Prof. Mingla **MAT1375-F22-Mingla-W058**

Assignment Absolute_Value_Inequalities due 09/14/2022 at 12:00pm EDT

Problem 1. **(1 point)** `CUNY/CityTech/Precalculus/setAbsValInequality/absValue-intro.pg`

Absolute Values

The "absolute value" of a number refers to the number's magnitude.

For example, the magnitude of -10 is 10, the same as the magnitude of 10 is also 10.

Symbolically, we'd say $|-10| = 10$, as well as $|10| = 10$.

Practice

Compute the following absolute values:

a. $|6| =$ ______

b. $|-8| =$ ______

c. $|19| =$ ______

d. $|22| =$ ______

e. $|-27| =$ ______

Hint: *(Instructor hint preview: show the student hint after the following number of attempts: 2)*

- The absolute value of **any** number different from 0 is always positive.

Correct Answers:

- 6
- 8
- 19
- 22
- 27

Problem 2. **(1 point)** `CUNY/CityTech/Precalculus/setAbsValInequality/absValue-equation-intro.pg`

Absolute Value Equations

We know that absolute values always result in zero or a positive number, regardless of whether we're finding the absolute value of a positive or negative number or zero in the first place. As in both: $|-3| = 3$ and $|3| = 3$.

This means that when solving an equation with absolute values, there may be multiple possible solutions.

For instance, if we know that the absolute value of an unknown number ends up being 3:

$$|x| = 3 \text{ has two solutions: } \quad x = 3 \text{ and } x = -3.$$

But since absolute values always result in zero or a positive number, we may have no solutions:

$$|x| = -3 \text{ has no solutions (because the absolute value is never negative).}$$

Practice

Find solutions to the following absolute value equations:

- If the equation has multiple solutions, enter them as a list, separated by commas.
- If the equation has no solutions, enter "no solution" as your answer. To enter "no solution": Step 1: type a quotation mark ("). Step 2: type the words: no solution.

a. $|x| = 8$ has solutions: $x =$ ______

b. $|x| = -6$ has solutions: $x =$ ______

c. $|x| = -17$ has solutions: $x =$ ______

d. $|x| = 29$ has solutions: $x =$ ______

e. $|x| = -57$ has solutions: $x =$ ______

Hint: *(Instructor hint preview: show the student hint after the following number of attempts: 2)*

- Is the absolute value of your unknown number **negative**, **zero** or **positive**?

- The absolute of **any** number is never negative.

- There is **only one** number whose absolute value is 0. What is it?

- **Two numbers** can have the same positive absolute value.

Correct Answers:

- $8, -8$
- no solution
- no solution
- $29, -29$
- no solution

Problem 3. **(1 point)** `CUNY/CityTech/Precalculus/setAbsValInequality/absValue-equation-monic-linear.pg`

Absolute Value Equations

We recognize the following situations:

$|x| = 7$ has two solutions: $x = 7$ and $x = -7$.

$|x| = -7$ has no solutions (because the absolute value is always zero or positive, and cannot be equal to a negative value).

But what if we have an absolute value of an expression, rather than just a variable? For example, what about $|x-3| = 7$?

Here we're considering the absolute value of $x-3$, so if $x-3$ has an absolute value of 7, then $x-3$ must be either 7 or -7.

Symbolically, either $x-3 = 7$ or $x-3 = -7$.

We solve each of these equations individually to conclude that either $x = 10$ or $x = -4$.

And in the case of being equal to a negative value, as in $|x-3| = -7$, we still have no possible solutions, as $|x-3|$ can never be equal to a negative value.

Practice

Solve the equations below.

- If the equation has multiple solutions, enter them as a list, separated by commas.
- If the equation has no solutions, enter "no solution" as your answer.

a. $|x+5| = 5$ has solutions: $x =$ ______

b. $|x+6| = -6$ has solutions: $x =$ ______

c. $|x-12| = -11$ has solutions: $x =$ ______

d. $|x-20| = 21$ has solutions: $x =$ ______

e. $|x+20| = 67$ has solutions: $x =$ ______

Hint: *(Instructor hint preview: show the student hint after the following number of attempts: 2)*

- Is the absolute value of your expression **negative**, **zero** or **positive**?

- The absolute value of **any** number is never negative.

- There is only one number whose absolute value is **0**. What is it? Set the expression inside the absolute value to be equal to this number. Then solve the equation.

- If **positive**, then there are two numbers, one positive and one negative, with that absolute value. Set up two equations: the expression inside the absolute value equal to the positive number, and that expression equal to the negative number. Solve each equation separately.

Correct Answers:

- $0, -10$
- no solution
- no solution
- $41, -1$
- $47, -87$

Problem 4. **(1 point)** `CUNY/CityTech/Precalculus/setAbsValInequality/absValue-equation-nonmonic-linear.pg`

Absolute Value Equations

We recognize the following situations:

$|x| = 9$ has two solutions: $x = 9$ and $x = -9$

$|x| = -9$ has no solutions (because the absolute value is always zero or positive, and cannot be equal to a negative value).

But what if we have an absolute value of an expression, rather than just a variable? For example, what about $|2x-5| = 9$?

Here we're considering the absolute value of $2x-5$, so if $2x-5$ has an absolute value of 9, then $2x-5$ must be either 9 or -9.

Symbolically, either $2x-5 = 9$ or $2x-5 = -9$.

We solve each of these equations individually to conclude that either $x = 7$ or $x = -2$.

And in the case of being equal to a negative value, as in $|2x-5| = -9$, we still have no possible solutions, as $|2x-5|$ must be zero or positive, and cannot be equal to a negative value.

Practice

Solve the equations below.

- If the equation has multiple solutions, enter them as a list, separated by commas.
- If the equation has no solutions, enter "no solution" as your answer.

a. $|6x-5| = 6$ has solutions: $x =$ ______

b. $|-(7x+8)| = 8$ has solutions: $x =$ ______

c. $|15-7x| = 16$ has solutions: $x =$ ______

d. $|3x+8| = -25$ has solutions: $x =$ ______

e. $|6x-10| = 97$ has solutions: $x =$ ______

Hint: *(Instructor hint preview: show the student hint after the following number of attempts: 2)*

- Is the absolute value of your expression **negative**, **zero** or **positive**?

- If **negative**, then that's not possible. The absolute value of **any** number is never negative.

- If **zero**, then there is only one number whose absolute value is 0. This number is 0. Set the expression inside the absolute value to be equal to 0. Then solve this equation.

- If **positive**, then there are two numbers, one positive and one negative, with that absolute value. Set up two equations: the expression inside the absolute value equal to the given positive number, and that expression equal to the negative of that number. Solve each equation separately.

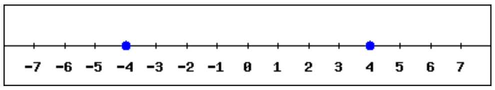
Correct Answers:

- $-\frac{1}{6}, \frac{11}{6}$
- $0, -\frac{16}{7}$
- $\frac{31}{7}, -\frac{1}{7}$
- no solution
- $-\frac{29}{2}, \frac{107}{6}$

Problem 5. **(1 point)** `CUNY/CityTech/Precalculus/setAbsValInequality/absValue-inequality-intro.pg`

Absolute Value Inequalities

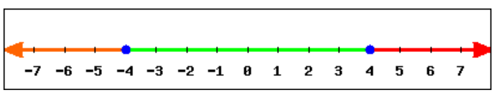
We recognize that the solutions to $|x| = 4$ are $x = -4$ and $x = 4$.



But what if we have an inequality instead of equality? For example, what about $|x| < 4$?

Notice that the points $x = -4$ and $x = 4$ divide the number line into three subintervals:

- $x < -4$ (in orange),
- $-4 < x < 4$ (in green),
- $4 < x$ (in red).



Let's pick a number in each subinterval and check the inequality $|x| < 4$ for it.

Check 1: $x = -5$ (in the orange subinterval)

$$|x| \text{ evaluated at } x = -5 \;\rightarrow\; |x| = |-5| = 5 > 4 \;\rightarrow\; \text{False}$$

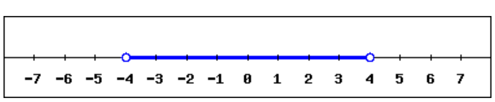
Check 2: $x = 1$ (in the green subinterval)

$$|x| \text{ evaluated at } x = 1 \;\rightarrow\; |x| = |1| = 1 < 4 \;\rightarrow\; \text{True}$$

Check 3: $x = 6$ (in the red subinterval)

$$|x| \text{ evaluated at } x = 6 \;\rightarrow\; |x| = |6| = 6 > 4 \;\rightarrow\; \text{False}$$

The only value that works is the one in the subinterval $-4 < x < 4$ or $(-4, 4)$. We now need to check whether the endpoints of this interval satisfy the inequality $|x| < 4$. Since the inequality is *strict*, the endpoints do not satisfy the inequality equation. Therefore, the solution set to $|x| < 4$ is $(-4, 4)$.



Practice

State your answer in **interval notation**.

- You can use "inf" and "-inf" for ∞ and $-\infty$.
- Use the *union* operator "U" to describe intervals with separate regions.
- If the inequality is true for all real numbers, use "R" to represent "all real numbers".
- If the inequality is never true, use "`{}`" or "EmptySet" (case-sensitive) to represent "no solutions".

a. $|x| > 7$ is true on the interval __________

b. $|x| < 7$ is true on the interval __________

c. $|x| > 19$ is true on the interval __________

d. $|x| > 26$ is true on the interval __________

e. $|x| \leq 69$ is true on the interval __________

Hint: *(Instructor hint preview: show the student hint after the following number of attempts: 2)*

- Replace your inequality symbol by $=$, keep the absolute value, and solve the equation.
- Graph your solutions on the number line, and identify the key subintervals.
- Choose a number in each subinterval and check to see whether or not it satisfies the original equation with the inequality. If so, the subinterval containing the number is part of the solution set.
- Check whether or not the endpoints satisfy the original equation with the inequality.
- Write your solution set using the interval notation.

Correct Answers:

- $(-\infty, -7) \cup (7, \infty)$
- $(-7, 7)$
- $(-\infty, -19) \cup (19, \infty)$
- $(-\infty, -26) \cup (26, \infty)$
- $[-69, 69]$

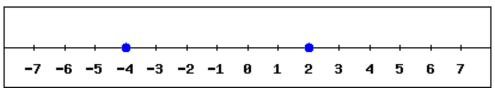
Problem 6. **(1 point)** CUNY/CityTech/Precalculus/setAbsValInequality/absValue-inequality-monic.pg

Absolute Value Inequalities

The equation $|x+1| = 3$ implies that

$$x+1 = -3 \quad \text{or} \quad x+1 = 3.$$

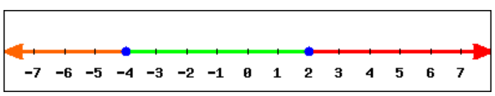
Solving each equation gives that $x = -4$ or $x = 2$.



But what if we have an inequality instead of equality? For example, what about $|x+1| \geq 3$?

Notice that the points $x = -4$ and $x = 2$ divide the number line into three subintervals:

- $x < -4$ (in orange),
- $-4 < x < 2$ (in green),
- $2 < x$ (in red).



Let's pick a number in each subinterval and check the inequality $|x+1| \geq 3$ for it.

Check 1: $x = -6$ (in the orange subinterval)

$$\begin{array}{c} |x+1| \text{ evaluated at} \\ x=-6 \ \rightarrow \ |x+1| = |-6+1| = |-5| = 5 \geq 3 \ \rightarrow \ \text{True} \end{array}$$

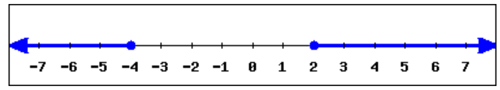
Check 2: $x = -1$ (in the green subinterval)

$$\begin{array}{c} |x+1| \text{ evaluated at} \\ x=-1 \ \rightarrow \ |x+1| = |-1+1| = |0| = 0 < 3 \ \rightarrow \ \text{False} \end{array}$$

Check 3: $x = 4$ (in the red subinterval)

$$\begin{array}{c} |x+1| \text{ evaluated at} \\ x=4 \ \rightarrow \ |x+1| = |4+1| = |5| = 5 \geq 3 \ \rightarrow \ \text{True} \end{array}$$

The values that work are -6 and 4 which are in the subintervals $x < -4$ and $2 < x$. This means that all numbers in the union $(-\infty,-4) \cup (2,\infty)$ satisfy our inequality. We now need to check whether the endpoints of these intervals satisfy the inequality $|x+1| \geq 3$. Since the inequality is *not strict*, the endpoints do satisfy our equation. So we need to include the endpoints in our solution. The solution set to $|x+1| \geq 3$ is $(-\infty,-4] \cup [2,\infty)$.



Practice

Solve the inequalities below.

- State your answer in **interval notation**.
- You can use "inf" and "-inf" for ∞ and $-\infty$.
- Use the *union* operator "U" to describe intervals with separate regions.
- If the inequality is true for all real numbers, use "R" to represent "all real numbers".
- If the inequality is never true, use "{ }" or "EmptySet" (case-sensitive) to represent "no solutions".

a. $|x+5| \geq 5$ is true on the interval ______

b. $|x+10| \leq 8$ is true on the interval ______

c. $|x+6| > 17$ is true on the interval ______

d. $|x-12| \leq 26$ is true on the interval ______

e. $|x+5| < 83$ is true on the interval ______

Hint: *(Instructor hint preview: show the student hint after the following number of attempts: 2)*

- Replace your inequality symbol by $=$, keep the absolute value, and solve the equation.
- Graph your solutions on the number line, and identify the key subintervals.
- Choose a number in each subinterval and check to see whether or not it satisfies the original equation with the inequality. If so, the subinterval containing the number is part of the solution set.
- Check whether or not the endpoints satisfy the original equation with the inequality.
- Write your solution set using the interval notation.

Correct Answers:

- $(-\infty,-10] \cup [0,\infty)$

- $[-18, -2]$
- $(-\infty, -23) \cup (11, \infty)$
- $[-14, 38]$
- $(-88, 78)$

Problem 7. **(1 point)** `CUNY/CityTech/Precalculus/setAbsValInequality/absValue-inequality-nonmonic.pg`

Absolute Value Inequalities

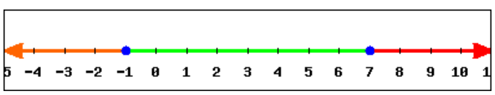
Let's solve $|2x-6|<8$. We first consider $|2x-6|=8$. This means that

$$2x-6=-8 \quad \text{or} \quad 2x-6=8.$$

Solving each equation gives that $x=-1$ or $x=7$.

Notice that the points $x=-1$ and $x=7$ divide the number line into three subintervals:

- $x<-1$ (in orange),
- $-1<x<7$ (in green),
- $7<x$ (in red).



Let's pick a number in each subinterval and check the inequality $|2x-6|<8$ for it.

Check 1: $x=-2$ (in the orange subinterval)

$$|2x-6| \text{ evaluated at}$$
$$x=-2 \;\rightarrow\; |2x-6|=|2(-2)-6|=|-10|=10>8 \;\rightarrow\; \text{False}$$

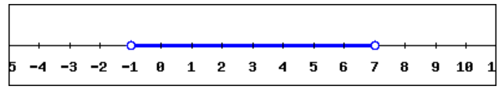
Check 2: $x=2$ (in the green subinterval)

$$|2x-6| \text{ evaluated at}$$
$$x=2 \;\rightarrow\; |2x-6|=|2(2)-6|=|-2|=2<8 \;\rightarrow\; \text{True}$$

Check 3: $x=9$ (in the red subinterval)

$$|2x-6| \text{ evaluated at}$$
$$x=9 \;\rightarrow\; |2x-6|=|2(9)-6|=|12|=12>8 \;\rightarrow\; \text{False}$$

The only value that works is 2, which is in the subinterval $-1<x<7$ or $(-1,7)$. We now need to check whether the endpoints of this interval satisfy $|2x-6|<8$. Since the inequality is *strict*, the endpoints do not satisfy our equation. We conclude that the solution set to $|2x-6|<8$ is $(-1,7)$.



Practice

Solve the inequalities below.

- State your answer in **interval notation**.
- You can use "inf" and "-inf" for ∞ and $-\infty$.
- Use the *union* operator "U" to describe intervals with separate regions.
- If the inequality is true for all real numbers, use "R" to represent "all real numbers".
- If the inequality is never true, use "{ }" or "EmptySet" (case-sensitive) to represent "no solutions".

a. $|5x+5|\geq 5$ is true on the interval ______

b. $|-3x-8|<7$ is true on the interval ______

c. $|3x-9|\geq 17$ is true on the interval ______

d. $|4-3x|\geq 22$ is true on the interval ______

e. $|-4x-20|>73$ is true on the interval ______

Hint: *(Instructor hint preview: show the student hint after the following number of attempts: 2)*

- Replace your inequality symbol by $=$, keep the absolute value, and solve the equation.
- Graph your solutions on the number line, and identify the key subintervals.
- Choose a number in each subinterval and check to see whether or not it satisfies the original equation with the inequality. If so, the subinterval containing the number is part of the solution set.
- Check whether or not the endpoints satisfy the original equation with the inequality.
- Write your solution set using the interval notation.

Correct Answers:

- $(-\infty,-2]\cup[0,\infty)$
- $\left(-5,\dfrac{-1}{3}\right)$
- $\left(-\infty,\dfrac{-8}{3}\right]\cup\left[\dfrac{26}{3},\infty\right)$
- $(-\infty,-6]\cup\left[\dfrac{26}{3},\infty\right)$

- $\left(-\infty, \frac{-93}{4}\right) \cup \left(\frac{53}{4}, \infty\right)$