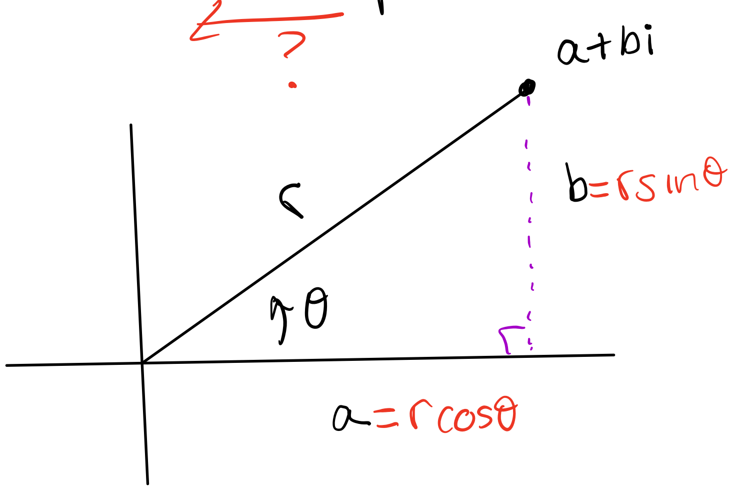
12/1/2021

Lesson #21: Complex Numbers

Last class we learned about complex numbers in standard + polar form.

Standard $\longrightarrow$ polar form
$\longleftarrow$?

SOHCAHTOA



$\sin\theta = \frac{b}{r}$

$\cos\theta = \frac{a}{r}$

can solve for a+b!

$a = r\cos\theta$
$b = r\sin\theta$

$a+bi$ standard form $\longrightarrow r\cos\theta + r\sin\theta\, i = r(\cos\theta + i\sin\theta)$

$\shortparallel$

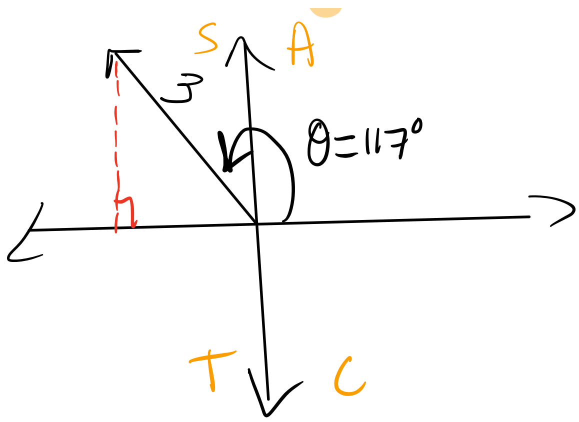
$re^{i\theta}$

These are both polar form

We can go back + forth between polar + standard form!

Ex Convert the number from polar form to standard form

$$3(\cos(117^\circ) + i\sin(117^\circ))$$



$a = r\cos\theta$

$= 3\cos 117^\circ = (-0.454)\cdot 3$

$b = r\sin\theta$

$\approx 3\sin 117^\circ = (+0.891)\cdot 3$

Standard form: $3(-0.454 + 0.891i)$

$= \boxed{-1.362 + 2.673i}$

Multiplication + division in polar form:

Example from Sample Final Exam in WeBWork:

Problem #5: z_1 with $r = 3$ $\theta = 302^\circ$

a) Write z_1 in polar form:

$z_1 = 3(\cos(302^\circ) + i\sin(302^\circ))$

b) z_2 $r = 4$ $\theta = 50^\circ$

z_2 in polar form

$z_2 = 4(\cos(50^\circ) + i\sin(50^\circ))$

c) Take the product

$$z_1 \cdot z_2 = r_1 \cdot r_2 \left(\cos(\theta_1+\theta_2) + i \sin(\theta_1+\theta_2)\right)$$

$$= \left(r_1 e^{i\theta_1}\right) \cdot \left(r_2 e^{i\theta_2}\right) = r_1 \cdot r_2 \, e^{i\theta_1 + i\theta_2}$$

Side note: This is why we add θ_1 & θ_2!

$$= 3 \cdot 4 \left(\cos(302^\circ + 50^\circ) + i \sin(302^\circ + 50^\circ)\right)$$

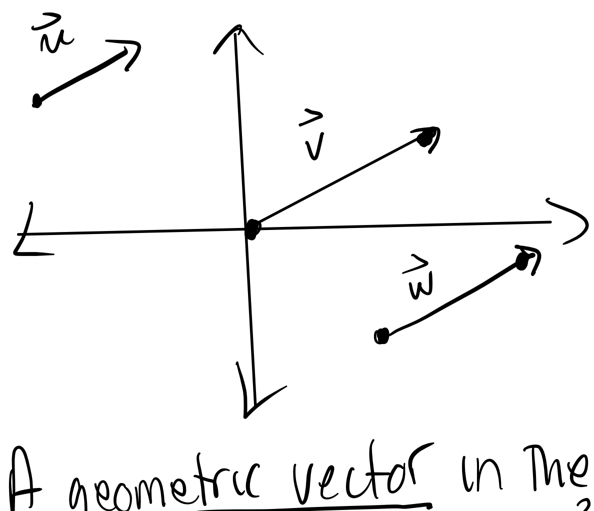
$$= 12\left(\cos(352^\circ) + i \sin(352^\circ)\right)$$

Modulus? 12

argument? 352°

polar form?

Next up: Vectors Lesson 22



Vector notation:

$\vec{v}$ $\mathbf{v}$

$\vec{v}$ $\dot{v}$

our textbook

$\vec{v}$

Def: A geometric vector in the plane is a geometric object in the plane $\mathbb{R}^2$ that is given by a direction and a magnitude.

We denote a vector by $\vec{v}$, its magnitude by $\|\vec{v}\|$ and its directional angle by θ.

Vectors are often represented by directed line segments $\vec{v} = \overrightarrow{PQ}$

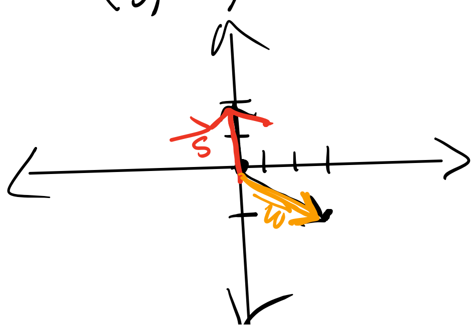
↑ ↑
start end
points

In particular, we can shift <u>any</u> vector so that its starting point is at the origin $(0,0)$. If the end point R is given by (a,b) Then we can also write $\vec{v} = \overrightarrow{OR}$

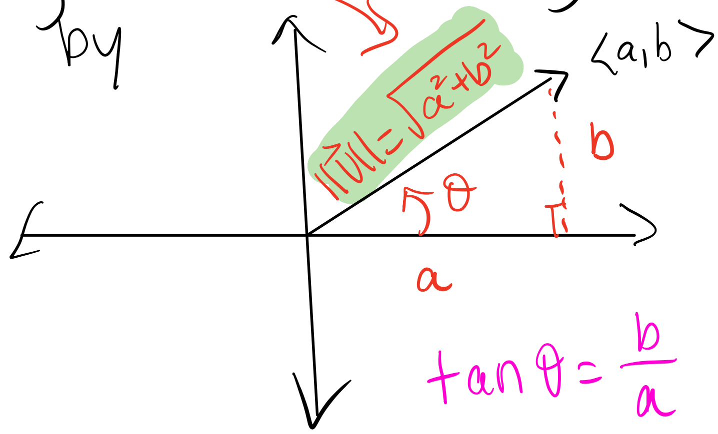
and

$\vec{v} = \underbrace{\langle a, b \rangle}_{\text{component notation}}$ or alternatively $\vec{v} = \begin{bmatrix} a \\ b \end{bmatrix}$

Ex Graph $\vec{w} = \langle 3, -1 \rangle$ and $\vec{s} = \langle 0, 2 \rangle$



Def: Let $\vec{v} = \langle a, b \rangle$ be a vector in $\mathbb{R}^2$. Then the magnitude and the angle of $\vec{v}$ are given by



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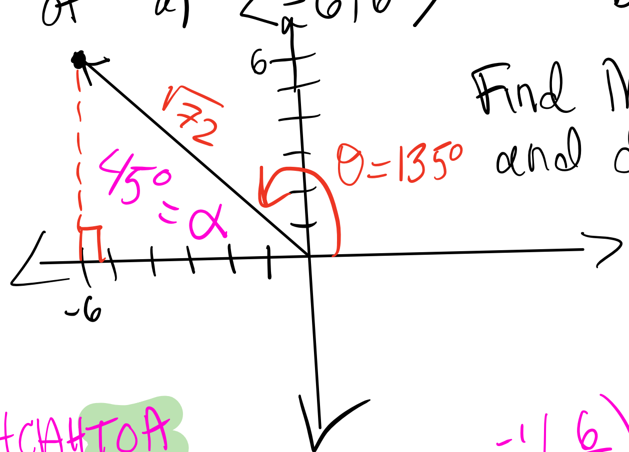
$\tan\theta = \frac{b}{a}$

so $\theta = \tan^{-1}\left(\frac{b}{a}\right)$

Ex Find the magnitude and direction of a) $\langle -6, 6 \rangle$ b) $\langle -2\sqrt{3}, -2 \rangle$

a)



Find the length = magnitude $\|\langle -6, 6 \rangle\|$ and direction = θ

$\|\langle -6, 6 \rangle\| = \sqrt{(-6)^2 + 6^2}$

$= \sqrt{36+36} = \sqrt{72}$

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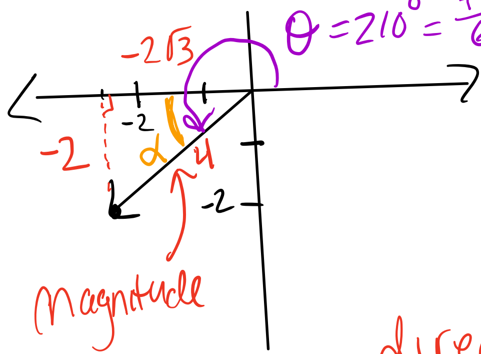
$\tan\alpha = \frac{6}{-6}$ $\longrightarrow$ $\tan^{-1}\left(\frac{6}{-6}\right) = \tan^{-1}(-1) = 45^\circ$

$\theta = 180^\circ - 45^\circ = 135^\circ$

$\langle -6, 6 \rangle$ has magnitude $\|\langle -6, 6 \rangle\| = \sqrt{72}$ and direction $\theta = 135^\circ$

b) $\langle -2\sqrt{3}, -2 \rangle$

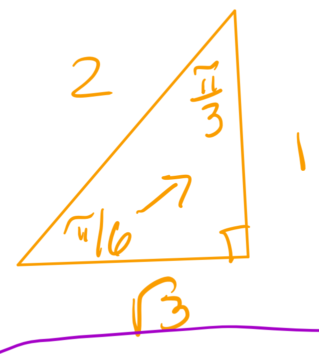


$\| \langle -2\sqrt{3}, -2 \rangle \| = \sqrt{(-2\sqrt{3})^2 + (-2)^2}$

$= \sqrt{12 + 4}$

$= \sqrt{16} = \boxed{4}$

direction $\alpha \Rightarrow$



$\tan\alpha = \frac{-2}{-2\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \alpha = \frac{\pi}{6} = 30°$

$\boxed{\theta = 180° + 30° = 210°}$ or $\boxed{\frac{6\pi}{6} + \frac{\pi}{6} = \frac{7\pi}{6}}$

So $\langle -2\sqrt{3}, -2 \rangle$ has magnitude 4 and direction $\frac{7\pi}{6} = \theta$

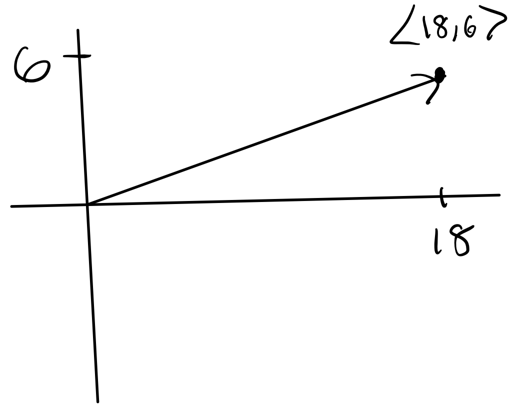
Operations on vectors:

Def: The scalar multiplication of a real # r with a vector $\vec{v} = \langle a, b \rangle$ is defined to be the vector

$$r\vec{v} = \langle r \cdot a, r \cdot b \rangle$$

Ex Multiply + graph $(-3) \cdot \langle -6, -2 \rangle$

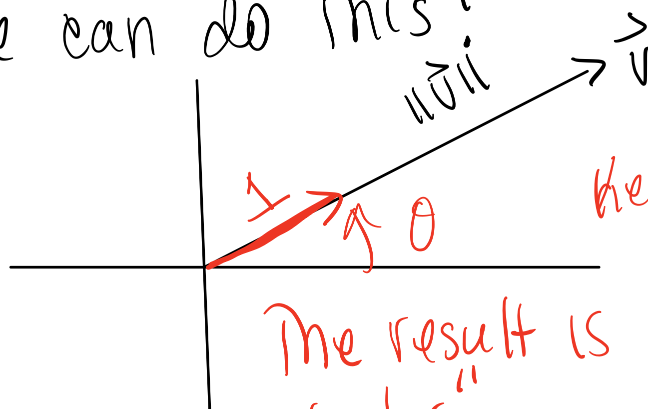
$= \langle +18, +6 \rangle$



Scalar → just a real #!

What if we want to keep the same direction of a vector but just change its magnitude to $\underline{\underline{1}}$?

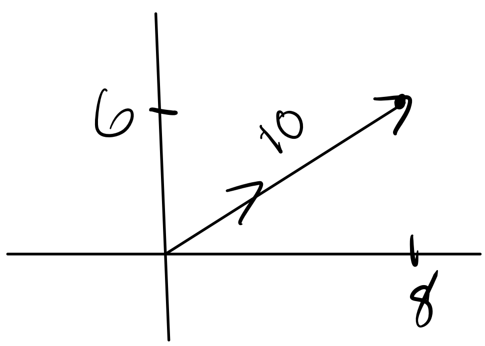
We can do this!



Keep θ, divide magnitude by itself!

The result is called a "unit vector."

Ex: Find a unit vector in the direction of $\vec{v} = \langle 8,6 \rangle$



$\|\vec{v}\| = \sqrt{8^2+6^2} = \sqrt{64+36}$

$= \sqrt{100} = 10$

Divide by $\underline{\underline{10}}$!

$\frac{1}{10} \cdot \langle 8,6 \rangle = \langle \frac{8}{10}, \frac{6}{10} \rangle$

by dividing by 10 the result will have magnitude 1! Check:

$$\left\|\left\langle \frac{8}{10}, \frac{6}{10} \right\rangle\right\| = \sqrt{\left(\frac{8}{10}\right)^2 + \left(\frac{6}{10}\right)^2}$$

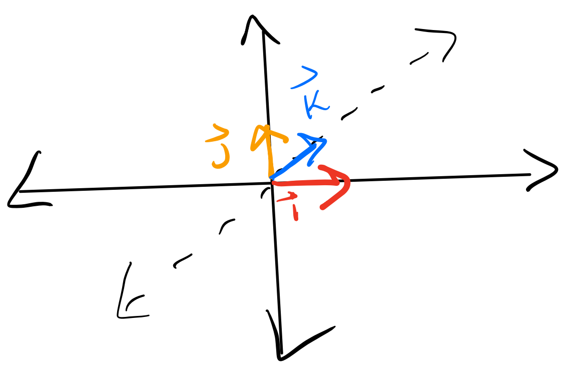
$$= \sqrt{\frac{64}{100} + \frac{36}{100}} = \sqrt{\frac{100}{100}}$$

$$= \frac{10}{10} = 1 \checkmark$$

Two special unit vectors are:

$\vec{i} = \langle 1, 0 \rangle$ and $\vec{j} = \langle 0, 1 \rangle$

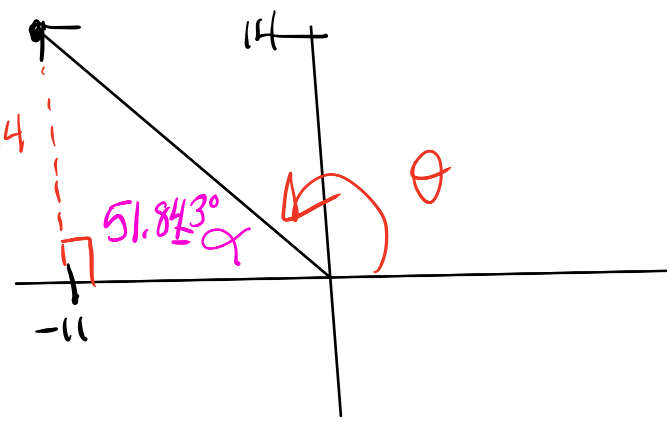
($\vec{k} = 3^{rd}$ dimension)



Next class vector addition.

Vectors Magnitude + Direction: WeBWorK

b) θ? $\langle -11, 14 \rangle$



$$\tan\alpha = \frac{14}{-11}$$

$$\tan^{-1}\left(\frac{-14}{11}\right) = 51.843^\circ$$

$$\theta = 180^\circ - 51.843^\circ = {}^\circ\boxed{128.157}$$