10/6/2021

Recall The Remainder Theorem: The remainder when dividing $f(x)$ by $(x-c)$ is $r=f(c)$.

In particular $f(c)=0$ if and only if $(x-c)$ is a factor of $f(x)$.

Ex Show That 5 is a root of $f(x)=x^3-19x-30$ and use This to factor $f(x)$ completely.

Language if $x-c$ is a factor $\Rightarrow$ c is called a "root" i.e. c is a solution to The equation $f(x)=0$.

Compute $f(5)=5^3-19(5)-30$

$=125-95-30=0$ ✓

Since There is zero remainder $\Rightarrow$ $x-5$ is a factor of $f(x)$. Now The goal is to factor $f(x)$ so let's divide $f(x)$ by $x-5$ to start.

$$x-5 \overline{\big)\, x^3 - 19x - 30}$$

notice we are missing an x^2 term so let's add in "$0x^2$"

$$\begin{array}{r} x^2+5x+6 \\ x-5 \overline{\big)\, x^3-0x^2-19x-30} \\ -(x^3-5x^2) \qquad\qquad \\ \hline 5x^2-19x \qquad \\ -(5x^2-25x) \qquad \\ \hline 6x-30 \\ -(6x-30) \\ \hline 0 \end{array}$$

$$f(x) = x^3-19x-30 = (x-5)\underbrace{(x^2+5x+6)}$$

Goal: Factor $f(x)$!

"prime"

can this be factored further?

$$\boxed{= \underline{(x-5)(x+2)(x+3)}}$$

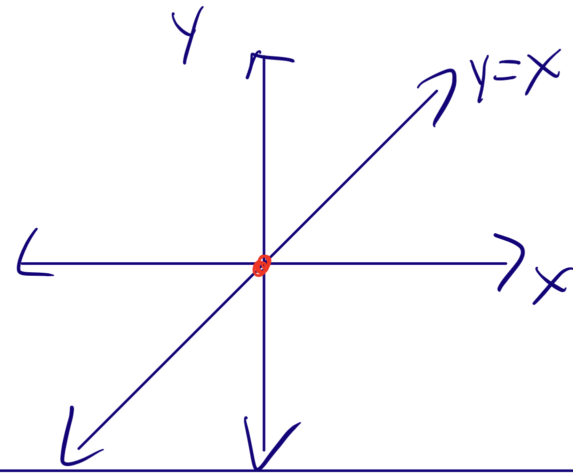
can't be factored any further!

Graphs of polynomials:

Degree 1: $f(x)=x$

Observe: $\boxed{1 \text{ root}}$

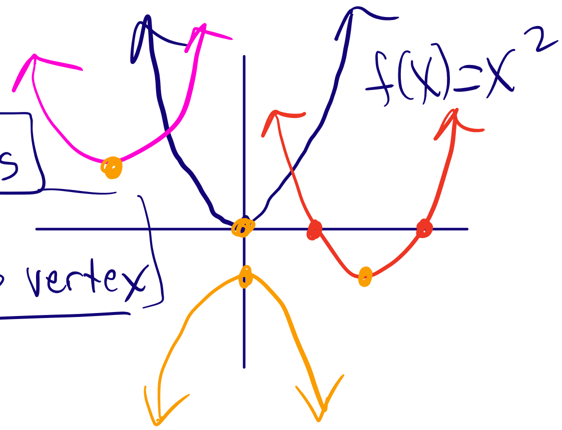
$\boxed{0 \text{ extreme points}}$



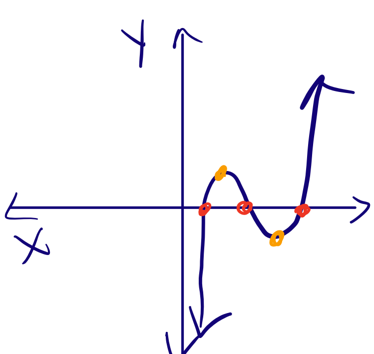
Degree 2:

Observe: $\boxed{0, 1 \text{ or } 2 \text{ roots}}$

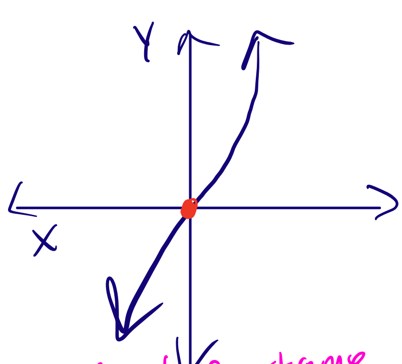
$\boxed{1 \text{ extreme point} \rightarrow \text{vertex}}$



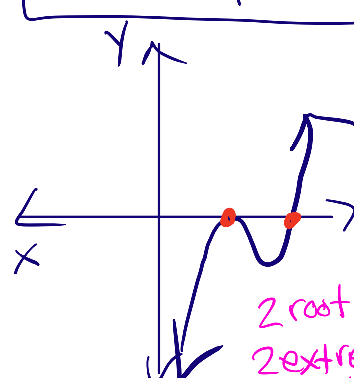
Degree 3: Observe: $\boxed{1, 2, 3 \quad \text{roots}}$, $\boxed{0, 2 \quad \text{extreme points}}$



3 roots, 2 extreme points



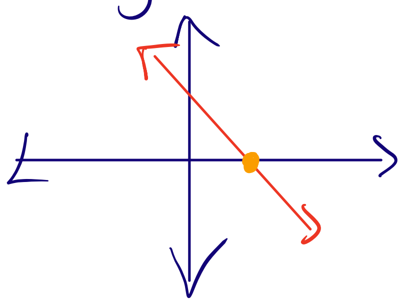
1 root, 0 extreme points



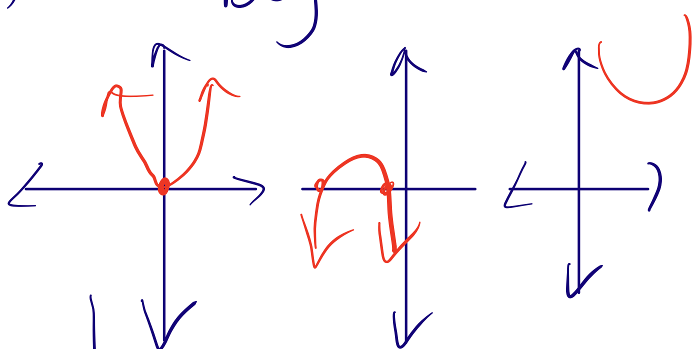
2 roots, 2 extreme points

Now consider: Is there a difference between functions of even degree and odd degree when it comes to the # of roots?

Degree 1 (odd)

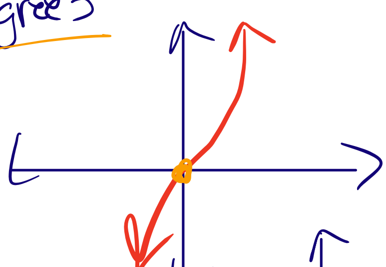


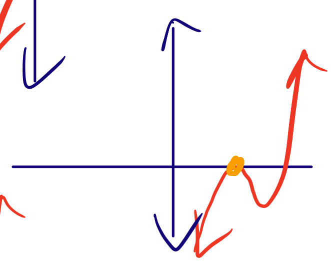
Degree 2 (even)

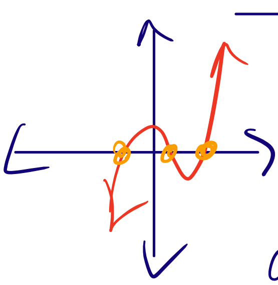


It is possible that there are no roots! Yes!

Degree 3







For odd degree we

We see that there are 2 broad categories:

* Even degree functions
* Odd degree functions

always have
at least 1 root!

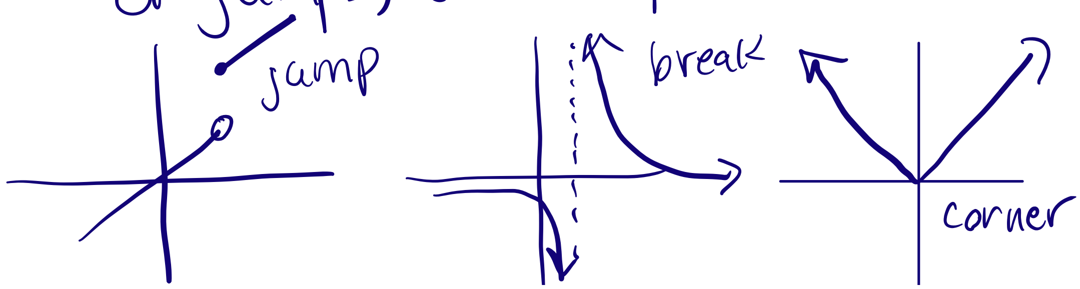
Question: Can an even degree function (polynomial) have an odd # of roots? Yes

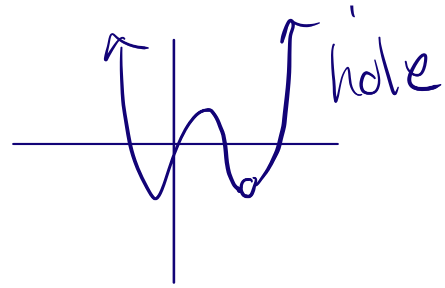
Check! Does the graph of a quartic function (degree 4) ever cross the x-axis in 1 or 3 places? Hmm Look at the graph

Question: Can an odd degree function (polynomial) have an even # of roots? Yes! Look at degree 3+5 graphs!

Notice: Graph of polynomials don't have any discontinuities (holes, breaks or jumps) or sharp corners

Ex




hole