Theory of Equations - Handout

1. **The Rational Root Test** If a rational number r/s (in lowest terms) is a root of the polynomial
$$a_n x^n + \ldots + a_1 x + a_0$$
where the coefficients $a_n, \ldots, a_1, a_0$ are integers with $a_n \neq 0$, $a_0 \neq 0$, then r is a factor of the constant term a_0 and s is a factor of the leading coefficient a_n.

2. **Fundamental Theorem of Algebra:** Every nonconstant polynomial has a root in the complex number system.

3. **Factorization over the Complex Numbers:** Let $f(x)$ be a polynomial of degree $n > 0$ with leading coefficient d. Then there are (not necessarily distinct) complex numbers $c_1, c_2, \cdots, c_n$ such that
$$f(x) = d(x - c_1)(x - c_2)\cdots(x - c_n).$$
Furthermore, $c_1, c_2 \cdots, c_n$ are the only roots of $f(x)$.

4. **Number of Roots:** Every polynomial of degree $n > 0$ has at most n different roots in the complex number system.

5. **A polynomial of degree n has exactly n roots.**

6. **Conjugate Roots Theorem:** Let $f(x)$ be a polynomial with real coefficients. If the complex number z is a root of $f(x)$, then its conjugate $\bar{z}$ is also a root of $f(x)$.

7. **Factorization over the Real Numbers:** Every nonconstant polynomial with real coefficients can be factored as a product of linear and quadratic polynomials with real coefficients in such a way that the quadratic factors, if any, have no real roots.