WeBWork

Rational Functions

Inequalities Solve for x

4) $\dfrac{3x^2-28x+49}{x^2-5x-14} \geq 0$

inequality

1) Find the roots → solve the equality!

$$\frac{3x^2-28x+49}{x^2-5x-14} = 0$$

Recall for $\frac{a}{b}=0$ if $a=0$

$$3x^2-28x+49=0$$

solve for x

roots are $x = 2\tfrac{1}{3}$ and $x = 7$

numerator

$$x = \boxed{\tfrac{7}{3}, 7} \rightarrow \left(x - \tfrac{7}{3}\right)(x-7)$$

Vertical asymptotes

$$\frac{3x^2 - 28x + 49}{x^2 - 5x - 14} = \frac{\left(x - 7/3\right)\cancel{(x-7)}}{(x+2)\cancel{(x-7)}}$$

Look at where the denominator is zero $\boxed{x = -2 \quad \cancel{x = +7}}$

Only vertical asymptote is $\boxed{x = -2}$

Where is the function non-zero?

$$(-\infty, -2) \cup \left(-2, \tfrac{7}{3}\right) \cup \left(\tfrac{7}{3}, 7\right) \cup (7, +\infty)$$

vertical asym.

root

hole

$$\boxed{(-\infty,-2)\cup[\tfrac{7}{3},7)\cup(7,+\infty)}$$

is where the rational function is ≥ 0

Applications of Exponential Functions:

Ex Let $f(x) = c \cdot b^x$. Determine the constant c and base b under the given conditions

$\boxed{a)\ f(0) = 5}$ $\boxed{b)\ f(-2) = 55,\ f(1) = 7}$

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a) $f(0) = c \cdot b^0 = 5$ Recall $x^0 = 1$

$c \cdot \underline{1} = 5 \Rightarrow c = 5$ so the function is

$$f(x) = 5 \cdot b^x$$

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b) $f(-2) = 55$ and $f(1) = 7$

trick form a quotient! This allows us to cancel the c!

$$\frac{f(-2)}{f(1)} = \frac{55}{7} = \frac{\cancel{c} \cdot b^{-2}}{\cancel{c} \cdot b^{1}}$$

Now can solve for b!

$$\frac{55}{7} = \frac{b^{-2}}{b^{1}}$$

Use rules of exponents!

$$b^{-5} = \frac{1}{b^{5}}$$

$$\frac{55}{7} = \frac{1}{b^{1} \cdot b^{2}}$$

take the reciprocal of both sides

$$\frac{55}{7} = \frac{1}{b^{3}} \Rightarrow b^{3} = \frac{7}{55}$$

take the cube root of both sides!

$$\left(b^{3}\right)^{1/3} = \left(\frac{7}{55}\right)^{1/3}$$

$$b = \left(\frac{7}{55}\right)^{1/3} \approx 0.503$$

$\Rightarrow$ Exponential function

$$f(x) = C \cdot (0.503)^{x}$$

Now let's find $\underline{\underline{C}}$

Use the given information $f(1) = 7$

$$f(1) = 7 = C(0.503)^{1}$$

$$C = \frac{7}{0.503} \approx 13.92$$

So the exponential function is

$$\boxed{f(x) = 13.92 \cdot (0.503)^x}$$

exponential model

Ex The mass of a bacteria sample is $\underset{\substack{\uparrow \\ \text{constant}}}{2} \cdot (\underset{\substack{\uparrow \\ \text{base}}}{1.02})^t$ after t hours.

a) What is the mass of the bacteria sample after 4 hours?

b) When will this mass reach 10_g?

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a) $f(t) = 2 \cdot (1.02)^t$

Let $t = 4$ hours

$$f(4) = 2 \cdot (1.02)^4 \approx 2.16 \text{ g}$$

b) What is t for $f(t) = 10g$?

$$\frac{10}{2} = \frac{2 \cdot (1.02)^t}{2}$$ ← solve for t!

$$5 = (1.02)^t$$ take the logarithm of both sides

$$\log(5) = \log((1.02)^t)$$ property of logarithms

$$\frac{\log(5)}{\log(1.02)} = \frac{t \log(1.02)}{\log(1.02)}$$

$$t = \frac{\log(5)}{\log(1.02)} \approx \boxed{81.27 \text{ hours}}$$

Def: An exponential function with a rate of growth r is a function

$f(x) = c \cdot b^x$ with base

$$b = (1+r)$$

$$\boxed{\text{i.e. } f(x) = c \cdot (1+r)^x}$$

Ex The # of PCs That are sold in The US in The year 2011 is approx 350 million with a rate of growth of 3.6% per year. Assuming the rate stays constant over the next years, how many PCs will be sold in the year 2015?

$$f(x) = \underset{\substack{\uparrow \\ 350m}}{c} \cdot (1 + \underset{\substack{\uparrow \\ 0.036}}{r})^{x \to 4}$$

start 2011
$\to$ 2015
$x = 4$ years

$$= 350(1.036)^4 \approx 403.19 \text{ million}$$

Note r = rate if positive $\to$ growing
if negative $\to$ decaying