9/20/2021

Operations on functions

Ex Let $f(x) = x^2 + 5x$ and $g(x) = 7x - 3$

Find the following functions + state their domain: $(f+g)(x)$, $(f-g)(x)$, $(f \cdot g)(x)$ + $\left(\frac{f}{g}\right)(x)$

We must make sure that we use values in both domains $\Rightarrow$ domain of f $\cap$ domain of g

$\uparrow$ intersection

This works for $+, -, \cdot, \div$

and just exclude any x-values where $g(x) = 0$ for division $\left(\frac{f(x)}{g(x)} \text{ exclude } x,\ g(x) = 0 \right)$

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a) $(f+g)(x) = f(x) + g(x) = (x^2 + 5x) + (7x - 3)$

$= \boxed{x^2 + 12x - 3}$

domain $f(x) = \mathbb{R}$ (all real #'s)

domain $g(x) = \mathbb{R}$

$\Rightarrow$ domain $(f+g)(x) = \mathbb{R}$

b) $(f-g)(x) = f(x) - g(x) = (x^2+5x) - (7x-3)$

$$= x^2+5x-7x+3$$

$$= \boxed{x^2-2x+3}$$

domain $(f-g)(x) = \mathbb{R}$

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c) $(f \cdot g)(x) = (x^2+5x)\cdot(7x-3) = 7x^3-3x^2+35x^2-15x$

$$= \boxed{7x^3+32x^2-15x}$$

domain $(f \cdot g)(x) = \mathbb{R}$

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d) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \boxed{\frac{x^2+5x}{7x-3}}$ → This is a "Rational Function"

We need to
be careful is there any value x
So that $g(x) = 0$?

$$7x-3=0$$

$x = 3/7$ ← Culprit!

We need to exclude $x = 3/7$ from the domain

How do we say this?

" "

$\text{domain}\left(\frac{f}{g}\right)(x) = \{x \mid x \neq 3/7\}$ "set builder notation"

or

$(-\infty, 3/7) \cup (3/7, +\infty)$ (interval notation $\rightarrow$ WeBWorK)

or

$\mathbb{R} - \{3/7\}$

Now we can also create new functions from old functions by using functions as <u>input</u> into other functions.

input $\quad$ output/input $\quad$ output/input

$$x \longrightarrow \boxed{f} \xrightarrow{f(x)} \boxed{g} \xrightarrow{g(f(x))} \boxed{h} \longrightarrow \boxed{h(g(f(x)))}$$

step 1 $\quad$ step 2 $\quad$ step 3

equivalently

$$\underset{3^{rd}}{h}\,(\underset{2^{nd}}{g}\,(\underset{1^{st}}{f}(x))) = (\underset{3^{rd}}{h} \circ \underset{2^{nd}}{g} \circ \underset{1^{st}}{f})(x)$$

$\boxed{\text{"}\bullet\text{" multiplication}}$

"$\circ$" means composition

$\rightarrow$ feed one function into the next

Ex Let $f(x) = x^2+1$ and $g(x) = x+3$. Find the following:

a) $(f \circ g)(3)$

b) $(g \circ f)(3)$

c) $(f \circ g)(x)$

d) $(g \circ f)(x)$

$f(x) = x^2+1$
$g(x) = x+3$

a) $(f \circ g)(3) = f(g(3))$

f: 2nd, g: 1st

$g(3) = 3+3 = 6$

$= f(6) = 6^2+1 = 37$

$\boxed{(f \circ g)(3) = 37}$

$3 \to \boxed{g} \to 6 \to \boxed{f} \to 37$

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b) $(g \circ f)(3) = g(f(3))$

g: 2nd, f: 1st

$3 \to \boxed{f} \to 10 \to \boxed{g} \to 13$

$= g(10) = 10+3 = 13$

$f(3) = 3^2+1 = 10$

$\boxed{(g \circ f)(3) = 13}$

$f(x) = x^2 + 1$

c) $(f \circ g)(x) = f(g(x))$

2nd (f), 1st (g)

$= f(x+3) = (x+3)^2 + 1$

$= (x+3)(x+3) + 1$

$= x^2 + 6x + 9 + 1$

$= \boxed{x^2 + 6x + 10}$

$x \rightarrow \boxed{g} \rightarrow x+3 \rightarrow \boxed{f} \rightarrow x^2 + 6x + 10$

domain for $(f \circ g)(x) = \mathbb{R}$

d) $(g \circ f)(x) = g(f(x))$

$f(x) = x^2 + 1$

$g(x) = x + 3$

2nd (g), 1st (f)

$= g(x^2+1) = x^2 + 1 + 3$

$= \boxed{x^2 + 4}$

$x \rightarrow \boxed{f} \rightarrow x^2 + 1 \rightarrow \boxed{g} \rightarrow x^2 + 4$

$(g \circ f)(x) = x^2 + 4$ domain $(g \circ f)(x) = \mathbb{R}$

From desmos activity: $f(x) = x^2$

$g(x) = \sqrt{x}$

a) $(f\circ g)(x) = f(g(x)) = f(\sqrt{x}) = (\sqrt{x})^2 = x$

$$x \longrightarrow \boxed{g} \longrightarrow \sqrt{x} \longrightarrow \boxed{f} \longrightarrow x$$

positive because squaring

b) $(g\circ f)(x) = g(f(x)) = g(x^2) = \sqrt{x^2} = \pm x$

positive + negative

$$x \longrightarrow \boxed{f} \longrightarrow x^2 \longrightarrow \boxed{g} \longrightarrow \pm x$$

$(f\circ g\circ h)(x) = f(g(h(x)))$

3rd 2nd 1st

$(g\circ h\circ f)(x) = g(h(f(x)))$

3rd 2nd 1st