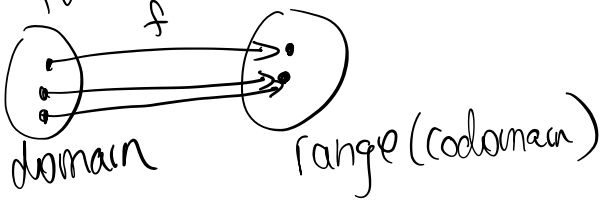
9/1/2021 Functions by formulas + graphs

Function notation

$f(x) = y$ ← output

x ← input

f ← name of the function



Ex For the given function f calculate the outputs $f(2)$, $f(-3)$ and $f(-1)$

a) $f(x) = \sqrt{x^2 - 3}$

$f(2) = \sqrt{2^2 - 3} = \sqrt{4-3} = 1$

$f(-3) = \sqrt{(-3)^2 - 3} = \sqrt{9-3} = \sqrt{6}$

$f(-1) = \sqrt{-2}$ ← not a real #!

b) $f(x) = \begin{cases} 5x-6, & -1 \le x \le 1 \quad (1) \\ x^3 + 2x, & 1 < x \le 5 \quad (2) \end{cases}$

$f(2) = 2^3 + 2 \cdot 2 = 8 + 4 = 12$

$f(-3) =$ undefined! -3 is not in the domain of f

$f(-1) = 5(-1) - 6 = -5 - 6 = -11$

Consider the piecewise function more closely $f(x) = \begin{cases} 5x-6 & \text{for } -1 \le x \le 1 \\ x^3 - 2x & \text{for } 1 < x \le 5 \end{cases}$

domain

Find the domain & range (codomain)

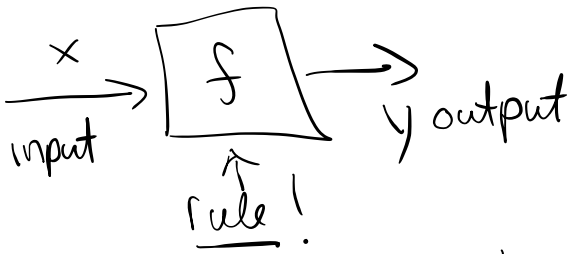
The domain is given → $[-1,1] \cup (1,5]$

For the range, looking at the graph helps!
Looking at the y values

Range = $[-11,-1] \cup (1,\infty)$

↑ always use a curved bracket for ∞ its symbolic no one knows its value!

Now take this to the next level!
Not just numerical input but algebraic input!



Ex Let f be the function given by

$$f(x) = x^2 + 2x - 3$$

Find the following function values

a) $f(\sqrt{5})$	d) $f(a)+5$
b) $f(\sqrt{3}+1)$	e) $f(x+h)$
c) $f(a)$	f) $f(x+h)-f(x)$
	g) $\frac{f(x+h)-f(x)}{h}$

Recall $f(x)=x^2+2x-3$ (every where we see an "x" replace it by "$\sqrt{5}$")

a) $f(\sqrt{5})=(\sqrt{5})^2+2\sqrt{5}-3$

$=5+2\sqrt{5}-3=\boxed{2+2\sqrt{5}}$

$\left(f(\blacksquare)=\blacksquare^2+2\blacksquare-3\right)$

b) $f(\sqrt{3}+1)=(\sqrt{3}+1)^2+2(\sqrt{3}+1)-3$

$=(\sqrt{3}+1)(\sqrt{3}+1)+2(\sqrt{3}+1)-3$

$=\sqrt{3}\cdot\sqrt{3}+1\cdot\sqrt{3}+1\cdot\sqrt{3}+1+2\sqrt{3}+2-3$

$=3+2\sqrt{3}+1+2\sqrt{3}-1$

$=\boxed{3+4\sqrt{3}}$

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c) $f(a)=a^2+2\cdot a-3$

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d) $f(a)+5 \neq f(a+5)$ Be careful those are not the same!

$f(a)+5=a^2+2a-3+5$

$=\boxed{a^2+2a+2}$

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e) $f(x+h)=(x+h)^2+2(x+h)-3$

$=(x+h)(x+h)+2(x+h)-3$

$\vdots$

$=\boxed{x^2+2xh+h^2+2x+2h-3}=f(x+h)$

$f(\blacksquare)=(\blacksquare)^2+2(\blacksquare)-3$

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f) $f(x+h)-f(x)$

$=x^2+2xh+h^2+2x+2h-3-(x^2+2x-3)$

$x^2+2xh+h^2+2x+2h-3-x^2-2x+3$

$=\boxed{2xh+h^2+2h}=f(x+h)-f(x)$

g) $\frac{f(x+h)-f(x)}{h}$) This is "The Difference Quotient"

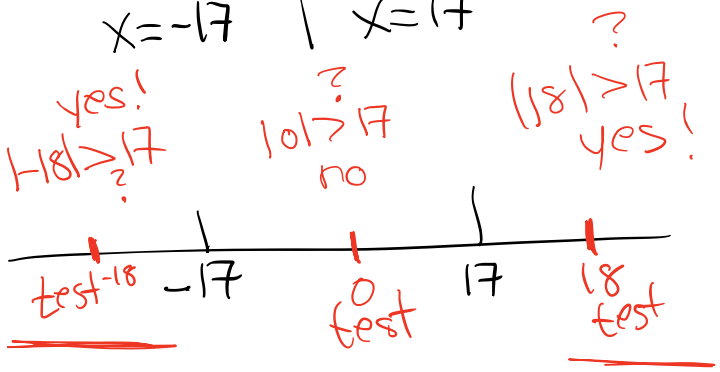
$= \frac{2xh+h^2+2h}{h} = \frac{h(2x+h+2)}{h} = \boxed{2x+h+2}$

Ex WeBWork Inequalities #5

c) $|x|>17$

$|x|=17$

$x=-17 \quad | \quad x=17$



$\boxed{(-\infty,-17)\cup(17,+\infty)}$