text{ roots}}$

0, or 2 extreme points

Polynomials

Degree 1 ex $y = 5x + 2$

Degree 2 ex $y = 16x^2 - 24x + 2$

Degree 3 ex $y = x^3 - 12x^2 - 14$

Degree 4 ex $y = -21x^4 - x^3 + 2x^2 + 1$

Desmos Activity :

Slide 14

Given roots : -4, 1 and 2

by the factor Theorem

we know we have factors

$$(x - (-4))(x - 1)(x - 2)$$

Reason : $(x - (-4))(x - 1)(x - 2) = 0$

Solve $x - (-4) = 0$ | $x - 1 = 0$ | $x - 2 = 0$

$x + 4 = 0$ | $+1 \; +1$ | $+2 \; +2$

| $x = 1$ | $x = 2$

Roots: $-4, 1, 2$ $\qquad$ $x = -4$ $\quad -4 \; -4$

Slide 1.6

Given roots: $-1, 1 + 3$

Corresponding factors $(x-(-1))(x-1)(x-3)$

Last Observation:

When we draw the graphs of polynomial functions there are no holes, jumps, breaks or sharp corners.

discontinuities

Ex