

Banksy Expo - Visit!

11/10/2021

Applications: Half-life + compound interest

Def: Let f be an exponential function
 $f(x) = c \cdot b^x$ (domain: \mathbb{R}). Then we say
that f has a half-life of h if
The base is given by

$$b = \left(\frac{1}{2}\right)^{\frac{1}{h}}$$

So re-writing the function

$$\boxed{f(x) = c \cdot \left(\frac{1}{2}\right)^{\frac{x}{h}}}$$

Nice property: $f(x+h) = \frac{1}{2} f(x)$

Ex Chromium-51 has a half-life of 27.7 days. How much of 3 grams of chromium-51 will remain after 90 days.

$$f(x) = 3 \cdot \left(\frac{1}{2}\right)^{\frac{x}{27.7}}$$

↑ initial amount ↖ half-life

$x \rightarrow$ time in days

$$f(90) = 3 \cdot \left(\frac{1}{2}\right)^{\frac{90}{27.7}} \approx \boxed{0.316 \text{ g}}$$

The half-life of carbon-14 is 5730 years!

Ex A dead tree trunk has 86% of its original carbon-14. Approximately how many years ago did the tree die?

Use $f(x) = C \cdot \left(\frac{1}{2}\right)^{\frac{x}{\text{half-life}}}$ half-life = 5730

$$0.86C = C \cdot \left(\frac{1}{2}\right)^{\frac{t}{5730}}$$
 left \downarrow initial amount start? \uparrow t solve for $t = \text{time elapsed!}$

$$0.86 = \left(\frac{1}{2}\right)^{\frac{t}{5730}}$$
 solve this exponential equation.

$$\ln(0.86) = \ln\left(\frac{1}{2}\right)^{\frac{t}{5730}}$$

$$\ln(0.86) = \left(\frac{t}{5730}\right) \ln\left(\frac{1}{2}\right)$$

$$\frac{t}{5730} = \frac{\ln(0.86)}{\ln(1/2)}$$

$$t = 5730 \cdot \left(\frac{\ln(0.86)}{\ln(1/2)}\right) \approx \boxed{1246.8 \text{ years}}$$