9/1/2021 Functions by formulas + graphs

For the given function f, calculate the outputs $f(2), f(-3), f(-1)$

a) $f(x)=\sqrt{x^2-3}$

b) $f(x)=\begin{cases} x^2-1 & \text{for } -1\le x\le 1 \quad (1) \\ x-1 & \text{for } 1< x\le 5 \quad (2) \end{cases}$

Recall function notation $f(x)=y$

rule

input (domain)

output (range, codomain)

a) $f(2)=\sqrt{2^2-3}=\sqrt{4-3}=1$

$f(-3)=\sqrt{6}$

$f(-1)=$ undefined $\rightarrow \sqrt{-2}$ complex #

b) $f(2)=2-1=1$ since $2\in(1,5]$ use rule (2)

$$f(x)=\begin{cases} x^2-1 & \text{for } -1\le x\le 1 \quad (1) \\ x-1 & \text{for } 1< x\le 5 \quad (2) \end{cases}$$

$f(-3)=$ f is undefined at -3

rule (1)

$f(-1)=1^2-1=1-1=0$

f is a "piecewise function"

What are the domain + range of f?

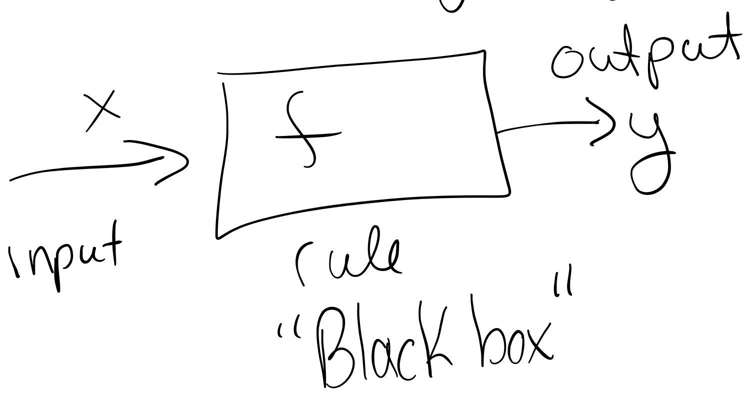
domain: $[-1,1] \cup (1,5]$
union

For the range we are looking for the outputs → y-values! Look at the graph to see which y-values appear

values from -1 to 4

range: $[-1,4]$

Ex Let f be the function given by $f(x)=x^2+2x-3$



$f(x)=x^2+2x-3$

Find

a) $f(\sqrt{5})$

b) $f(\sqrt{3}+1)$

c) $f(a)$

d) $f(a)+5$

e) $f(x+h)$

f) $f(x+h)-f(x)$

g) $\dfrac{f(x+h)-f(x)}{h}$

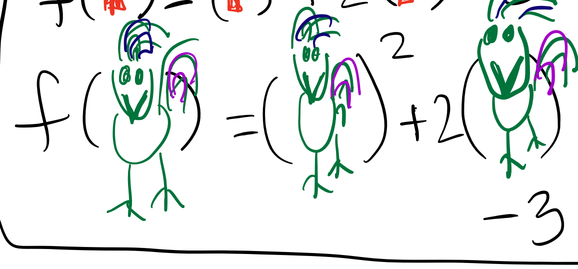
a) $f(\sqrt{5}) = (\sqrt{5})^2 + 2(\sqrt{5}) - 3$

$= 5 + \boxed{2\sqrt{5}} - 3$

$= \boxed{2 + 2\sqrt{5}}$

$f(x) = x^2 + 2x - 3$

$f(\triangle) = (\triangle)^2 + 2(\triangle) - 3$



b) $f(\sqrt{3}+1)$

$= (\sqrt{3}+1)^2 + 2(\sqrt{3}+1) - 3$

$= (\sqrt{3}+1)(\sqrt{3}+1) + 2(\sqrt{3}+1) - 3$

$= \sqrt{3}\cdot\sqrt{3} + 1\cdot\sqrt{3} + 1\cdot\sqrt{3} + \cancel{1} + 2\sqrt{3} + \cancel{2} - \cancel{3}$

$= \boxed{3 + 4\sqrt{3}}$

c) $f(a) = a^2 + 2a - 3$

d) $f(a) + 5 \neq f(a+5)$

here input is only a

here the input is $a+5$

$f(a) + 5 = (\underbrace{a^2 + 2a - 3}_{f(a)}) + 5 = a^2 + 2a - 3 + 5$

$= \boxed{a^2 + 2a + 2}$

e) $f(x+h) = (x+h)^2 + 2(x+h) - 3$

$= (x+h)(x+h) + 2(x+h) - 3$

$= x^2 + hx + hx + h^2 + 2x + 2h - 3$

$= \boxed{x^2 + 2xh + h^2 + 2x + 2h - 3} = f(x+h)$

$f(\ ) = (\ )^2 + 2(\ ) - 3$

$f(x) = x^2 + 2x - 3$

given!

f) $f(x+h) - f(x)$

$= \underbrace{x^2 + 2xh + h^2 + 2x + 2h - 3}_{f(x+h)} - \underbrace{(x^2 + 2x - 3)}_{f(x)}$

$= \cancel{x^2} + 2xh + h^2 + \cancel{2x} + 2h \cancel{-3} \cancel{-x^2} \cancel{-2x} \cancel{+3}$

$= \boxed{2xh + h^2 + 2h} = f(x+h) - f(x)$

g) $\dfrac{f(x+h) - f(x)}{h} = \dfrac{2xh + h^2 + 2h}{h} = \dfrac{\cancel{h}(2x + h + 2)}{\cancel{h}}$

$= 2x + \cancel{h}^{\,0} + 2 \Leftarrow$ final answer!

This is called "The Difference Quotient"

If we let $h \to 0$ result

$2x+2 \to$ This is called "The derivative" of f

$$f(x) = 4x^{2} + 2x^{1} - \cancel{3}x^{0}$$

$$= \underline{\underline{2x + 2}}$$