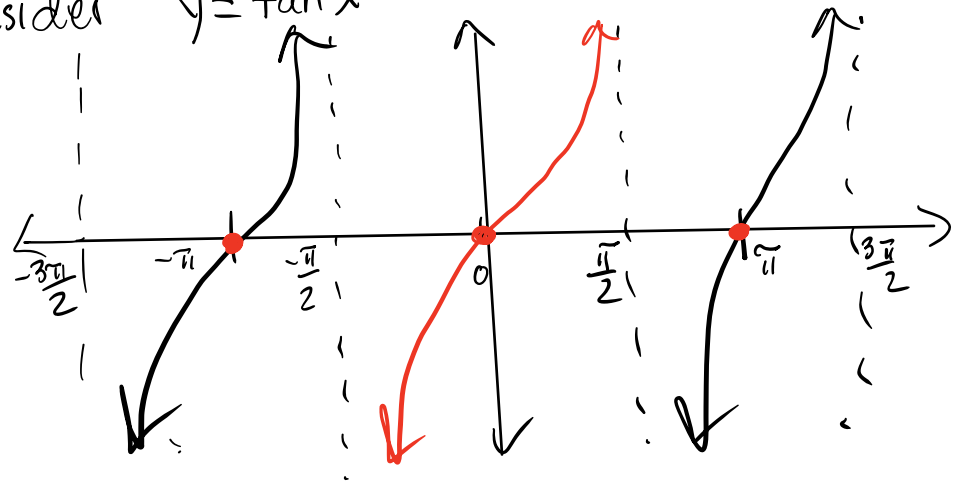
Mon	Wed
11/22 (19)	11/24 (20)
11/29 (21)	12/1 (22)
12/6 (23)	12/8 (24)
12/13 (25)	12/15 Review for final

Final Exam 12/20

Lesson 19: Inverse Trig Functions

Consider $y = \tan x$



$y = \tan(x)$

- y: output numerical value
- x: input angle

$x = \tan^{-1}(y)$

- x: output angle
- y: input numerical value

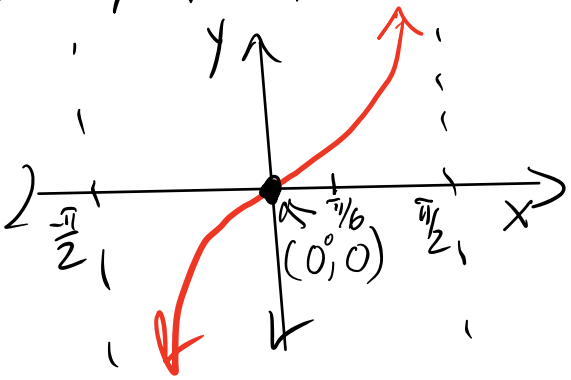
Take # 0 →

$$x = \tan^{-1}(0)$$

↑

What is the x?
We can't tell! There are many x's which work!
$x = 0°, \pi, -\pi, \ldots$

Solution: Restrict the domain of $y = \tan x$ to be $(-\pi/2, \pi/2)$



Now if we ask the same question, we can answer it!

$\tan^{-1}(0) = 0°$

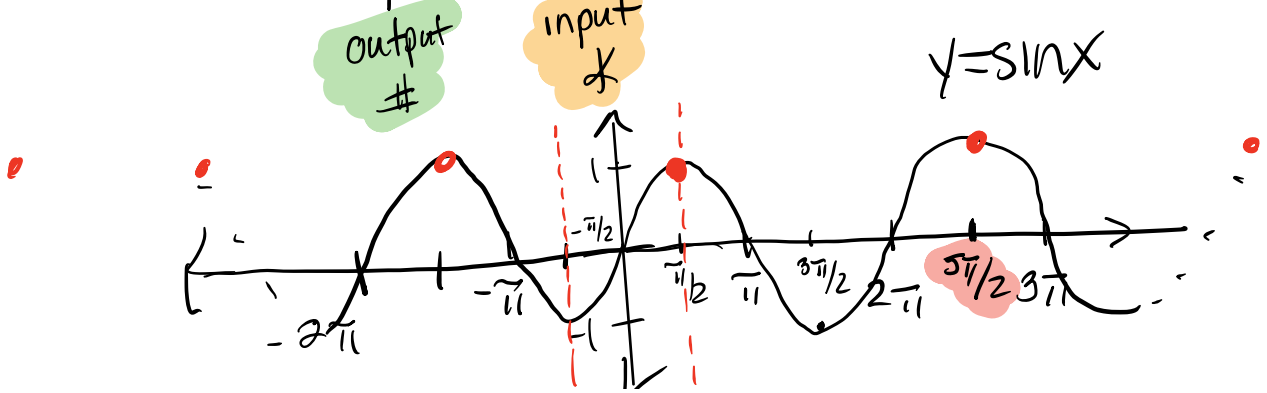
↑ #

↑ x

Consider $y = \sin x$

↑ output #

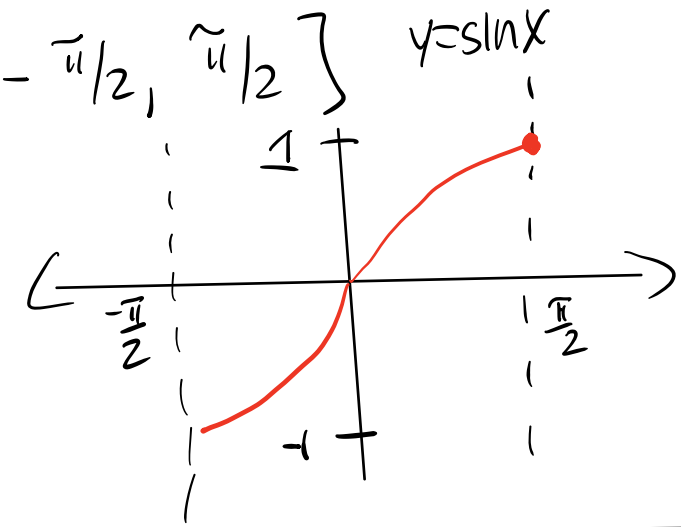
↑ input x



Given output # 1 → what x did it come from? i.e. $\sin(1) = x$?

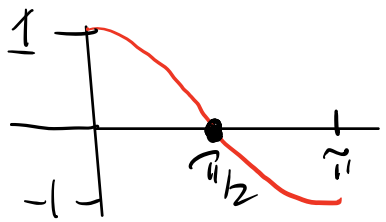
There are ∞-many options!

Trick: Restrict the domain to $\left[-\pi/2, \pi/2\right]$



$\sin(1) = \pi/2$
↑ ↑
x

Similarly we can restrict $y = \cos x$ to $[0, \pi]$

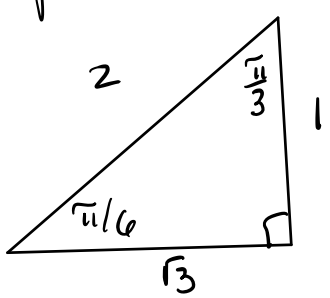


$\cos^{-1}(0) = \pi/2$
↑ ↑
x

here it is 1-1 and we can find its inverse.

Now lets practice!

SOHCAHTOA

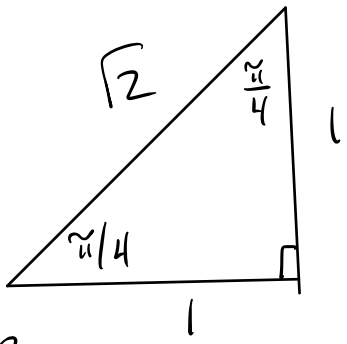


$\tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$
↑ x
x

$\sin^{-1}\left(\frac{1}{2}\right) = \pi/6$

$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

$\tan^{-1}(1) = \frac{\pi}{4}$



$\cos^{-1}(.3245) = 71.06°$ → for something like this use a calculator!

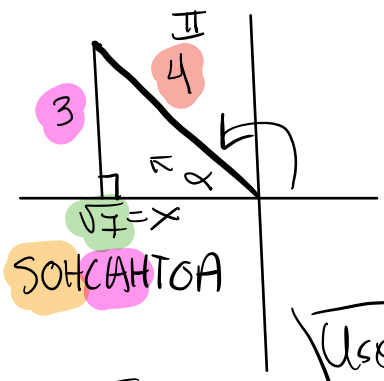
WeBWorK set: Double + 1/2 ∡ Formulas

Problem #1: Given that α is in QII and $\sin\alpha = \frac{3}{4}$, give an exact answer for the following:

a) $\sin(2\alpha) =$

b) $\cos(2\alpha) =$

c) $\tan(2\alpha) =$



Use the pythagorean Thm to find x!

$x^2 + 3^2 = 4^2$ $\quad$ $leg^2 + leg^2 = hyp^2$

$x^2 = 4^2 - 3^2$

$x = \sqrt{16-9} = \sqrt{7}$

II	
S	A
T	C

Given $\sin\alpha = +\frac{3}{4}$

$\cos\alpha = -\frac{\sqrt{7}}{4}$

$\tan\alpha = -\frac{3}{\sqrt{7}}$

a) $\sin(2\alpha) = 2\sin\alpha\cos\alpha$

$= \cancel{2}^{1}\left(\frac{3}{\cancel{4}^{2}}\right)\left(-\frac{\sqrt{7}}{4}\right) = \boxed{-\frac{3\sqrt{7}}{8}}$

b) $\cos(2\alpha) = 1 - 2\sin^2(\alpha)$

$= 1 - 2\left(\frac{3}{4}\right)^2 = 1 - 2 \cdot \frac{9}{16} = 1 - \frac{9}{8} = \frac{8}{8} - \frac{9}{8}$

$= \boxed{-\frac{1}{8}}$

---

c) $\tan(2\alpha) = \frac{2\tan\alpha}{1 - \tan^2(\alpha)} = \frac{2\left(-\frac{3}{\sqrt{7}}\right)}{1 - \left(\frac{-3}{\sqrt{7}}\right)^2}$

$= \frac{-6/\sqrt{7}}{1 - \frac{9}{7}} = \frac{-6/\sqrt{7}}{-2/7} = -\frac{6}{\sqrt{7}} \div \frac{-2}{7}$

$= \frac{-6}{\sqrt{7}} \cdot \frac{7}{-2} = \boxed{\frac{42}{2\sqrt{7}}}$

Note: Can rationalize

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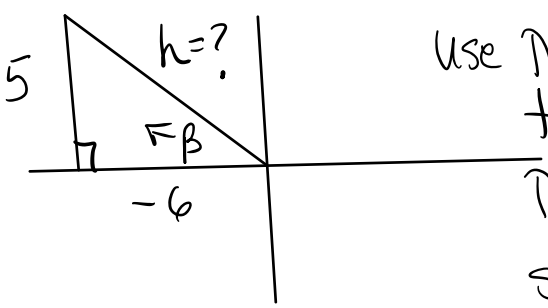
2. β in QII $\tan\beta = \frac{-5}{6}$



SOHCAHTOA

Use the pythagorean Thm to find hyp

Then write all

$\sin\beta =$ $\cos\beta =$

$\tan\beta = -5/6$

Last, plug into double α formulas

Could Project: WeBWorkt #1

$P(t) = C \cdot (b)$

A

Some
Constant
initial amount = 684

$P(t) = 684 \cdot b^{t}$ ← exponential model

info
81 days = t use this!
35,107

what is b?

$P(81) = 684 \cdot b^{81} = 35107$

solve for b!
exponential equation.

b) plug in a value for t

c) doubling time? $t = 0, 684$

$t = d, 2 \cdot 684$

$2 \cdot 684 = 684\, b^{d}$

solve for this!

$2 = (b)^{d}$

take log of both sides & solve for d = doubling time.