10/20/21

Rational Functions (cont.)

In addition to vertical + horizontal asymptotes, some rational functions have "removable discontinuities" or "holes." This happens when there are common factors in the numerator + denominator which can be cancelled.

$$\text{Ex } f(x) = \frac{\cancel{(x+1)}(x+2)}{\cancel{(x+1)}(x-1)}$$

Session 12

Polynomial + Rational Inequalities

a) $2x+5 \geq 4x-11$

$\quad -4x \qquad -4x$

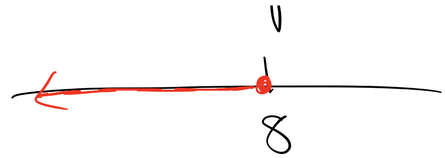
$-2x+5 \geq -11$

$\quad -5 \qquad -5$

$$\frac{-2x}{-2} \geq \frac{-16}{-2}$$

Reminder:

If we multiply or divide both sides of an inequality by a negative value we must flip the direction of the inequality.

$\boxed{x \le 8}$ or  or $(-\infty, 8]$

① ② ③

Ex Solve $-2x-1 \le 3x+4 < 4x-20$

Method: Split into 2 inequalities & solve each piece!

① $-2x-1 \le 3x+4$

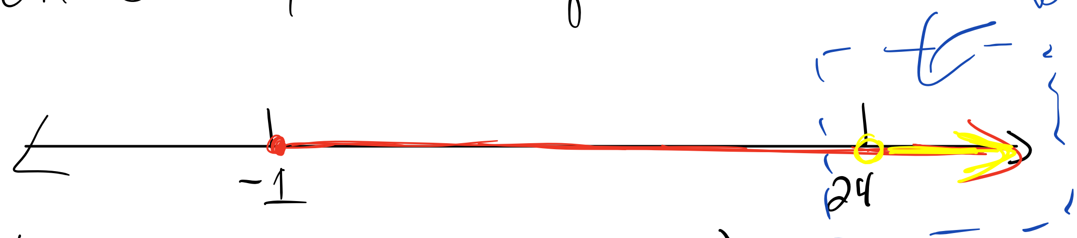
$$\begin{aligned} -2x-1 &\le \cancel{3x}+4 \\ -3x \quad & \quad \cancel{-3x} \\ -5x-\cancel{1} &\le 4 \\ \cancel{+1} \quad & \quad +1 \\ -5x &\le 5 \\ \frac{-5x}{-5} &\ge \frac{5}{-5} \\ \boxed{x \ge -1} \end{aligned}$$

② $3x+4 < 4x-20$

$$\begin{aligned} 3x+4 &< 4\cancel{x}-20 \\ -4x \quad & \quad \cancel{-4x} \\ -x+4 &< -20 \\ -4 \quad & \quad -4 \\ -x &< -24 \\ \frac{-x}{-1} &> \frac{-24}{-1} \\ \boxed{x > 24} \end{aligned}$$

Need to include all the values (interval) which satisfy both inequalities.

satisfy both



Solution: $x > 24$ or $(24, +\infty)$

Now increase the degree!

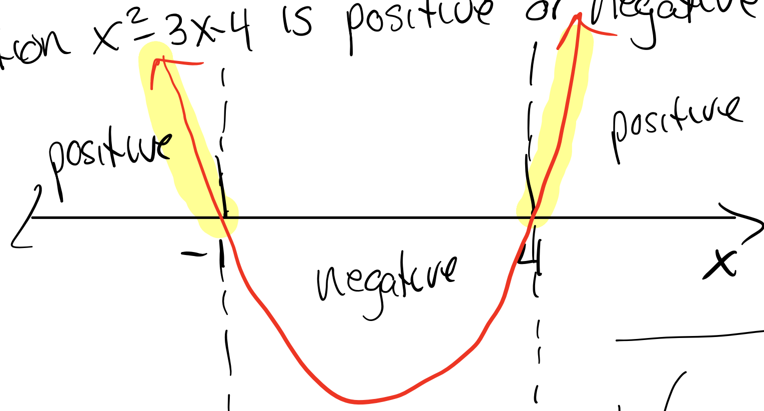
Ex Solve for x

a) $x^2-3x-4 \geq 0$

1) First solve the equality

$$x^2-3x-4=0$$
$$(x-4)(x+1)=0$$
$$\boxed{x=4} \quad \boxed{x=-1}$$

2) Partition the numberline, test whether the quadratic function x^2-3x-4 is positive or negative on each piece.

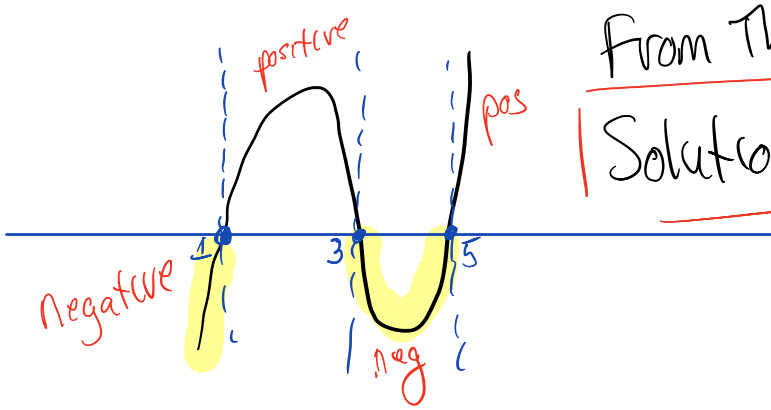


Recall: $x^2-3x-4 \geq 0$ from the graph $\boxed{(-\infty,-1] \cup [4,+\infty)}$

"Where is the graph above or hitting the x-axis?"

b) Solve: $x^3-9x^2+23x-15 \leq 0$ "≤ 0" means negative or $=0$

"Where is the graph below or hitting the x-axis?"



From The graph:

$\boxed{\text{Solution: } (-\infty, 1] \cup [3,5]}$

Ex Solve

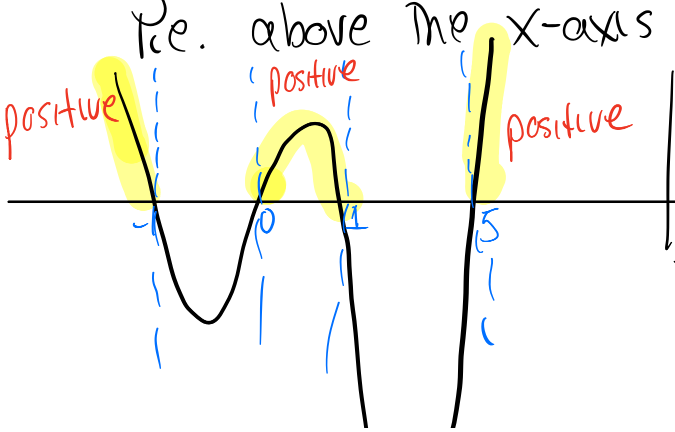
$$x^4 - x^2 > 5(x^3 - x)$$

First Thing: Rewrite This inequality so There is a zero on one side.

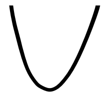
$$x^4 - x^2 > 5x^3 - 5x$$
$$-5x^3 + 5x \qquad -5x^3 + 5x$$

$$\underbrace{x^4 - 5x^3 - x^2 + 5x}_{} > 0$$

Graph The function and look where it is greater Than zero → where it is positive i.e. above The x-axis!



$\boxed{\text{Solution: } (-\infty, -1) \cup (0,1) \cup (5, +\infty)}$



Now use the same strategy to solve rational inequalities!

Ex Solve

$$\frac{x^2-5x+6}{x^2-5x} \geq 0$$

where is the graph positive or zero?

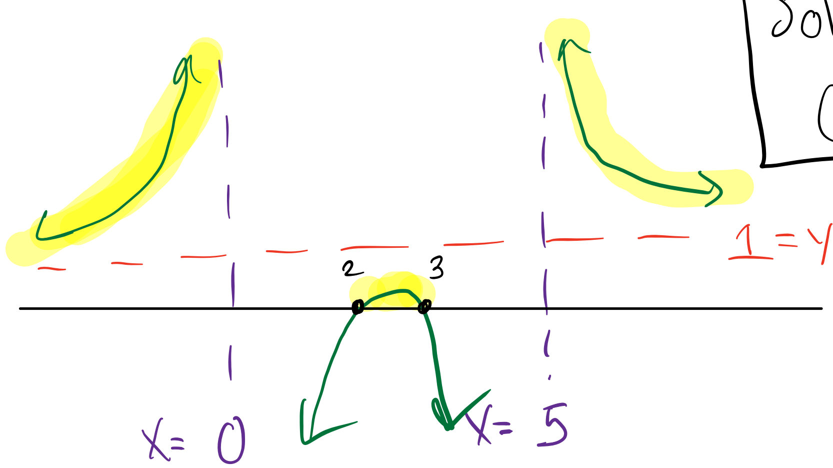
$$\frac{(x-2)(x-3)}{x(x-5)} \geq 0$$

horizontal asymptote at $y=1$

$x=0$ $\quad x=5$ → vertical asymptotes

Solution:

$(-\infty,0) \cup [2,3] \cup (5,\infty)$



Sometimes being able to solve a polynomial (or rational) inequality allows us to solve questions related to the domain of a function.

Ex Find the domain of the given functions

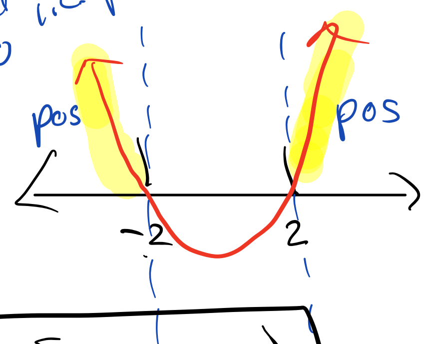
a) $f(x) = \sqrt{x^2-4}$ b) $g(x) = \sqrt{x^3-5x^2+6x}$

a) $f(x) = \sqrt{x^2-4}$

want $x^2-4 \geq 0$ where is x^2-4 greater than or $=0$ i.e positive!

$(x+2)(x-2)=0$

$x=-2 \quad x=2$



$\boxed{\text{domain of } f(x) = (-\infty,-2] \cup [2,+\infty)}$

b) $g(x) = \sqrt{x^3-5x^2+6x}$

$x^3-5x^2+6x \geq 0$ where is this graph above or hitting the x-axis

$\boxed{\text{domain of } g(x) = [0,2] \cup [3,+\infty)}$