9/27/2021

Dividing Polynomials:

Def: A monomial is a #, a variable, or a product of #'s & variables: Ex $2x^2$, 2, y, $-\frac{14}{3}x^2y^4$

A polynomial is a sum or difference of monomials.

Ex $2x + 15cr^2 + 3$

Catch: cannot have a negative or fraction power & still be a polynomial

Ex $2x^{1/2} + 15y^{-1}$ ← not a poly!

Def: A polynomial is a function f of the form:

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \dots + a_1 x + a_0$$

for some real (or complex) numbers

Called "coefficients" → $a_n, \dots, a_0$. The domain of a poly is all real #'s. For each k, the number a_k is the coefficient of x^k.

The number a_n (where n has the highest value) is called the "leading coefficient."

$$2x^5 + x^4 + 16x^3 + 7$$

"leading term"

2 → "leading coefficient

5 → degree

The zeros of a polynomial are referred to as "roots".

Def: A rational function is a fraction of polynomials

$$f(x) = \frac{g(x)}{h(x)} \qquad h(x) \neq 0$$

Ex $\dfrac{x^2+3}{2x^2-7x+4}$

Ex Divide the following fractions:

a) $\frac{3571}{11}$

$$\begin{array}{r} 324 \\ 11\,\overline{)\,3571} \\ -33 \\ \hline 27 \\ -22 \\ \hline 51 \\ -44 \\ \hline 7 \end{array}$$

324 ← quotient

3571 ← dividend

11 — divisor

7 ← remainder

$3571 \div 11 = 324 + \frac{7}{11}$

324 — quotient; 7 — remainder; 11 — divisor

b) $\frac{x^3+5x^2+4x+2}{x+3}$

$$\begin{array}{r} x^2+2x-2 \\ x+3\,\overline{)\,x^3+5x^2+4x+2} \\ -(x^3+3x^2) \\ \hline 2x^2+4x \\ -(2x^2+6x) \\ \hline -2x+2 \\ -(-2x-6) \\ \hline 8 \end{array}$$

x^2+2x-2 ← quotient

8 ← remainder

Side note:

$x+3 = 15$

called a "prime" or irreducible poly

$\frac{x^3+5x^2+4x+2}{x+3} = x^2+2x-2 + \frac{8}{x+3}$

Remainder Theorem: Special case dividing by $(x-c)$

The remainder when dividing $f(x)$ by $(x-c)$ is $f(c)$.

our example

$x+3$

$= x-(-3)$

$c=-3$

$f(x)=x^3+5x^2+4x+2$

Check $f(-3) \stackrel{?}{=} 8$

$f(-3) = (-3)^3 + 5(-3)^2 + 4(-3)+2$

$= -27 + 45 - 12 + 2$

$= -39 + 47 = 8$ ✓

$$\frac{x^2+2}{x^4+x^3+x+7}$$

no x^2 term! Insert "$0x^2$"

set up

$$\begin{array}{r} x^2 \\ x^2+2 \overline{\big) \boxed{x^4} \boxed{+x^3} \boxed{+0x^2} +x+7} \\ -\big(\boxed{x^4} \boxed{+0x^3} \boxed{+2x^2}\big) \\ \hline \end{array}$$

First Desmos Activity slide:

$$\begin{array}{r|l} & \boxed{} \\ \hline x+9 & x^2+11x+18 \end{array}$$

Graph 2 Things: $x^2+11x+18$ (done orange)

$(x+9)\boxed{}$

check graphs are the same

$$\begin{array}{r|l} & \boxed{x+2} \leftarrow \text{quotient} \\ \hline x+9 & x^2+11x+18 \\ & -(x^2+9x) \\ \hline & \quad 2x+18 \\ & -(2x+18) \\ \hline & \qquad 0 \end{array}$$

$$\begin{array}{r|l} & 7x^3-4x^2+6x-9 \\ \hline 3x+8 & 21x^4+44x^3-14x^2+21x-72 \\ & -(21x^4+56x^3) \\ \hline & \quad -12x^3-14x^2 \\ & \quad -(-12x^3-32x^2) \\ \hline & \qquad 18x^2+21x \\ & \qquad -(18x^2+48x) \\ \hline & \qquad\qquad -27x-72 \\ & \qquad\qquad (-27x-72) \end{array}$$