WeBWork HW Functions Operations
Problem 6 c) & d)

$f(x) = 2x - 7 \qquad g(x) = \frac{1}{x^2 + 5x - 24}$

c) Simplify: $(g \circ f)(x)$

2nd 1st

$= g(f(x)) = g(2x-7) = \frac{1}{(2x-7)^2 + 5(2x-7) - 24}$

$= \frac{1}{(2x-7)(2x-7) + 10x - 35 - 24}$

$= \frac{1}{4x^2 - 14x - 14x + 49 + 10x - 35 - 24}$

$= \frac{1}{4x^2 - 18x - 10}$

$(g \circ f)(x) = \boxed{\frac{1}{4x^2 - 18x - 10}}$

To find the domain: exclude any values which give us a zero denominator

set $4x^2 - 18x - 10 = 0$

Choose one
3 methods: 1) Use quadratic formula
2) Use graph to find x-intercepts = roots

3) Factor & solve

From 2) Desmos
When $x = -\frac{1}{2}$ and 5 denominator $= 0$

so domain: $(-\infty, -1/2) \cup (-1/2, 5) \cup (5, +\infty)$

WeBWork
Symmetries #2:

(x_1, y_1)
$f(x)$ passes through $(4,7)$

1) Even $f(-x) = f(x)$
$f(4) = f(-4)$ → also passes through the point
$(-4, 7)$

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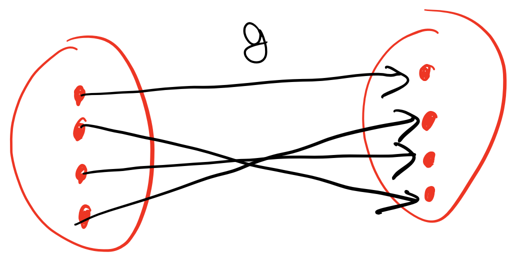
2) Odd passes through what other 2 points?
$(4,7)$

odd $f(-x) = -f(x)$
$f(-x) = f(-4)$ & $\boxed{(4, -7)}$

$\boxed{(-4, -7)}$

One-to-one functions

Def: A function f is called one-to-one (1-1) or injective, if two different inputs $x_1 \neq x_2$ always have different outputs $f(x_1) \neq f(x_2)$.

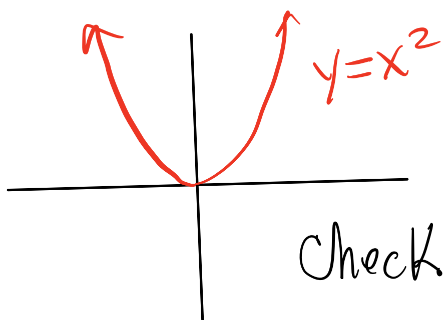


Is g 1-1?

yes!

different inputs $\Rightarrow$ different outputs

Is $y=x^2$ 1-1?



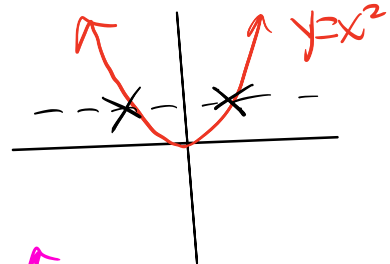
Check:

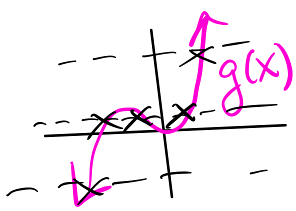
$x_1 = 1 \quad x_2 = -1$

$$\Rightarrow f(1) = 1^2 = f(-1)$$

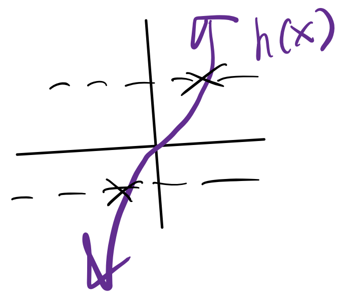
different inputs $\Rightarrow$ same output

If you only have the graph of a function an easy way to check if a function is 1-1 is to draw a horizontal line. If a horizontal line intersects the graph in more than 1 spot then the function is <u>NOT</u> 1-1!





1-1? No!



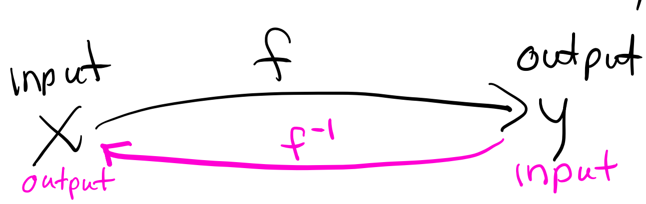
1-1? yes!

One-to-one functions have inverse!

The roles of input and output are reversed!

<u>Def</u>: Let f be a function with domain D_f and range R_f, and assume f is 1-1, then the inverse of f is f^{-1} so that

$f(x)=y$ means precisely that $f^{-1}(y)=x$.



Consider again $y = x^2$

Given the output $y = 4$
can you tell me what the input value was?

We don't only $\underline{\text{one}}$ value!

$x_1 = +2$ $\underline{\text{or}}$ $x_2 = -2$

This is why going in reverse breaks down.

Ex Find the inverse of the following functions

a) $g(x) = \sqrt{x+2}$

b) $h(x) = \dfrac{1}{x+4}$

c) $j(x) = \dfrac{x+1}{x+2}$ ← Challenge! $j^{-1}(x) = \dfrac{1-2x}{x-1} = \dfrac{2x-1}{1-x}$

Strategy:

1) Swap x and y

2) Solve for y

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a) $g(x) = \sqrt{x+2}$ i.e. $y = \sqrt{x+2}$

swap

1) $x = \sqrt{y+2}$

2) solve for y

$$x^2 = \left(\sqrt{y+2}\right)^2$$

$$x^2 = y+2$$
$$-2 \qquad\quad -2$$

$$y = x^2 - 2$$ Notation $\boxed{g^{-1}(x) = x^2 - 2}$

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2) $h(x) = \frac{1}{x+4}$ i.e. $y = \frac{1}{x+4}$

1) Swap x & y

$$x = \frac{1}{y+4}$$ cross multiply

2) solve for y

$$x(y+4) = 1$$

$$xy + 4x = 1$$
$$\quad -4x \quad -4x$$

$$\frac{xy}{x} = \frac{1-4x}{x}$$

$$y = \frac{1-4x}{x} = \frac{1}{x} - 4 \qquad \boxed{h^{-1}(x) = \frac{1}{x} - 4}$$

How do we know if the inverse is correct?

$$\begin{tikzcd} X \arrow[r, bend left, "f"] & Y \arrow[l, bend left, "f^{-1}"] \end{tikzcd}$$

f and f^{-1}
"undo"
eachother
so we get back
to our starting point!

$$\begin{tikzcd} x \arrow[r, bend left, "f^{-1}"] & y \arrow[l, bend left, "f"] \end{tikzcd}$$

Check $f(x) = \sqrt{x+2}$ $\quad f^{-1}(x) = x^2 - 2$

$f^{-1}(f(x)) = x$ and $f(f^{-1}(x)) = x$

$$f^{-1}(\sqrt{x+2}) = (\sqrt{x+2})^2 - 2$$

$$= x + \cancel{2} - \cancel{2}$$

$$= x$$

$$f(x^2-2) = \sqrt{(x^2-2)+2}$$

$$= \sqrt{x^2 - \cancel{2} + \cancel{2}}$$

$$= \sqrt{x^2} = x$$