11/17/2021

Trig Review:

WeBWork: Trig Unit Circle

Problem #1: No calculator → exact value

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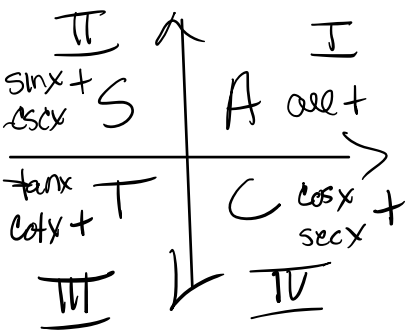
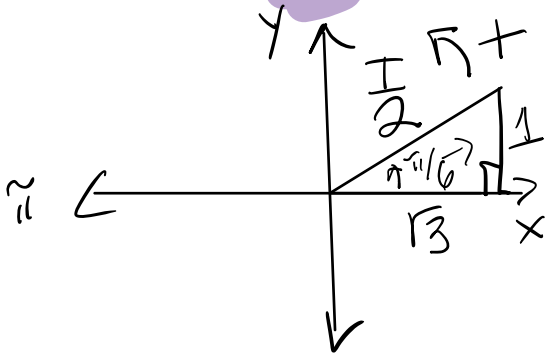
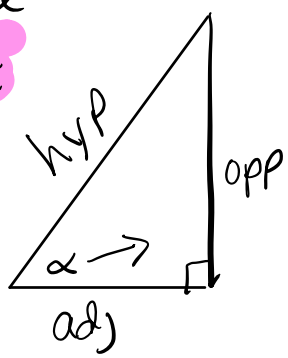
a) $\sin\left(\frac{\pi}{6}\right) = \frac{+1}{2}$



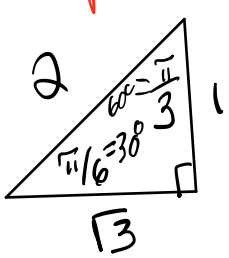


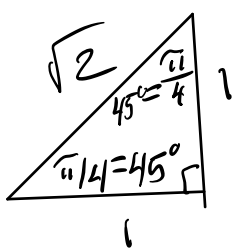


$\pi = 180^\circ$

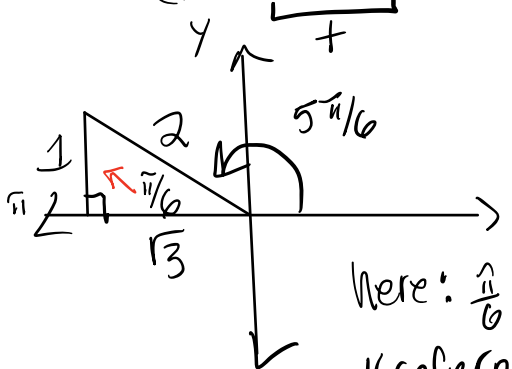
$2\pi = 360^\circ$

Special Δ's





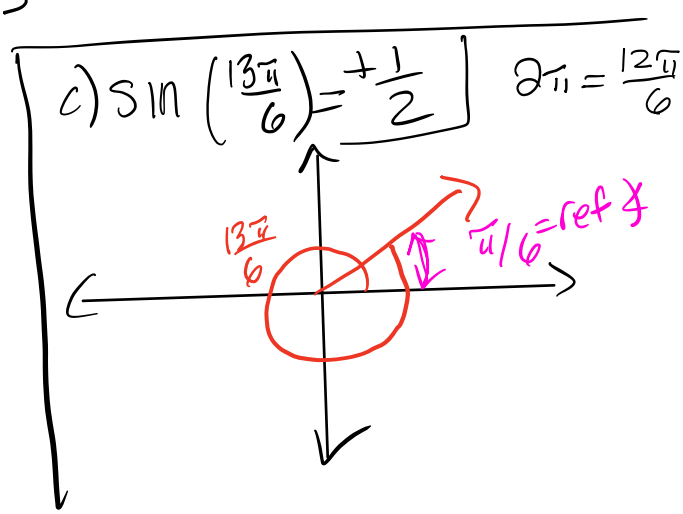
b) $\sin\left(\frac{5\pi}{6}\right) = \frac{+1}{2}$



here: $\frac{\pi}{6}$ is our "reference $\measuredangle$"

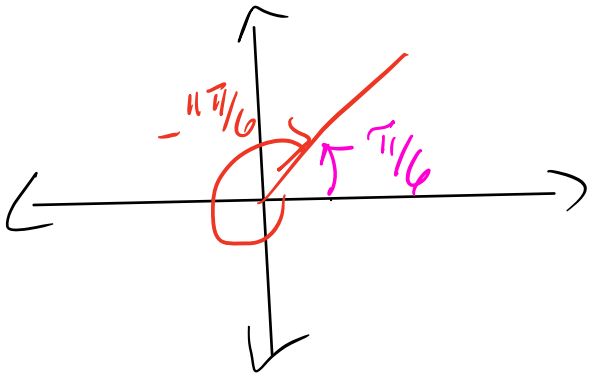
S	A
T	C

c) $\sin\left(\frac{13\pi}{6}\right) = \frac{+1}{2}$ $\quad 2\pi = \frac{12\pi}{6}$



d) $\sin\left(\frac{-11\pi}{6}\right) = +\frac{1}{2}$ ↘ $\pi = \frac{6\pi}{6}$

$2\pi = \frac{12\pi}{6}$



Given $f(x) = a\cos(bx+c)$ or $f(x) = a\sin(bx+c)$

$|a|$ = amplitude

$\left|\frac{2\pi}{b}\right|$ = period

$-\frac{c}{b}$ = phase shift

WeBWorK Problem Set: Trig Graphing Comprehensive

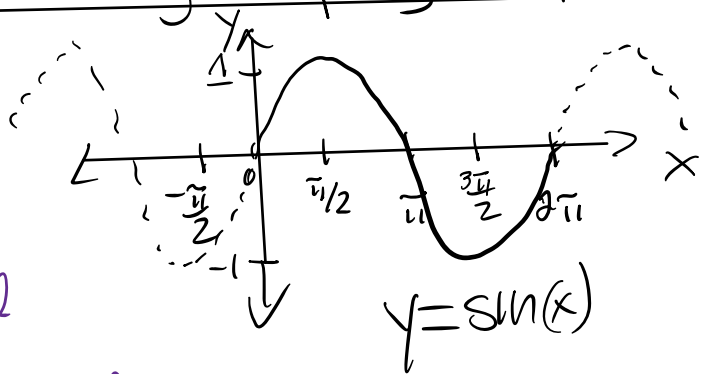
Problem #1:

$f(x) = -2\sin(Hx)$

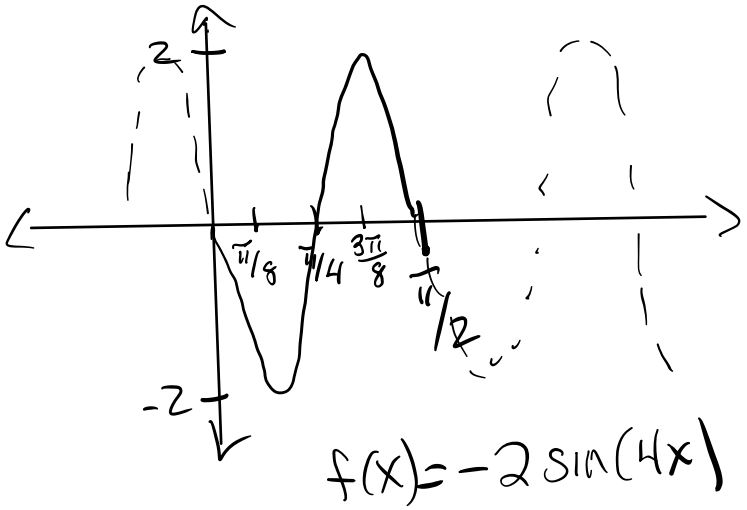
vertical stretch (2 = a)

reflect in x-axis (−)

horizontal stretch/compression (H = b)



amplitude $= |-2| = 2$ period $= \left|\frac{2\pi}{H}\right| = \frac{\pi}{2}$



$f(x) = -2\sin(4x)$

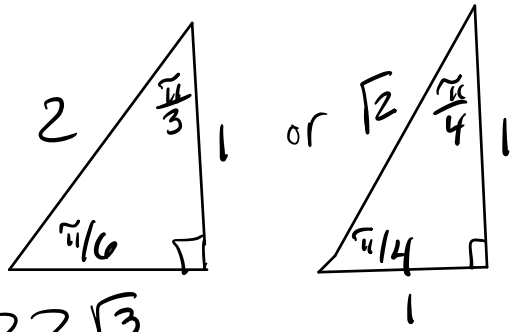
Session 18: Addition/Subtraction & Double/Half Angle Formulas

Look in Text pg 252 → 257 ↗ for formulas

Ex Find the exact values of the trig functions:

a) $\cos\left(\frac{\pi}{12}\right)$ b) $\tan\left(\frac{5\pi}{12}\right)$

Want to use θ's for which we know the values of the trig functions:



Can we write $\frac{\pi}{12}$ as a sum or difference of $\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{4}$??

$\frac{\pi}{6} = 30°$ $\frac{\pi}{3} = 60°$ $\frac{\pi}{4} = 45°$ $\frac{\pi}{12} = 15°$

Yes! $\frac{\pi}{3}-\frac{\pi}{4}=\frac{\pi}{12}$ (i.e. $60°-45°=15°$!!)

(or $\frac{\pi}{4}-\frac{\pi}{6}$ i.e. $45°-30°$!!)

$\cos\left(\frac{\pi}{3}-\frac{\pi}{4}\right)$ (α = $\frac{\pi}{3}$, β = $\frac{\pi}{4}$)

$= \cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4}$

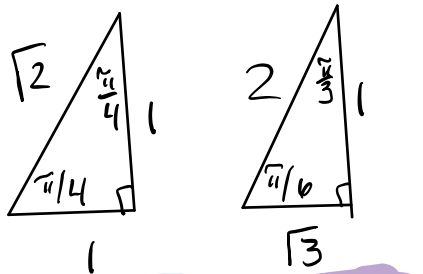
$= \frac{1}{2}\cdot\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}}$

rationalize

$= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$

$\cos(\alpha-\beta)=\cos\alpha\cos\beta + \sin\alpha\sin\beta$



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$\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{2}}{2}$ rationalize

b) $\tan\left(\frac{5\pi}{12}\right)$

$\frac{5\pi}{12}$ = sum or difference of $\frac{\pi}{3}, \frac{\pi}{6}, \frac{\pi}{4}$

$\frac{4\pi}{12}, \frac{2\pi}{12}, \frac{3\pi}{12}$

$\boxed{\frac{5\pi}{12} = \frac{2\pi}{12}+\frac{3\pi}{12} = \frac{\pi}{6}+\frac{\pi}{4}}$

$\tan\left(\frac{5\pi}{4}\right) = \tan\left(\frac{\pi}{6}+\frac{\pi}{4}\right) = \frac{\tan\frac{\pi}{6}+\tan\frac{\pi}{4}}{1-\tan\frac{\pi}{6}\tan\frac{\pi}{4}}$

$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\alpha}{1-\tan\alpha\tan\beta}$

$= \frac{\left(\frac{1}{\sqrt{3}}+1\right)}{\left(1-\frac{1}{\sqrt{3}}\cdot 1\right)} = \boxed{\frac{\frac{1}{\sqrt{3}}+1}{1-\frac{1}{\sqrt{3}}}}$

(to rationalize multiply top + bottom by conjugate: $1+\frac{1}{\sqrt{3}}$)

Ex Find using the 1/2 angle formula

$\sin\left(\frac{\pi}{8}\right)$

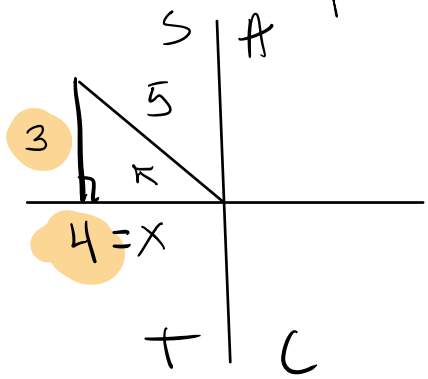
Notice: $\frac{\pi}{8} = \frac{(\pi/4)}{2}$ (α)

$\sin\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1-\cos\alpha}{2}}$

$= \pm\sqrt{\frac{1-\cos\frac{\pi}{4}}{2}}$

$= \pm\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}} = \pm\sqrt{\frac{1}{2}-\frac{\sqrt{2}}{4}}$

Ex Find the trig functions of 2α when α has the properties: $\sin(\alpha)=\frac{3}{5}$ and α is in quadrant II



use pythagorean theorem:

$x^2+3^2=5^2$

$x^2=5^2-3^2$

$x^2=25-9=16$

$x=4$

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$\sin(2\alpha)=2\sin\alpha\cos\alpha \rightarrow 2\cdot\left(\frac{3}{5}\right)\left(-\frac{4}{5}\right)=-\frac{24}{25}$

$\cos(2\alpha)=2\cos^2\alpha-1 \rightarrow 2\left(-\frac{4}{5}\right)^2-1=\frac{2\cdot 16}{25}-1=\frac{32}{25}-1=\frac{7}{25}$

$\tan(2\alpha)=\frac{2\tan\alpha}{1-\tan^2\alpha} \rightarrow \frac{2\cdot\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)^2}=\frac{\frac{3}{2}}{1-\frac{9}{16}}=\frac{3/2^{1}}{7/16_{8}}=\frac{3}{7/8}=24/7$