9/27/2021

<u>Def</u>: A <u>monomial</u> is a # a variable or a product of #'s & variables. A polynomial is a sum or difference of <u>monomials</u>

Ex $2x^2$, $3xy^5$, $-128s^{15}r^3$ are monomials.

Ex $2x^2 - 24y^5x^6 + 36$ is a polynomial

<u>Catch</u>: polynomials cannot have rational or negative exponents.

Ex $2x^{1/2} - y^{-27}$.
is <u>not</u> a polynomial.

<u>Def</u>: A polynomial is a function of the form
$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$
for some $a_n, \dots, a_0$ real (or complex) numbers. The domain of f is all real #'s ($\mathbb{R}$).

The numbers $a_n, \ldots, a_0$ are called "coefficients"
For each # k, a_k is the coefficient for x^k.
The number a_n is the "leading coefficient"
and n is the degree of the polynomial.

Ex $16x^5 + 2x^4 + 20x^3 - 5x^2 - 1$

* $16x^5$ The leading term
* 16 The leading coefficient
* 5 The degree

<u>Def</u>: A <u>rational function</u> $f(x)$ is a fraction of two polynomials

$$f(x) = \frac{g(x)}{h(x)} \text{ where } h(x) \neq 0$$

Ex $\dfrac{-3x^2+7x-5}{2x^3+4x^2+3x+1}$, $\dfrac{1}{x}$

Long Division:

Ex Divide the following fractions via long division

a) $\frac{3571}{11}$

$$11\overline{)3571}$$

quotient!

divisor

dividend

$$\begin{array}{r}
324 \\
11\overline{)3571} \\
-(33) \\
\hline
27 \\
-(22) \\
\hline
51 \\
-(44) \\
\hline
7
\end{array}$$

7 ← remainder

$$\frac{3571}{11} = 324 + \frac{7}{11}$$

remainder

divisor

quotient

Ex b) $\frac{x^3+5x^2+4x+2}{x+3}$

Sidenote: $x+3$ is called an "irreducible" or "prime" polynomial

$$
\begin{array}{r}
x^2+2x-2 \\
x+3\ \overline{)\,x^3+5x^2+4x+2} \\
-(x^3+3x^2) \qquad\qquad \\
\hline
2x^2+4x \qquad \\
-(2x^2+6x) \qquad \\
\hline
-2x+2 \\
-(-2x-6) \\
\hline
8
\end{array}
$$

remainder

$$\frac{x^3+5x^2+4x+2}{x+3} = x^2+2x-2 + \frac{8}{x+3}$$

remainder

quotient

divisor

Dividing by $(x-c)$

The remainder when dividing a polynomial $f(x)$ by $x-c$ is $f(c)$.

In our example

$x+3 = x-(-3)$

$\Rightarrow c=-3$

So $\boxed{r = 8 \stackrel{?}{=} f(-3)}$

$f(-3) = (-3)^3+5(-3)^2$
$+4(-3)+2 =$
$= -27+45-12+2 = 8$

Ex $f(x) = x^3+5x^2+4x+2$

Ex $\dfrac{x^3 + 2x + 1}{x - 1}$ no x^2 term

to keep the division from being messy, can add in a "$0x^2$" term

$x-1 \overline{)\, x^3 + 0x^2 + 2x + 1}$

Desmos Activity ~~Slide~~ #

$x+9 \overline{)\, x^2 + 11x + 18}$

Graph 2 Things: 1) $x^2 + 11x + 18$ (done orange)

2) $(x+9)\boxed{}$ find quotient first

$$
\begin{array}{r}
x + 2 \\
x+9 \overline{)\, x^2 + 11x + 18} \\
-(x^2 + 9x) \\
\hline
2x + 18 \\
-(2x + 18) \\
\hline
0
\end{array}
$$

$\boxed{x+2}$

Slide: 3 Desmos Activity

$$7x^3-4x^2+6x-9$$

$$3x+8\ \overline{)\,21x^4+44x^3-14x^2+21x-72}$$

$$-(21x^4+56x^3)$$

$$-12x^3-14x^2$$

$$-(-12x^3-32x^2)$$

$$+18x^2+21x$$

$$-(18x^2+48x)$$

$$-27x-72$$

$$-(-27x-72$$

$$0$$