NYC College of Technology, CUNY — Mathematics Department/Fall 2015

Midterm Exam — Name: ______________________.

1) Evaluate the following limits if they exist:

a) $\lim_{x\to 1} \frac{x^2+x-2}{x^2-x}$

b) $\lim_{x\to 0} \frac{\sqrt{x^2+100}-10}{x^2}$

c) $\lim_{\theta\to 0} \frac{\sin\theta}{\sin 2\theta}$

d) $\lim_{x\to \infty} \frac{\sqrt{x^2+1}}{x+1}$

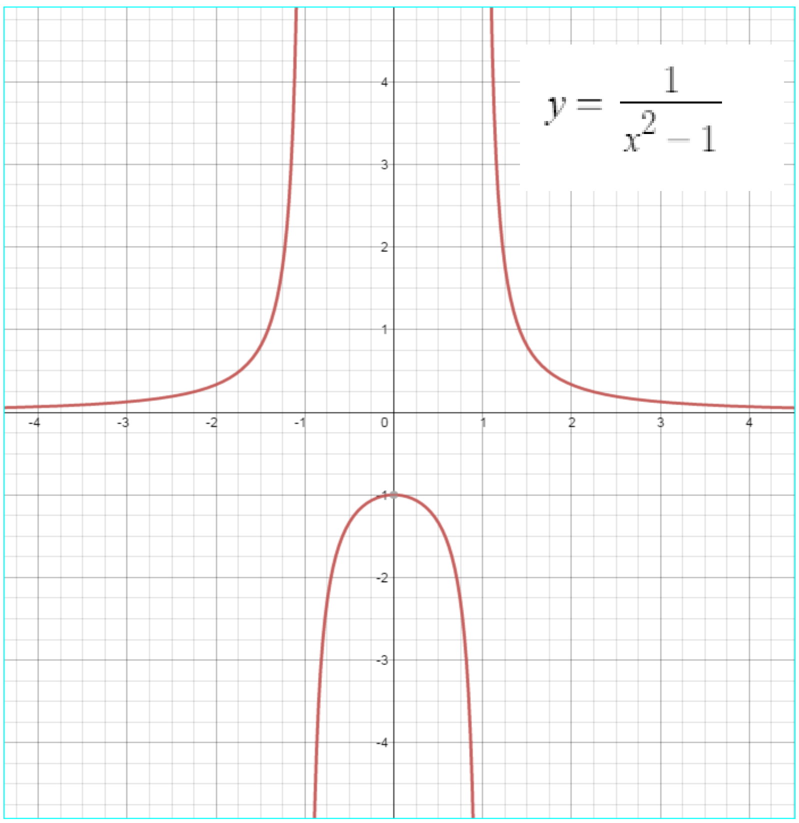
e) $\lim_{x\to 3}(sec^2(x) - tan^2(x))$

f) $\lim_{x\to 0} \frac{e^{-6x}-e^{3x}}{-6x}$

2) The graph of the function: $f(x) = \frac{1}{x^2-1}$ is given below.

Determine graphically and analytically:
* Horizontal and vertical asymptotes
* The domain and the range of this function
* $\lim_{x\to -1} f(x)$
* $\lim_{x\to 1} f(x)$
* Whether this function is continuous. If yes state the interval(s) where the function is continuous.

Based on the theorems that you know justify your answers.



3) Approximate analitically and graphycally: (sketch the graph in the open inteval $(0,\pi)$)

$\lim_{x\to\pi/2} f(x)$, where

$$f(x) = \begin{cases} \sin x & x \le \pi/2 \\ \cos x & x > \pi/2 \end{cases}.$$

4) Determine intervals in which the function is continuous:

$f(x) = \sqrt{\ln x}$

5)

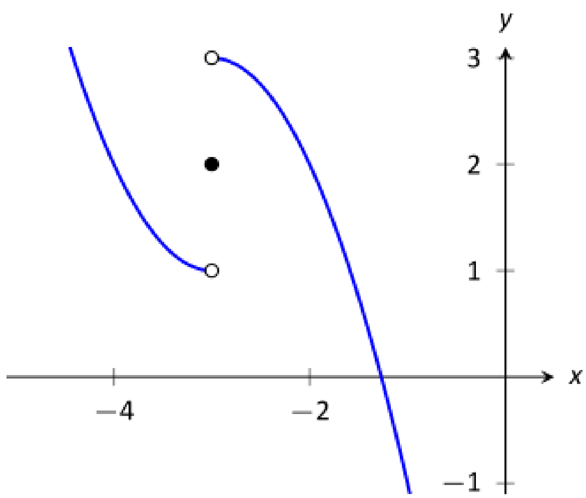
Use the graph of $f(x)$ provided to answer the following.

(a) $\lim_{x \to -3^-} f(x) = ?$

(b) $\lim_{x \to -3^+} f(x) = ?$

(c) $\lim_{x \to -3} f(x) = ?$

(d) Where is f continuous?



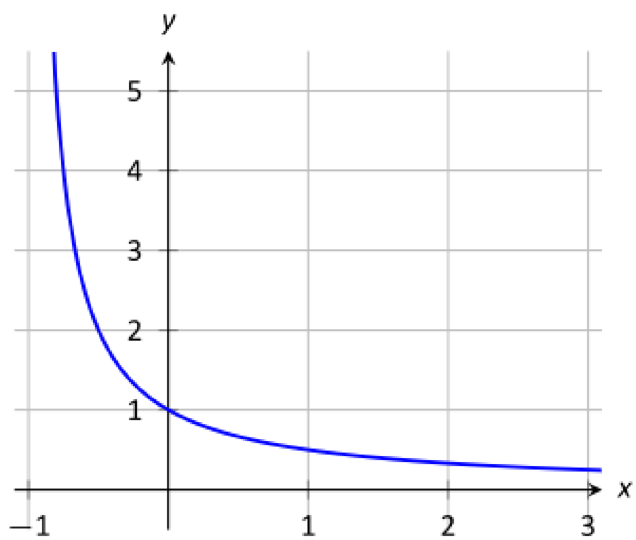
6)

The graph of $f(x) = \dfrac{1}{x+1}$ is shown.

(a) Use the graph to approximate the slope of the tangent line to f at the following points: $(0, 1)$ and $(1, 0.5)$.

(b) Using the definition, find $f'(x)$.

(c) Find the slope of the tangent line at the points $(0, 1)$ and $(1, 0.5)$.



7) Find the equation of the tangent line, in slope y-intercept form to the curve:

$f(x) = 4x - x^2$, at (1,3)

8) Using implicit differentiation, find the equation of the tangent line to the given point:
$y^2 - 7xy + x^3 - 2x = 9,\ at\ (0,3)$

9) Use L'Hopital's Rule to evaluate the limit:

$$\lim_{x \to -2} \frac{x^3 + 4x^2 + 4x}{x^3 + 7x^2 + 16x + 12}$$

$$\lim_{x \to \infty} \frac{\ln x}{x}$$

$$\lim_{x \to \infty} \frac{\ln(x^2)}{x}$$

10) Compute the derivatives of the given functions:

Pick three functions from each group to differentiate.

a)

$$f(t) = \sqrt[5]{t}(\sec t + e^t)$$

$$f(x) = \frac{\sin x}{\cos x + 3}$$

$$g(x) = e^2\left(\sin(\pi/4) - 1\right)$$

$$g(t) = 4t^3e^t - \sin t \cos t$$

$$h(t) = \frac{2^t + 3}{3^t + 2}$$

$$f(x) = x^2e^x \tan x$$

b)

$$f(x) = (4x^3 - x)^{10}$$

$$f(t) = (3t - 2)^5$$

$$g(\theta) = (\sin\theta + \cos\theta)^3$$

$$h(t) = e^{3t^2+t-1}$$

$$f(x) = \left(x + \tfrac{1}{x}\right)^4$$

$$f(x) = \cos(3x)$$

c)

. $f(x) = 2\ln x - x$

. $p(s) = \frac{1}{4}s^4 + \frac{1}{3}s^3 + \frac{1}{2}s^2 + s + 1$

. $h(t) = e^t - \sin t - \cos t$

. $f(x) = \ln(5x^2)$

. $f(t) = \ln(17) + e^2 + \sin \pi/2$

. $g(t) = (1 + 3t)^2$

d)

$g(x) = \tan^{-1}(2x)$

$f(x) = x\sin^{-1} x$

$g(t) = \sin t \cos^{-1} t$

$f(t) = \ln te^t$

$$h(x) = \frac{\sin^{-1} x}{\cos^{-1} x}$$

$g(x) = \tan^{-1}(\sqrt{x})$