11/29/2021 More Trig Equations

Revisit:

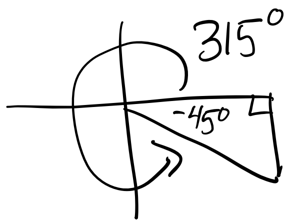
$\tan x = -1$ → part of $\tan^2 x + 2\tan x + 1 = 0$

$(\tan x + 1)(\tan x + 1) = 0$

$\Rightarrow \tan x = -1$

$\tan^{-1}(-1) = x$

get



Be careful $\tan x$ has a period of π not 2π!

So the solution:

$$\boxed{\begin{aligned} &x = 315^\circ + 180^\circ \cdot n \\ &n = 0, \pm 1, \pm 2, \ldots \\ &\text{or} \\ &x = \frac{7\pi}{4} + \pi \cdot n \end{aligned}}$$

WeBWork Set: Trig Equations

Problem #6: Give all solutions

$$2\cos x \sin x + \sqrt{3} \sin x = 0$$

We can factor this → factor out $\sin x$ from both terms. $\sin x(2\cos x + \sqrt{3}) = 0$

$\Rightarrow \sin x = 0$ $\quad\quad$ $2\cos x + \sqrt{3} = 0$

Recall "zero-product property"

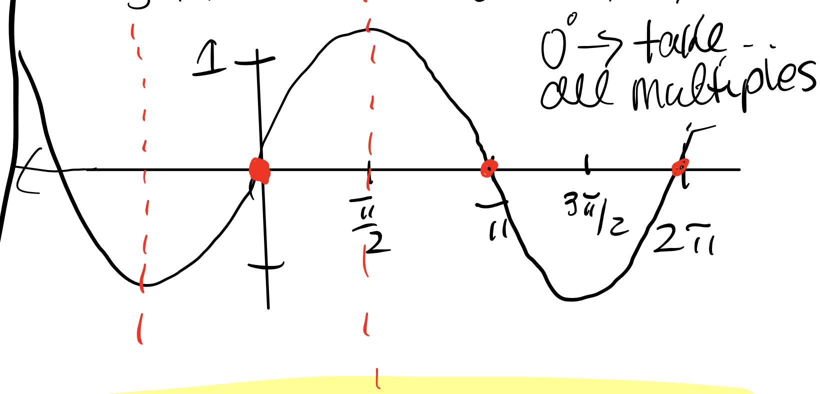
If $A \cdot B = 0$

$A = 0$ or $B = 0$

or both

$\sin x = 0 \rightarrow \sin^{-1}(0) = x$

$0° \rightarrow$ take all multiples



from $\sin x = 0 \Rightarrow 0 + n \cdot \pi \quad n = 0, \pm 1, \pm 2, \ldots$

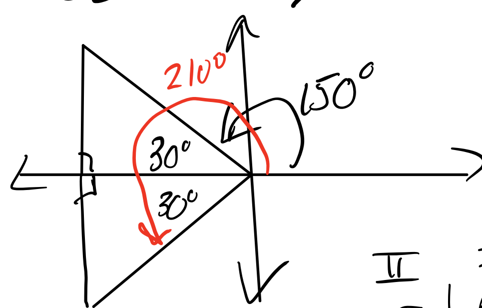
$2\cos x + \sqrt{3} = 0$

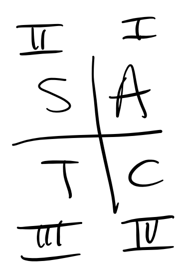
$\quad - \sqrt{3} \quad - \sqrt{3}$

$2\cos x = -\sqrt{3}$

$\cos x = -\dfrac{\sqrt{3}}{2}$

$\rightarrow \cos^{-1}\left(-\dfrac{\sqrt{3}}{2}\right) = 150°$





from $2\cos x + \sqrt{3} = 0$

$\Rightarrow \quad 150° + 2\pi \cdot n$

$\quad 210° + 2\pi \cdot n \quad n = 0, \pm 1, \pm 2, \ldots$

or

$$\boxed{\begin{array}{l} \dfrac{5\pi}{6} + 2\pi \cdot n \\ \dfrac{7\pi}{6} + 2\pi \cdot n \end{array}}$$

Complete Solution: $0 + \pi n$

$\dfrac{5\pi}{6} + 2\pi n \quad n = 0, \pm 1, \pm 2, \ldots$

$7\pi/6 + 2\pi n$

Problem #7: $\tan^2 x + \sqrt{3}\tan x = 0$

let $u = \tan x$

$$u^2 + \sqrt{3}\,u = 0$$

Factor out u: $\quad u(u+\sqrt{3}) = 0$

$\Rightarrow \quad u = 0$

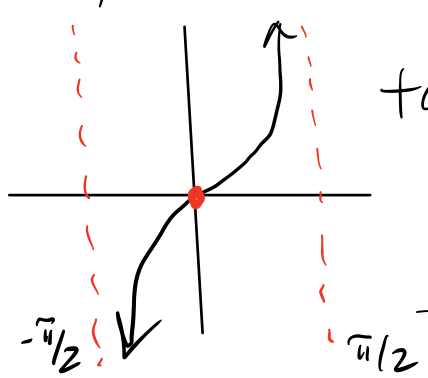
$\tan x = 0$

$\tan^{-1}(0) = x$

$\Rightarrow 0 + n\cdot\pi$

$n = 0, \pm 1, \pm 2, \ldots$

y=tanx



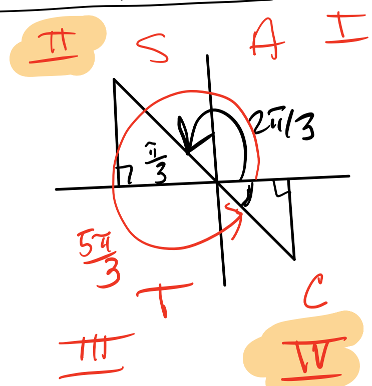
$u + \sqrt{3} = 0$

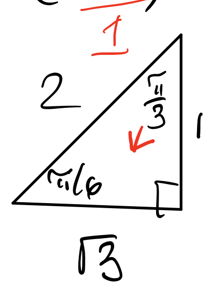
$\tan x + \sqrt{3} = 0$

$\quad - \sqrt{3} \quad - \sqrt{3}$

$\tan x = -\sqrt{3}$

$\tan^{-1}(-\sqrt{3}) = 0$





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$$X = \frac{2\pi}{3} + n\cdot\pi$$

$n = 0, \pm 1, \pm 2, \ldots$

Lesson #21: Complex #'s

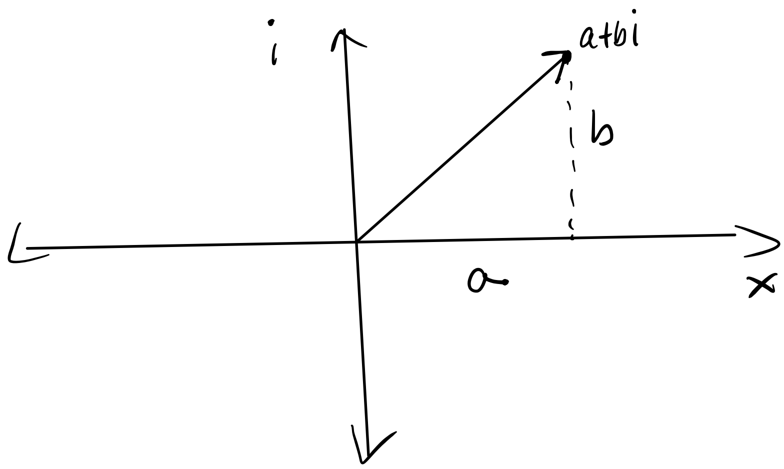
Recall: $\sqrt{-1} = i$ (imaginary)

$i^2 = -1$

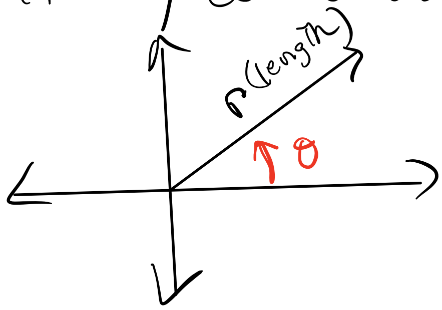
Complex # : Standard form:

$a + bi$ where $a, b \in \mathbb{R}$

(a,b in the set of real #'s)



Alternatively could use polar form



$re^{i\theta}$

Some basic operations:

a) $(2-3i) + (-6+4i)$ add real and imaginary parts

$= (2-6) + (-3+4)i$

$= \boxed{-4+i}$

b) $(3+5i)(-7+i)$ use FOIL

$= -21 + 3i - 35i + 5i^2$

$= -21 - 32i + 5(-1)$ $\quad (i^2 = -1)$

$= \boxed{-27 - 32i}$

c) $\dfrac{5+4i}{3+2i}$ $\leftarrow$ Look at the denominator, take the "complex conjugate".

$3+2i \longrightarrow 3-2i$

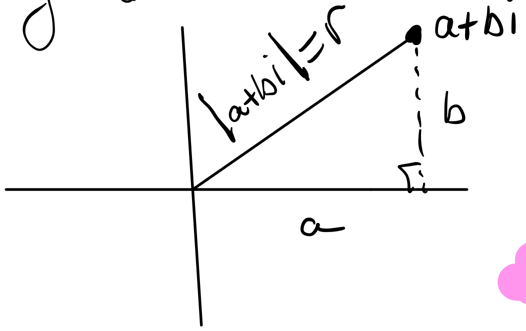
Now multiply top + bottom by complex conjugate

$$\frac{(5+4i)(3-2i)}{(3+2i)(3-2i)} = \frac{15 - 10i + 12i - 8i^2}{9 - \cancel{6i} + \cancel{6i} - \underset{+4}{4i^2}}$$

$$= \frac{15 + 2i + 8}{13} = \frac{23 + 2i}{13}$$

$$= \underbrace{\frac{23}{13} + \frac{2}{13}i}_{\text{standard form}} \qquad \boxed{a = \frac{23}{13}} \boxed{b = \frac{2}{13}}$$

Going between standard form + polar form



$r^2 = a^2 + b^2$
just the pythagorean theorem!

$$|a+bi| = \sqrt{a^2 + b^2}$$

Find the absolute value

$$|5-3i| = \sqrt{5^2+(-3)^2}$$
$$= \sqrt{25+9}$$
$$= \sqrt{34}$$

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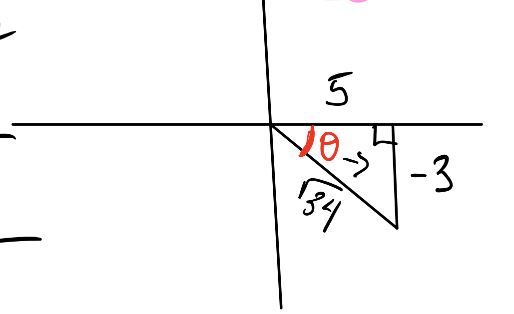
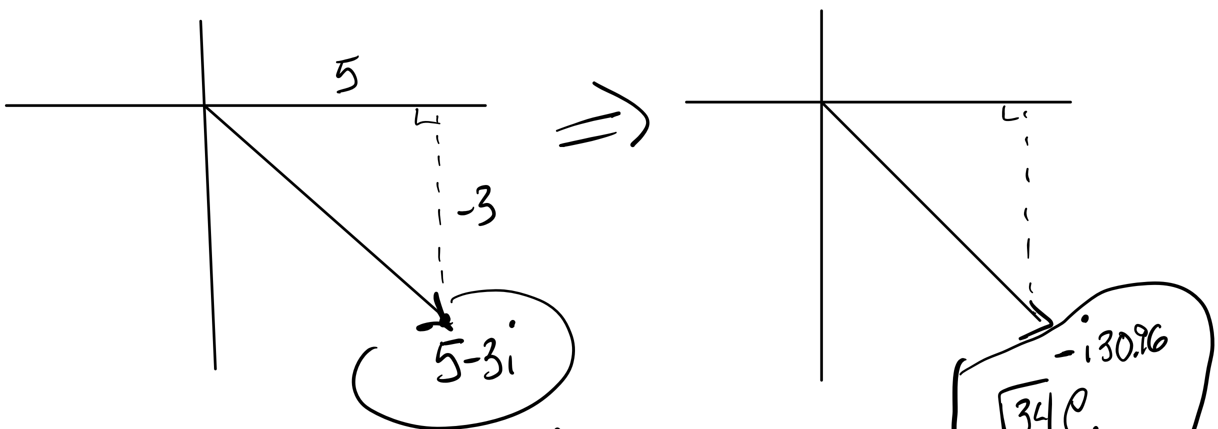


One step further.... what is the ∡?

$$\sin\theta = \frac{opp}{hyp} = \frac{-3}{\sqrt{34}} \Rightarrow \sin^{-1}\left(\frac{-3}{\sqrt{34}}\right) = \theta \;!!$$

$$\theta = -30.96^\circ$$



standard form

Polar form

Next up: Polar form
$\Rightarrow$ standard form