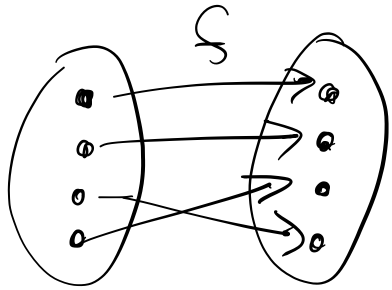
9/22/2021

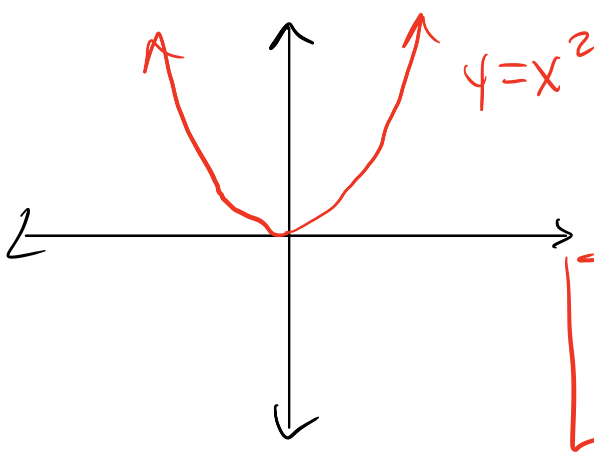
"One-to-one" or "injective"

if <u>two different</u> inputs <u>always</u> have two different outputs.

one-to-one?




yes!




one-to-one?

Two different inputs

$x_1 = 2 \Rightarrow y_1 = 2^2 = 4$

$x_2 = -2 \Rightarrow y_2 = (-2)^2 = 4$

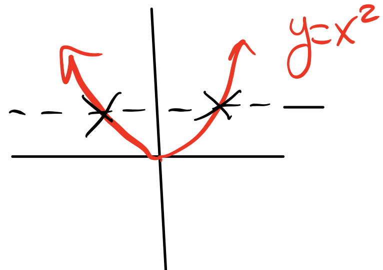
We get the same output!

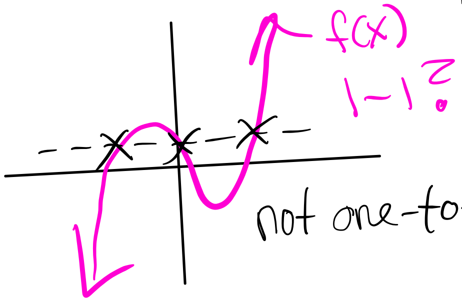
So $y=x^2$ is <u>NOT</u> 1-1

Quick way to test?

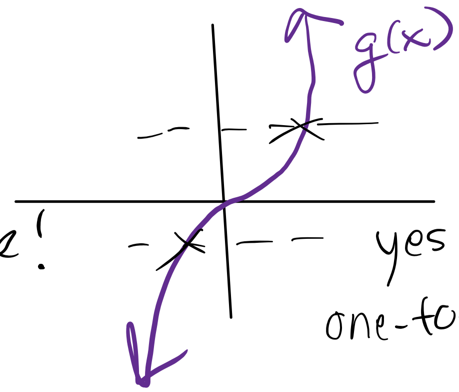
<u>Horizontal line test</u>: If a horizontal

line intersects the graph more than once, then the function is <u>NOT</u> 1-1!

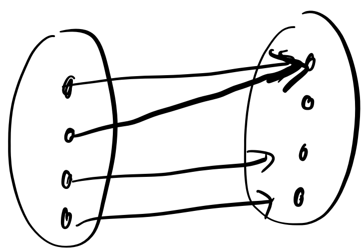



not one-to-one!




yes! one-to-one



<u>not</u> one-to-one
two different inputs go to the same <u>output</u>

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<u>If</u> a function is 1-1, then we can find its <u>inverse</u>. So, for every element in the range which came from an element in the domain, we can identify where it came from.

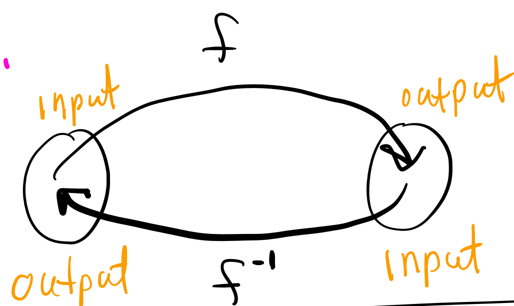
Ex Consider $y=x^2$

4 (output), can I figure out which element it came from? (Input?)

So 4 could have come from $x_1=2$

OR could have come from $x_2=-2$

We can't find the inverse in this case.




Def: Let f be a function with domain $D_f$ and range $R_f$, an assume f is 1-1. The inverse of f is a function $f^{-1}$ so that $f(x)=y$ then $f^{-1}(y)=x$.

The inverse function reverses the roles of the inputs and the outputs.

Ex Find the inverse (via algebra)

a) $f(x) = 2x - 7$

b) $g(x) = \sqrt{x+2}$

c) $h(x) = \frac{1}{x+4}$

d) $j(x) = \frac{x+1}{x+2}$

e) $K(x) = (x-2)^2 + 3$ for $x \geq 2$

Step 1 for all:
swap x and y

Step 2 for all
solve for y

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a) $\boxed{f(x) = 2x - 7}$ rewrite This: $\boxed{y = 2x - 7}$

1) Now swap x and y

$$x = 2y - 7$$

2) Now solve for y

$$\frac{x+7}{2} = \frac{2y}{2}$$

$$\boxed{y = \frac{x+7}{2}}$$ Notation $$\boxed{f^{-1}(x) = \frac{x+7}{2}}$$

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b) $g(x) = \sqrt{x+2}$, i.e. $y = \sqrt{x+2}$

1) Swap x + y

$$x = \sqrt{y+2}$$

2) Solve for y

Square both sides of the equation to get y out from under the radical!

$$x^2 = (\sqrt{y+2})^2 \quad \text{(solving for y)}$$

$$x^2 = y+2$$
$$-2 \qquad -2$$

$$\boxed{y = x^2 - 2} \quad \text{or} \quad \boxed{g^{-1}(x) = x^2 - 2}$$

c) $h(x) = \frac{1}{x+4}$ i.e. $y = \frac{1}{x+4}$

1) swap

$$x = \frac{1}{y+4}$$

2) solve for y: (Hint: cross multiply)

$$\rightarrow x(y+4) = 1$$

$$xy + 4x = 1$$
$$-4x \quad -4x$$

$$\frac{xy}{x} = \frac{1-4x}{x}$$

$$\boxed{y = \frac{1-4x}{x}} = \frac{1}{x} - \frac{4x}{x} = \boxed{\frac{1}{x} - 4}$$

$$\boxed{h^{-1}(x) = \frac{1}{x} - 4}$$

d) $j(x) = \frac{x+1}{x+2}$ find $j^{-1}(x) = ?$

$$y = \frac{x+1}{x+2}$$

swap $x = \frac{y+1}{y+2}$

$$x(y+2) = y+1$$

$$xy + 2x = y+1$$
$$-y \quad -2x \qquad -y \quad -2x$$

move all terms with y onto one side of the equation!

$$xy - y = 1 - 2x$$

factor out the y!

$$\frac{y(x-1)}{(x-1)} = \frac{1-2x}{x-1}$$

$$\boxed{y = \frac{1-2x}{x-1}}$$

$$\boxed{j^{-1}(x) = \frac{1-2x}{x-1}}$$

Only for Richard ☺

$$\left(\frac{-1(1-2x)}{-1(x-1)} = \frac{-1+2x}{-x+1}\right)$$

How can we check if we found the <u>CORRECT</u> inverse?

$$\begin{tikzcd} x \arrow[r, bend left, "f"] & y \arrow[l, bend left, "f^{-1}"] \end{tikzcd}$$

$$f^{-1}(f(x)) = x$$

$$f(f^{-1}(x)) = x$$

f and $f^{-1}$ undo each other so I should come back to the same starting point

Check

a) $f(x) = 2x-7 \qquad f^{-1}(x) = \dfrac{x+7}{2}$

$$f^{-1}(f(x)) = f^{-1}(2x-7) = \frac{(2x-7)+7}{2}$$

$$= \frac{2x\cancel{-7}\cancel{+7}}{2} = \frac{\cancel{2}x}{\cancel{2}} = x$$

start and end

$$\begin{tikzcd} x \arrow[r, bend left, "f"] & 2x-7 \arrow[l, bend left, "f^{-1}"] \end{tikzcd}$$

b) start/end

$$\begin{tikzcd} x \arrow[r, bend left, "f^{-1}"] & \dfrac{x+7}{2} \arrow[l, bend left, "f"] \end{tikzcd}$$

so $f(f^{-1}(x)) = f\left(\frac{x+7}{2}\right) = \cancel{2}\left(\frac{x+7}{\cancel{2}}\right) - 7$

$= x + \cancel{7} - \cancel{7} = \boxed{x}$ ✓