**Problems from Lecture 7:**

1. Consider an experiment consists of drawing 5 cards from 52-card deck. Compute

the probability of following hands:

1. 5 face cards.
2. 4 aces.
3. Royal flush (10, J, Q, K, A all in one suit).
4. 2 aces and 3 queens.
5. 2 kings and 3 aces.
6. There are 12 face cards in a 52 card deck.

First compute the total # of 5 card hands: Since order DOESN’T matter here we use combinations to count: C(52,5)

Next compute the # of ways a 5 card hand can have only face cards: C(12,5)

Now form the probability: **C(12,5)/C(52,5)=792/2,595,960**

1. Our denominator remains the same: C(52,5) but now our numerator is different. How many 5 card hands have 4 aces?

Since there are only 4 aces we must choose all four of them to be in our hand:

C(4,4)\*C(48,1) (this is our numerator)

Probability of having a 5 card hand with 4 aces:

**C(4,4)\*C(48,1)/C(52,5) =48/2,595,960**

1. For a Royal Flush (10,J,K,Q,A ALL THE SAME SUIT): This can only happen 4 ways since there are only 4 suits. That is our numerator.

Probability of getting a royal flush: **4/C(52,5)=4/2,595,960**

1. For 2 aces and three queens: Probability **= C(4,2)\*C(4,3)/C(52,5) = 24/2,595,960**
2. For 2 kings and 3 aces: Prob.= **24/2,595,960**
3. From a standard 52-card deck, what is the probability of 5-card hand having at least one face card?

“at least one” means 1, 2, 3, 4 or 5. Instead let’s find the probability of having 0 face cards. The complement of this is our solution.

Probability of a 5 card hand having 0 face cards: C(40,5)/C(52,5)

Probability of a 5 card hand having at least one face card is 1- C(40,5)/C(52,5)=

=**1- (658008/2,595,960) approximately 75% chance**

1. A sports club has 120 members, of whom 44 play tennis, 30 play squash, and 18 play both tennis and squash. If a member is chosen at random, find the probability that this person.
2. Does not play tennis.
3. Does not play squash.
4. Plays neither tennis nor squash.
5. A shipment of 55 precision parts, including 12 that are defective, is sent to an assembly plant. The quality control division selects 10 at random for testing and rejects the entire shipment if one or more in the sample are found defective. What is the probability that the shipment will be rejected?

Total # parts 55

12 defective

43 are not defective

Probability that one or more of 10 parts is defective = 1- Probability that 0 of ten are defective.

=1-C(43,10)/C(55,10)=1-(1917334783/ 29248649430) = approximately 93.4%

1. A shipment of 55 precision parts, including 12 that are defective, is sent to an assembly plant. The quality control division selects 10 at random for testing and rejects the entire shipment if one or more in the sample are found defective. A shipment gets rejected only if 8 or more parts are defective. What is the probability that the shipment gets rejected?

Under what circumstances will a shipment get rejected? If there are 8 or 9 or 10 defective parts. We must compute the three separate probabilities and SUM them.

Total # parts 55

12 defective

43 are not defective

P(8 defective, 2 non-defective) + P(9 def., 1 non-def.) + P(10 def., 0 non-def.) =

**(C(12,8)\*C(43,2) + C(12,9)\*C(43,1)+C(12,10)\*C(43,0) )/C(55,10)**

**Definitions**

The probability of the occurrence of an event *A*, given the occurrence of another event *B*, is called a ***conditional probability*** and is denoted by $P(B|A)$

For example:

*B*=adult has lung cancer

*A*=adult is a heavy smoker

Then $P(B|A)$ represents the probability of an adult having lung cancer, given that he/she is a heavy smoker.

**Motivational Example:**

Consider rolling a fair die.

1. What is the probability of rolling a prime number?
2. What is the probability that a prime number has turned up if we are given the additional information that an odd number has turned up?

4

1

6

3

5

2

For events *A* and *B* in an arbitrary sample space *S*, we define the conditional probability of *B* given *A* by

$$P\left(A\right)=\frac{P(A∩B)}{P(A)} ;where P(A)\ne 0$$

Note that since we know that event *B* has occurred, it becomes our new sample space shown in figure below.



We obtain the following multiplication rule from conditional probability:

$$P\left(A∩B\right)=P(B)P\left(A\right)$$

Independent Events:

Two events A and B in a sample space S are said to be independent if and only if

$P\left(A∩B\right)=P(A)P(B)$.

If $P\left(A\right)=P(B)$, then the probability of *B* is not effected by occurrence of *A*.

**Examples:**

1. One of two urns is chosen at random with one as likely to be chosen as the other. Then a ball is withdrawn from the chosen urn. Urn 1 contains 1 white and 4 red balls, and urn 2 has 3 white and 2 red balls. If a white ball is drawn, what is the probability that it came from urn 2? (Hint: Draw a tree diagram)
2. A card is drawn at random from a standard 52-card deck. Events *A* and *B* are:

 *A* = the drawn card is a club.

 *B* = the drawn card is even (face cards are not valued).

1. Find $P(B|A)$.
2. Test *A* and *B* for independence.
3. Use the given tree diagram to answer the following questions.

 0.2 A

 M

 0.6 0.8 B

 0.4 0.7 A

 N

 0.3 B

(a) What is $P(A|M)$? (b) What is $P(A)$? (c) What is$ P(M|A)$?

**In-class Activity #4: Let’s make a deal!**

In the popular game shot “Let’s Make a Deal,” contestants are shown three curtains. Behind one door is a high-value prize, for example a car, and behind the other two are less desirable prizes, for example, goats. The contestant picks one of the three doors. Then the host (who knows what is behind each door) opens another door to reveal a goat. The host then asks the contestant whether he or she want to stay or switch to the other curtain before revealing what is behind the contestant’s chose door.

There are three possibilities: The contestant should stay; The contestant should switch; It doesn’t matter what is behind the contestant’s chosen curtain.

Form groups of three and choose roles: host, contestant, and recorder. The host lays the cards face down. The contestant chooses a card. The host then turns over a “goat” (but not the one just selected). The contestant now must choose a strategy: stay or switch. After that decision is made, the host reveals whether the contestant won a goat or a car. Repeat this several times, giving the contestant several opportunities to try both strategies.

For each trial, the recorder will put a tally mark in one of **four** **categories:** Stayed and Won; Stayed and Lost; Switched and Won; Switched and Lost.

After a few games, change goals so that everyone gets to try all roles. We will compile the results at the end of class.

**Before the activity:** **Which strategy do you think will be better: staying or switching? Or does it make no difference? Why?**

**After the activity:** **Judging on the basis of your class data, what is the empirical probability of winning if the contestant stays? What is it if the contestant switches? Do these probabilities convince you to change your strategy? Explain.**