10/27/2021

From last lecture:

Exponential functions:

$$y = \underset{\substack{\uparrow \\ \text{some} \\ \#}}{c} \cdot \underset{\substack{\uparrow \\ \text{base}}}{b}^{x}$$

Logarithmic functions: Inverses of

$$\log_{\underset{\substack{\uparrow \\ \text{base}}}{b}}(x) = y$$

$$\boxed{\log_{\underset{\substack{\uparrow \\ \text{base} \\ '' \\ 10}}{}}(100) = \underset{\substack{\uparrow \\ \text{exponent}}}{2}}$$

("The log is the exponent!")

rewrite this: $\boxed{10^{2} = 100}$

Session 14: Properties of Exponential and logarithmic functions

Recall some properties of exponents:

1) $b^{x+y} = b^x \cdot b^y$

2) $b^{x-y} = \frac{b^x}{b^y}$

3) $(b^m)^n = b^{m \cdot n}$

Properties for logarithms:

1) $\log_b(x \cdot y) = \log_b(x) + \log_b(y)$

2) $\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$

3) $\log_b(x^n) = n \log_b(x)$

Ex Combine the terms using the properties of logarithms so as to write one logarithm

a) $\frac{1}{2}\ln(x) + \ln(y) = \ln(x^{1/2}) + \ln(y)$

(property 3)

$= \boxed{\ln(x^{1/2} \cdot y)}$ (by property 1)

b) $\frac{2}{3}(\underbrace{\log(x^2y)-\log(xy^2)}_{\text{first}})$

$= \frac{2}{3}\left(\log\left(\frac{x^{\cancel{2}}\cancel{y}}{\cancel{x}y^{\cancel{2}}}\right)\right) = \frac{2}{3}\left(\log\left(\frac{x}{y}\right)\right) = \log\left(\left(\frac{x}{y}\right)^{2/3}\right)^{*}$

(from property 2) (by property 3) equivalent $*$

$= \log\left(\sqrt[3]{\left(\frac{x}{y}\right)^2}\right)$

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c) $5+\log_2(a^2-b^2)-\log_2(a-b)$

rewrite This as a logarithm! How?

$5=\log_2(2^5)$ "$\log_2$" and "2^x" undo each other!

$=\underbrace{\log_2(2^5)+\log_2(a^2-b^2)}-\log_2(a-b)$

$\boxed{2^5=32}$

work from left to right (use property 1)

$=\log_2(32\cdot(a^2-b^2))-\log_2(a-b)$ (now use property 2)

$=\log_2\left(\frac{32(a^2-b^2)}{(a-b)}\right)=\log_b\left(\frac{3\cancel{(a-b)}(a+b)}{\cancel{(a-b)}}\right)$

$\boxed{a^2-b^2=(a+b)(a-b)}$

$=\boxed{\log_b(3(a+b))}$

Solving exponential and logarithmic equations:

1) $x=y \iff b^x=b^y$ ← (goes both ways)

2) $x=y \iff \log_b(x)=\log_b(y)$

Ex Solve for x

a) $2^{x+7}=\underset{\uparrow}{32}=2^5 \Rightarrow x+7=5$

$$\boxed{x=-2}$$

Check in original equation $2^{(-2+7)} \overset{?}{=} 32$

$2^5 \overset{\checkmark}{=} 32$

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b) $5^{(3x+1)}=25^{(4x-7)}$ $\qquad 25=5^2$

$5^{(3x+1)}=\left(5^2\right)^{(4x-7)}$ ← by property of exponents multiply the powers

$5^{(3x+1)}=5^{2(4x-7)} \Rightarrow 3x+1=2(4x-7)$

Solve for x!

$3x+1=8x-14$

$-3x \qquad -3x$

$1=5x-14$

$+14 \qquad +14$

Check!

$5^{3\cdot3+1} \overset{?}{=} 25^{4\cdot3-7}$

✓

$15 = 5x$

$\Rightarrow \boxed{x=3}$

c) $\ln(3x-5) = \ln(x-1)$

$\Rightarrow 3x-5 = x-1$ Now solve for x!

$\boxed{x=2}$

d) $\log_2(x+5) = \log_2(x+3) + 4$

$\uparrow$ rewrite $4 = \log_2(2^4)$

$$\log_2(x+5) = \underbrace{\log_2(x+3) + \log_2(2^4)}$$

remember we can rewrite this as a single log! use property 1 of logs.

$$\log_2(x+5) = \log_2((x+3) \cdot 2^4)$$

$$\log_2(x+5) = \log_2(16(x+3))$$

$\Rightarrow x+5 = 16(x+3)$ solve for x! $\boxed{x = -43/15}$

e) $\log_3(x-2) + \log_3(x+6) = 2$

$2 = \log_3(3^2)$

$$\underbrace{\log_3(x-2)+\log_3(x+6)}_{\text{use property of logarithms}}=\log_3(3^2)$$

$$\log_3\left((x-2)(x+6)\right)=\log_3(3^2)$$

$$\Rightarrow (x-2)(x+6)=3^2$$ ← This is a quadratic equation!

$$(x-2)(x+6)=9$$

$$x^2+\underline{6x-2x}-12=9$$

$$x^2+4x-12=9$$

$$\quad\quad -9 \quad -9$$

$$x^2+4x-21=0$$ ← solve for x!

$$(x+7)(x-3)=0$$

$$x+7=0 \qquad x-3=0$$

$$\boxed{x=-7} \qquad \boxed{x=3}$$

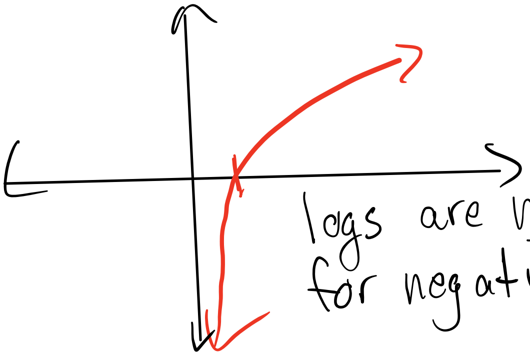
Must check in the original equation!

$$\log_3(x-2)+\log_3(x+6)=2$$

Check: $x=\cancel{-7}$ → $\log_3(-7-2)+\log_3(-7+6)\overset{?}{=}2$

reject

$$\log_3(-9)+\log_3(-1)\overset{?}{=}2$$



logs are not defined for negative values!

Check: $\boxed{x=3}$ Keep!

→ $\log_3(3-2)+\log_3(3+6)\overset{?}{=}2$

$$\log_3(1)+\log_3(9)\overset{?}{=}2$$

$$\log_3(1\cdot 9)\overset{?}{=}2$$

$$\log_3(9)\overset{?}{=}2 \quad ✓$$

→ $3^2 \underset{?}{=} 9$

What if we don't have the same base on each side of the equation?!

Ex Solve

a) $3^{x+5}=8$ here we cannot rewrite the sides to have the same base — we need a different strategy! Apply logarithms to both sides of the equation.

$$\log(3^{x+5}) = \log(8)$$

now apply properties of logs

↑ apply property 3

don't forget that log(3), log(8) are just #'s!

$$(x+5)\log(3) = \log(8)$$

solve for x!

$$x\log(3) + 5\log(3) = \log(8)$$

$$-5\log(3) \qquad -5\log(3)$$

$$\frac{x\cancel{\log(3)}}{\cancel{\log(3)}} = \frac{\log(8) - 5\log(3)}{\log(3)}$$

$$\boxed{x = \frac{\log(8) - 5\log(3)}{\log(3)} \approx -3.107}$$

b) $5^{x-7} = 2^{x}$

here we cannot rewrite either side so we matching bases however we can take the logarithm of both sides! We natural logarithm "ln"

$$\ln(5^{x-7}) = \ln(2^{x})$$

use property 3

ln(5)+ ln(2) are just numerical values!

$$(x-7)\ln(5) = x\ln(2)$$

$$x\ln(5) - 7\ln(5) = x\ln(2)$$

solve for x!

$-x\ln(2)$ $+7\ln(5)$ $\qquad$ $-x\ln(2)$ $+7\ln(5)$

$$x\ln(5) - x\ln(2) = 7\ln(5)$$

$$\frac{x\cancel{(\ln(5)-\ln(2))}}{\cancel{(\ln(5)-\ln(2))}} = \frac{7\ln(5)}{\ln(5)-\ln(2)}$$

$$x = \frac{7\ln(5)}{\ln(5)-\ln(2)} \approx 12.3$$