9/20/2021

Operations on functions:

We can do arithmetic with functions to build <u>new</u> functions:

1) $(f+g)(x) = f(x) + g(x)$
2) $(f-g)(x) = f(x) - g(x)$
3) $(f \cdot g)(x) = f(x) \cdot g(x)$
4) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

The domain of must have values that are in <u>both</u> f and g

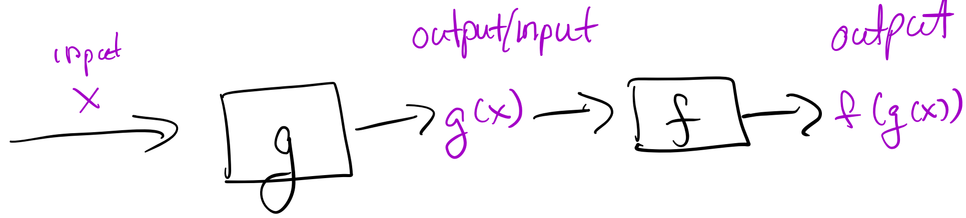
Additionally for the domain of

$$\frac{f}{g} = \{ x \mid x \in D_f \cap D_g ,\ g(x) \neq 0 \}$$

intersection

"is in" "domain of f" "domain of g"

5) One step further: $(f \circ g)(x)$
$= f(g(x))$ "$\circ$" compostion



Ex Let $f(x) = x^2+5x$ and $g(x) = 7x-3$

Find the following functions and state their domain

a) $(f+g)(x)$, b) $(f-g)(x)$, c) $(f\cdot g)(x)$, d) $\left(\frac{f}{g}\right)(x)$

a) $(f+g)(x) = f(x) + g(x)$

$= (x^2+5x)+(7x-3) = \boxed{x^2+12x-3}$

b) $(f-g)(x) = f(x) - g(x)$

$(x^2+5x)-(7x-3) = x^2+5x-7x+3$

$= \boxed{x^2-2x+3}$

c) $(f\cdot g)(x) = f(x)\cdot g(x)$

$= (x^2+5x)(7x-3) = 7x^3-3x^2+35x^2-15x$

$= \boxed{7x^3+32x^2-15x}$

d) $\left(\frac{f}{g}\right)(x) = \frac{x^2+5x}{7x-3} = \frac{f(x)}{g(x)}$

This is an example of a "rational function"

What about the domains?

domain of $f = \mathbb{R}$ domain of $g = \mathbb{R}$

all real #'s

for a) → c) domains $= \mathbb{R}$

d) All values so that $g(x) \neq 0$ $\quad \dfrac{f(x)}{g(x)}$

Find where $g(x)=0$ and exclude that value from the domain.

$g(x) = 7x-3 = 0$

$7x-3=0 \Rightarrow x = 3/7$

domain of $\left(\frac{f}{g}\right)(x) = \{ x \mid x \neq 3/7 \}$

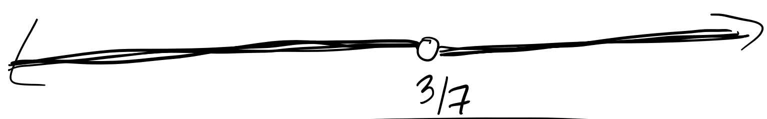
"set builder notation"

$(-\infty, 3/7) \cup (3/7, +\infty)$ in interval notation

(WeBWork asks for this)

yet another way to say this:

$\mathbb{R} - \{3/7\}$



Ex Let $f(x) = \sqrt{x+2}$ $\quad g(x) = x^2 - 5x + 4$

Find $\left(\frac{g}{f}\right)(x)$ and its domain

$$\left(\frac{g}{f}\right) = \frac{g(x)}{f(x)} = \frac{x^2-5x+4}{\sqrt{x+2}}$$

domain of $g(x) = \mathbb{R}$ or $(-\infty, +\infty)$

domain of $f(x) \Rightarrow$ since there is a radical we must make sure

$$x+2 \geq 0$$
$$\quad -2 \quad -2$$
$$x \geq -2$$

$$[-2, +\infty)$$

domain $\Rightarrow$ values in both $\rightarrow \boxed{[-2, +\infty)}$

Remember we are looking for the domain of the quotient so must remove any value x so that $g(x) = 0$

$$\sqrt{x+2} = 0$$
$$x = -2$$

Final answer: domain of $\left(\frac{g}{f}\right)(x) = \frac{x^2-5x+4}{\sqrt{x+2}}$

$$= (-2, +\infty)$$

You try: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

Ex Let $f(x) = x^2+1$ $g(x) = x+3$. Find the following compositions

a) $(f \circ g)(3)$

b) $(g \circ f)(3)$

c) $(f \circ g)(x)$

d) $(g \circ f)(x)$

$$x \to \boxed{g} \to g(x) \to \boxed{f} \to f(g(x))$$

$f(x) = x^2+1$

$g(x) = x+3$

a) $(f \circ g)(3) = f(\boxed{g(3)})$

2nd 1st

$g(3) = 3+3 = 6$

$$3 \to \boxed{g} \to 6 \to \boxed{f} \to 37$$

$f(6) = 6^2+1 = 37$

$$\boxed{(f \circ g)(3) = 37}$$

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b) $(g \circ f)(3) = g(\underline{f(3)})$

$f(3) = 3^2+1 = 10$

$g(10) = 10+3 = 13$

$$3 \to \boxed{f} \to 10 \to \boxed{g} \to 13$$

$$\boxed{(g \circ f)(3) = 13}$$

$f(x) = x^2 + 1$

$f(\Uparrow) = \Uparrow^2 + 1$

c) $(\underset{\substack{\uparrow \\ 2^{nd}}}{f} \circ \underset{\substack{\uparrow \\ 1^{st}}}{g})(x) = f(g(x))$

$= f(x+3)$

$= (x+3)^2 + 1$

$= (x+3)(x+3) + 1$

$= x^2 + 6x + 9 + 1$

$= x^2 + 6x + 10$

$$\boxed{(f \circ g)(x) = x^2 + 6x + 10}$$

$$x \longrightarrow \boxed{g} \longrightarrow g(x) \longrightarrow \boxed{f} \longrightarrow f(g(x))$$

check that $g(x) \in$ domain of $f(x)$

$g(x) = x+3$ domain $\mathbb{R}$

$\longrightarrow$ range real values

$f(x) = x^2 + 1 \rightarrow$ domain $\mathbb{R}$

domain of $(f \circ g)(x) = \mathbb{R}$

$f(x) = x^2 + 1$

$g(x) = x + 3$

$g(\Uparrow) = \Uparrow + 3$

d) $(\underset{\substack{\uparrow \\ 2^{nd}}}{g} \circ \underset{\substack{\uparrow \\ 1^{st}}}{f})(x) = g(f(x))$

$= g(x^2 + 1) = (x^2 + 1) + 3$

$= x^2 + 4$

$(g \circ f)(x) = x^2+4$

$x \to \boxed{f} \to f(x) \to \boxed{g} \to g(f(x))$

make sure $f(x)$ in the domain of $g(x)$

domain $f(x) = \mathbb{R}$

$\to$ outputs real values

$g(x)$ accepts all real values

domain of $(g \circ f)(x) = \mathbb{R}$

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a) $(f \circ g \circ h)(x) = f(g(h(x)))$

3rd 2nd 1st

$x \to \boxed{h} \to h(x) \to \boxed{g} \to g(h(x)) \to \boxed{f} \to f(g(h(x)))$

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b) $(g \circ h \circ f)(x)$

3rd 2nd 1st

$= g(h(f(x)))$

$x \to \boxed{f} \to f(x) \to \boxed{h} \to h(f(x)) \to \boxed{g} \to g(h(f(x)))$