11/24/2021 : Trig Equations

Basic Trig Equations : $\cos x = c$, $\sin x = c$, $\tan x = c$

Ex Solve for x : $\tan x = \sqrt{3}$

rewrite it ↗ ↗ (x, #)

Could take : $\tan^{-1}(\sqrt{3}) = x$

This gives one solution however, the trig equation is asking for all solutions! The tan x function repeats its pattern ∞-many times.

Need to be aware that $\tan^{-1}(\sqrt{3}) = x$ gives us only one solution, the one that is in the restricted domain : $(-\frac{\pi}{2}, \frac{\pi}{2})$

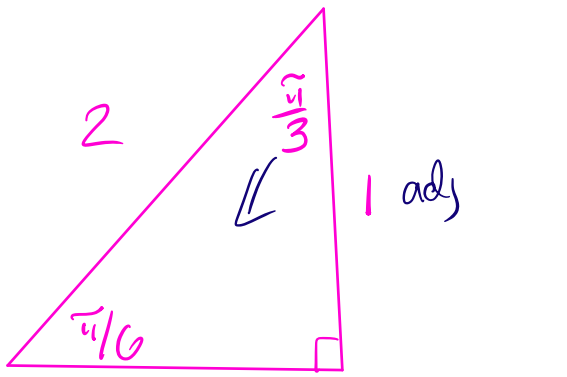
That is $\frac{\pi}{3} = 60°$

To get all solutions we must add integer multiples of its period : π

$= \{ x \mid x = \frac{\pi}{3} + n\pi$ where $n \in \mathbb{Z} \}$

set builder notation

↑ set of integers



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$x = \frac{\pi}{3} + n\pi \quad n = 0, \pm 1, \pm 2, \pm 3, \ldots$

$x = 60^\circ + n \cdot 180^\circ$ where $n = 0, \pm 1, \pm 2, \ldots$

Ex Solve for x: $\cos(x) = \frac{1}{2}$

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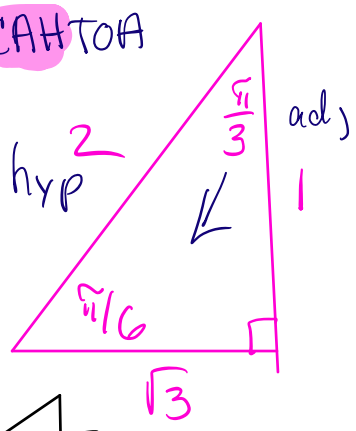
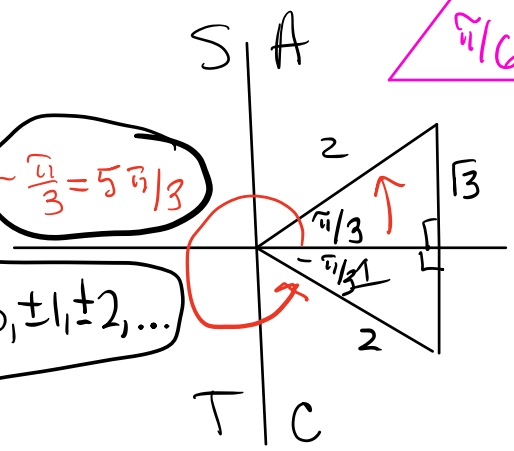


$\cos^{-1}\left(\frac{1}{2}\right) = x$

$\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$



Solutions:

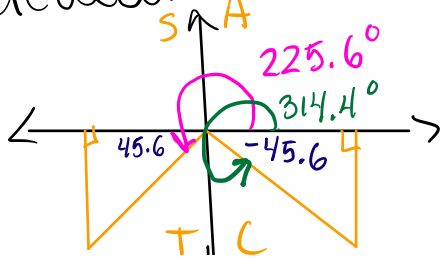
$x = \frac{\pi}{3} + 2\pi \cdot n \in n = 0, \pm 1, \pm 2, \ldots$

and

$x = \frac{5\pi}{3} + 2\pi \cdot n$

What happens if the angle is <u>not</u> from a special Δ? Still solve the equation!

Ex Solve $\sin x = -\frac{5}{7}$ In this case use a calculator: $\sin^{-1}\left(-\frac{5}{7}\right) = -45.6^\circ$



$x = 225.6^\circ + 360^\circ \cdot n$

or $n = 0, \pm 1, \pm 2, \ldots$

$x = 314.4^\circ + 360^\circ \cdot n$

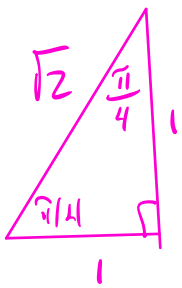
Ex Solve for x

$$\sec x = -\sqrt{2}$$

$$\frac{1}{\cos x} = -\sqrt{2}$$

$$\cos x = -\frac{1}{\sqrt{2}}$$

$$\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$



Identities: (remember cs!)

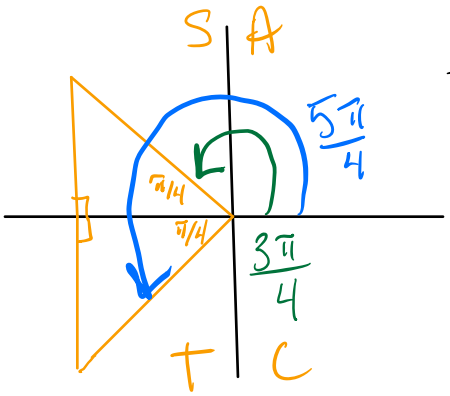
reciprocal

$\cos x \rightarrow \frac{1}{\cos x} = \sec x$

$\sin x \rightarrow \frac{1}{\sin x} = \csc x$

$\tan x \rightarrow \frac{1}{\tan x} = \cot x$

where is $\cos x$ negative?!



Solutions:

$$x = \frac{5\pi}{4} + 2\pi \cdot n$$

and

$$x = \frac{3\pi}{4} + 2\pi \cdot n$$

$n = 0, \pm 1, \pm 2, \ldots$

Solve for x

$$\tan^2(x) + 2\tan(x) + 1 = 0$$

Use a dummy variable = $u = \tan(x)$

$$u^2 + 2u + 1 = 0$$

This is just a quadratic equation

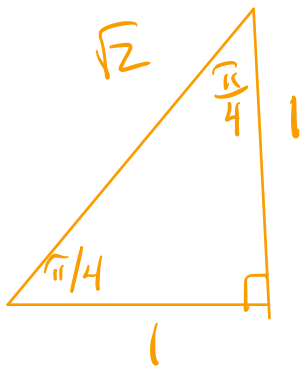
$$(u+1)(u+1) = 0$$

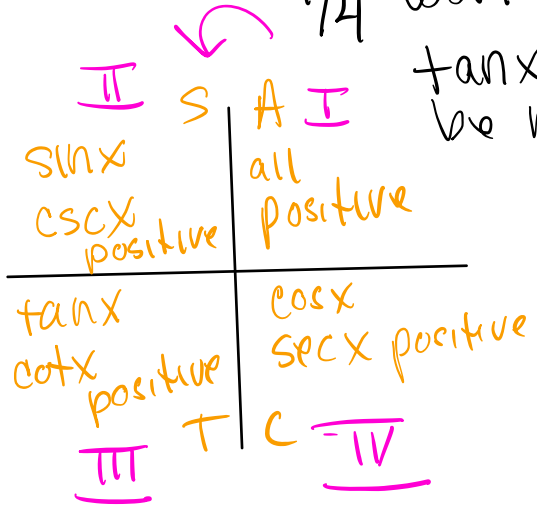
$u = -1 \quad u = -1$ → remember that $u = \tan x$!

$$\tan x = -1$$

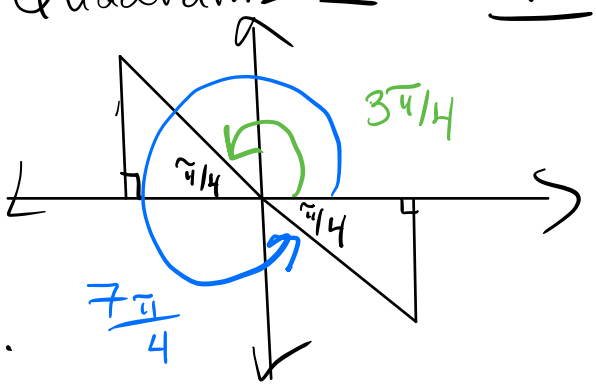
really $\tan^{-1}(-1) = x$?

$\pi/4$ works but $\tan x$ needs to be negative





$\tan x$ is negative in Quadrants II and IV



$$x = \frac{7\pi}{4} + \pi \cdot n$$

or

$n = 0, \pm 1, \pm 2 \ldots$

$$x = 315^\circ + 180^\circ n$$