10/20/2021

Polynomial and Rational Inequalities:

Ex Solve for x:

a) $2x+5 \geq 4x-11$

$\quad -4x \qquad -4x$

$-2x+5 \geq -11$

$\quad -5 \qquad -5$

$-2x \geq -16$

$\frac{-2x}{-2} \quad \frac{-16}{-2}$

$\boxed{x \leq 8}$

Recall: When multiplying or dividing both sides of an inequality by a negative # we must flip the direction of the inequality.

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b) $-2x-1 \leq 3x+4 < 4x-20$

① ②

① $-2x-1 \leq 3x+4$

$\quad +1 \qquad +1$

$-2x \leq 3x+5$

$-3x \qquad -3x$

$-5x \leq 5$

$\frac{-5x}{-5} \quad \frac{5}{-5}$

$\boxed{x \geq -1}$

② $3x+4 < 4x-20$

$-4x \qquad -4x$

$-x+4 < -20$

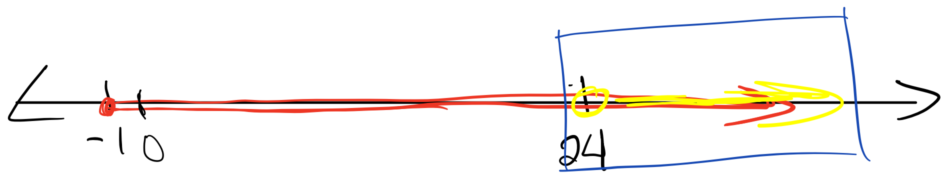
$\quad -4 \qquad -4$

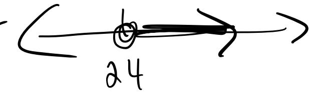
$-x < -24$

$\frac{-x}{-1} \quad \frac{-24}{-1}$

$\boxed{x > 24}$

The solution to the original problem are all the values which are true for both inequalities.



Solution: $x > 24$ or $(24, +\infty)$ or 

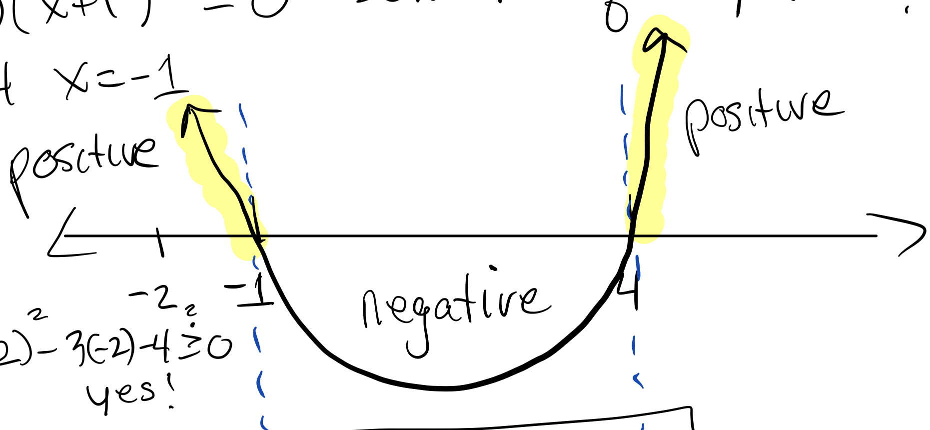
Ex Solve:

a) $x^2 - 3x - 4 \geq 0$

Where is the quadratic function positive or zero? Where is the graph above or hitting the x-axis?

$(x-4)(x+1) = 0$ Solve the equality first!

$x = 4 \quad x = -1$



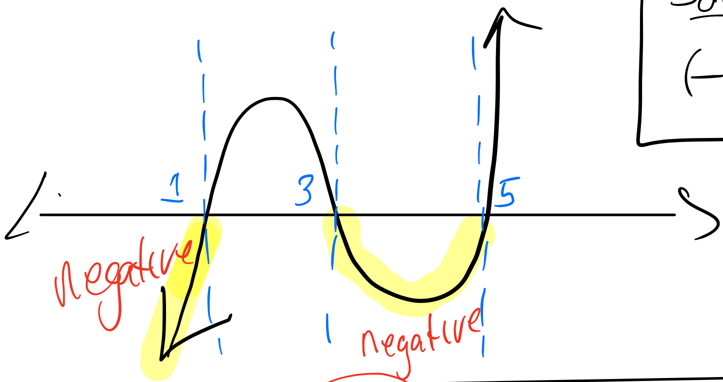
$(-2)^2 - 3(-2) - 4 \stackrel{?}{\geq} 0$

yes!

$$\boxed{\text{Solution: } (-\infty, -1] \cup [4, +\infty)}$$

b) $x^3 - 9x^2 + 23x - 15 \leq 0$

Tells us: Where is the graph of the function negative or zero? i.e. Where is the graph below (or hitting) the x-axis?



Solution:

$(-\infty, 1] \cup [3, 5]$

c) $x^4 - x^2 > 5(x^3 - x)$

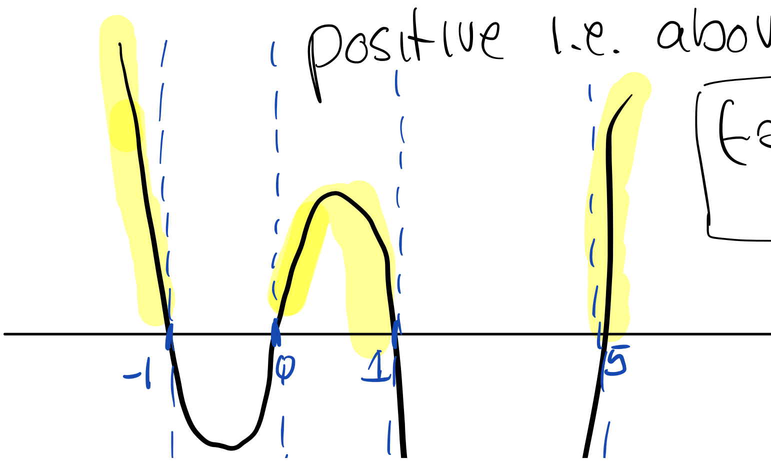
Sometimes the inequality is presented without a "zero" on one side. No worries! We can just rewrite it!

$$x^4 - x^2 > 5x^3 - 5x$$
$$-5x^3 + 5x \qquad -5x^3 + 5x$$

$$x^4 - 5x^3 - x^2 + 5x > 0$$

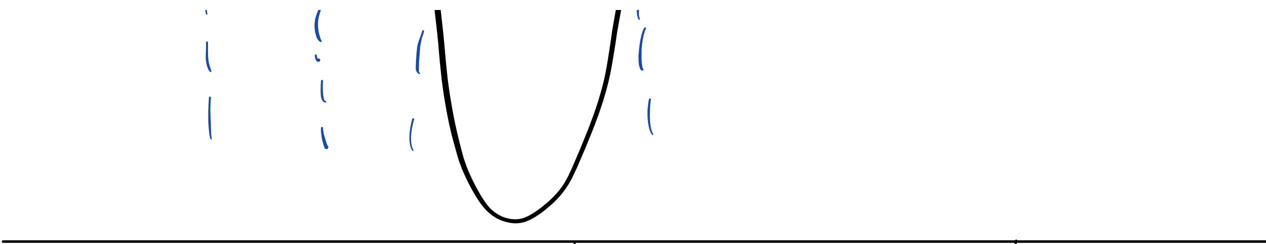
Now we can solve.

Look for where the graph is positive i.e. above the x-axis!



$(-\infty, -1) \cup (0, 1) \cup (5, +\infty)$

Solution:



Solve

Looking for where the graph is positive or zero, i.e. where it is above or hitting the x-axis.

a) $\dfrac{x^2-5x+6}{x^2-5x} \geq 0$

factor

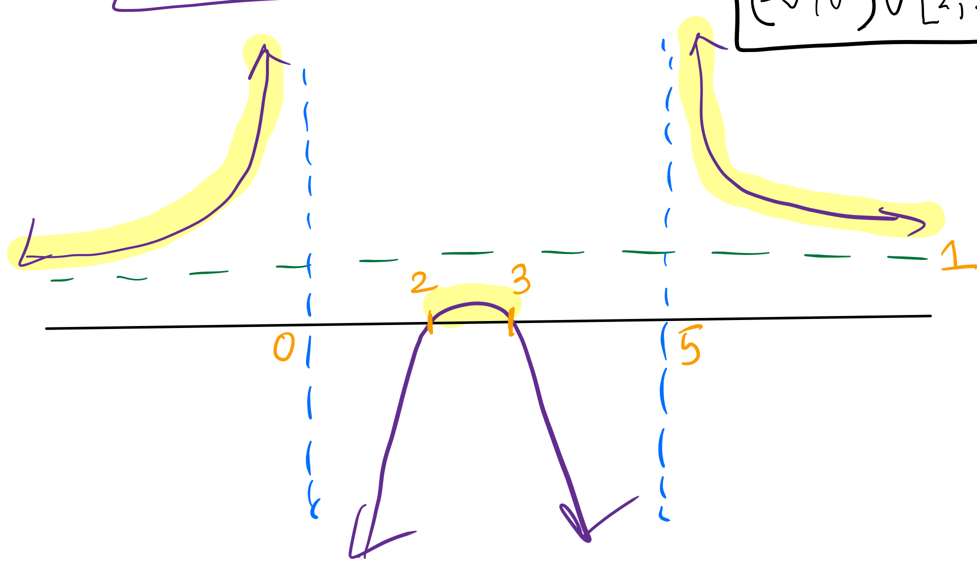
$\dfrac{(x-2)(x-3)}{x(x-5)} \geq 0$

horizontal asymptote $y=\frac{1}{1}=1$

Know that there are x-intercepts at $x=2$ and $x=3$

vertical asymptotes at $x=0$ $x=5$

Solution:
$(-\infty,0)\cup[2,3]\cup(5,\infty)$



c) $\frac{4}{x+5} < \frac{3}{x-3}$ Rewrite first!

$-\frac{3}{x-3} \quad -\frac{3}{x-3}$

$\underbrace{\frac{4}{x+5} - \frac{3}{x-3}}$ < 0 is $\boxed{\frac{x-27}{(x+5)(x-3)} < 0}$ where is the graph below the x-axis?

Add these expressions - rewrite over a common denominator

$\frac{4}{x+5} - \frac{3}{x-3}$ Common denominator: $(x+5)(x-3)$

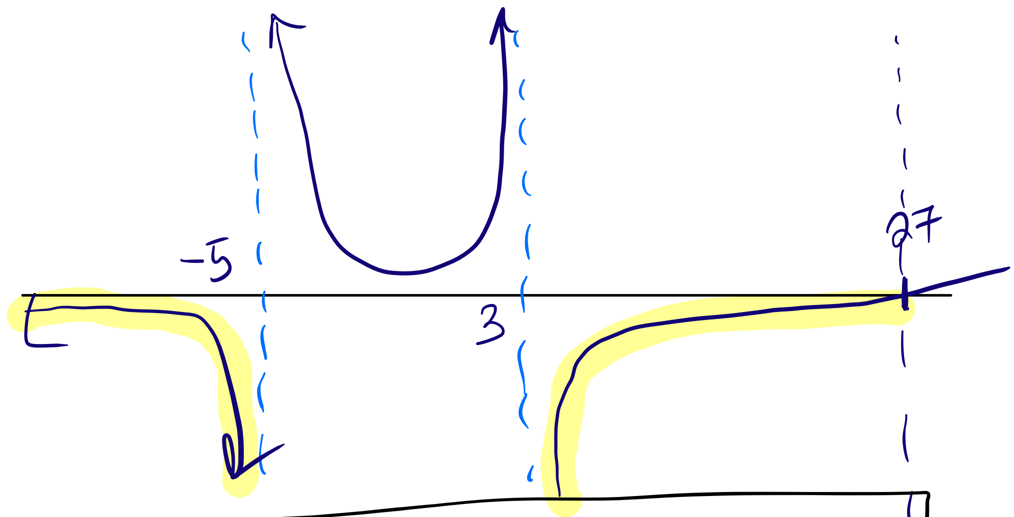
Rewrite each part:

$\frac{4}{x+5} \times (x-3) = \frac{4(x-3)}{(x+5)(x-3)}$ $\quad \times (x-3)$

$\frac{3}{(x-3)} \times (x+5) = \frac{3(x+5)}{(x+5)(x-3)}$ $\quad \times (x+5)$

$\frac{4(x-3)}{(x-3)(x+5)} - \frac{3(x+5)}{(x+5)(x-3)} = \frac{4(x-3) - 3(x+5)}{(x+5)(x-3)}$

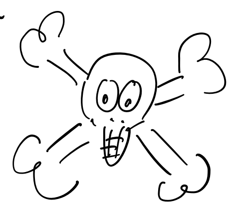
$= \frac{4x - 12 - 3x - 15}{(x+5)(x-3)} = \boxed{\frac{x-27}{(x+5)(x-3}}$



$$\boxed{\text{Solution: } (-\infty,-5)\cup(3,27)}$$

EX $g(x)=\sqrt{x^3-5x^2+6x}$

Find the domain ....

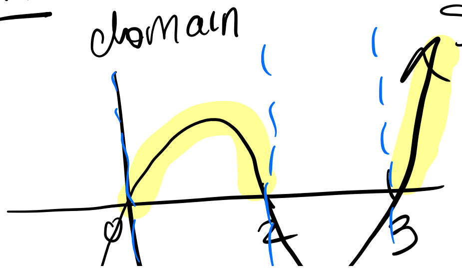


Can't have a negative under square root

So.... want is

The solution to this is the domain → $\underbrace{x^3-5x^2+6x}_{\text{Solve like before!}} \geq 0$

Where is graph above x-axis (or hitting)?



domain: $[0,2]\cup[3,+\infty)$