Exponential and Logarithmic Functions

Def: A function f is called an exponential function if it has the form

$$f(x) = c \cdot b^x$$

where c positive real value and b is called the "base."

Graphs: $f(x) = 2^x$, $g(x) = 3^x$, $h(x) = 10^x$

$k(x) = \left(\frac{1}{2}\right)^x$, $l(x) = \left(\frac{1}{10}\right)^x$

Observations: * The graph of an exponential function has y-intercept $(0,1)$.

* " " " " " "
has a horizontal asymptote at $y=0$ (the x-axis).

Def: The Euler number e is an irrational number that is approximately

$$e \approx 2.718281828\ldots$$

Graph: $f(x) = e^x$

Ex Graph a) $y = 2^x$ b) $y = 3 \cdot 2^x$ c) $y = (-3) \cdot 2^x$

d) $y = (0.2) \cdot 2^x$ e) $y = (-0.2) \cdot 2^x$

Ex Graph a) $y = 3^x - 5$

b) $y = e^{x+4}$

c) $y = \frac{1}{4} e^{x-3} + 2$

Def: Let $0 < b \neq 1$ be a positive real #

For $x > 0$, the logarithm of x with base b is defined by the equivalence

$$y = \log_b(x) \iff b^y = x$$

For a particular base, $b = 10$, we use the short form $\log(x) := \log_{10}(x)$

For the particular base $e = b$ (e = Euler #), we call the logarithm with base e the

"natural logarithm" and write

$$\ln(x) := \log_e(x)$$

The <u>logarithmic function</u> is the function

$y = \log_b(x)$ with Domain

$\{x \in \mathbb{R} \mid x > 0\}$ or $(0, +\infty)$

<u>It</u> is the <u>inverse</u> of the exponential function

$y = b^x$ with base b.

Ex Rewrite the equation as a logarithmic equation:

a) $3^4 = 81$ b) $10^3 = 1000$

c) $e^x = 17$ d) $2^{7a} = 53$

---

a) $\log_3(81) = 4$ b) $\log(1000) = 3$

c) $\ln(17) = x$ d) $\log_2(53) = 7a$

Ex Evaluate the expression by rewriting it as an exponential expression.

a) $\log_2(16) = x \iff 2^x = 16$ $\boxed{x=4}$

b) $\ln(e^7) = x \iff e^x = e^7 \Rightarrow \boxed{x=7}$

Inverses they undeach other!

c) $\log(10{,}000) = x \iff 10^x = 10{,}000 \Rightarrow \boxed{x=4}$

d) $\log_4(4) = x \iff 4^x = 4$ $\boxed{x=1}$

Ex $\log_b(b^x) = x$ $\log_b(b) = 1$ $\log_b(1) = 0$

Ex: a) Graph the function $p(x) = -3\ln(x) + 4$
What is the domain?

b) Graph the function $g(x) = \ln(5-x)$.
What is the domain?

a) Graph on desmos domain: $(0, +\infty)$

b) " " " " : $(-\infty, 5)$