

$$x^4 - 2x^3 - x^2 + 2x < 0$$

$$x(x^3 - 2x^2 - x + 2) < 0$$


$$x(x - 2)(x^2 - 1) < 0$$

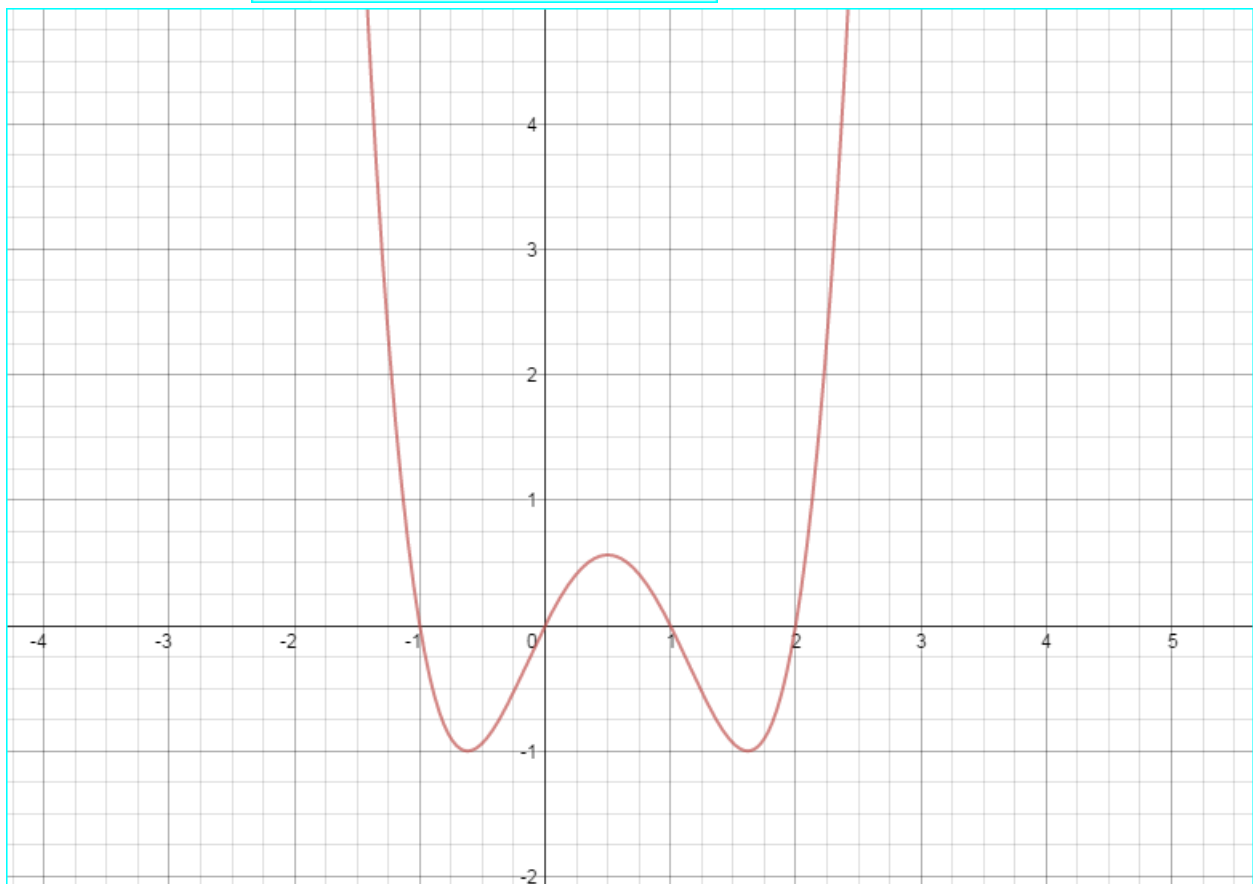
$$x(x - 2)(x - 1)(x + 1) < 0$$

So for the equation :

$$x(x - 2)(x - 1)(x + 1) = 0$$

Roots are 0, 2, -1, 1


$$y = x^4 - 2x^3 - x^2 + 2x$$



The solution for the inequality is $S = [-1, 0] \cup [1, 2]$

95

100

Math 1375

Exam 2

Spring, 2013

Name Diamonique Johnson

Directions: Show all steps and label and simplify answers.

1. Divide by long division and check:

$$\frac{x^3 - 4x^2 + 3x - 4}{x - 2} = x^2 - 2x - 1 + \frac{-6}{x-2}$$

(6)

$$\begin{array}{r} x^2 - 2x - 1 \\ x-2 \overline{) x^3 - 4x^2 + 3x - 4} \\ \underline{-x^3 + 2x^2} \\ -2x^2 + 3x \\ \underline{-2x^2 + 4x} \\ -x + 4 \\ \underline{-x + 2} \\ -6 \end{array}$$

• (x-2) goes into (x³-4x²) x² times, so multiply x²(x-2) then subtract from x³-4x²

• Bring down +3x

• (x-2) goes into (-2x²+3x) -2x times, so multiply -2x(x-2) then subtract from -2x²+3x

• Bring down -4

• (x-2) goes into (-x+4) -1 times so multiply

-1(x-2) then subtract from -x+4.

check

$$(x-2)(x^2 - 2x - 1 + \frac{-6}{x-2})$$

$$x(x^2 - 2x - 1 + \frac{-6}{x-2}) + -2(x^2 - 2x - 1 + \frac{-6}{x-2})$$

$$(x^3 - 2x^2 - x + \frac{-6x}{x-2}) + (-2x^2 + 4x + 2 + \frac{12}{x-2})$$

(combine like terms) Put the remainder (-6) over (x-2) and add that fraction

$$x^3 - 4x^2 + 3x + 2 + \frac{-6x+12}{x-2}$$

$$-6 = \frac{-6x+12}{x-2} \quad x-2 \overline{) \frac{-6x+12}{-6x+12} } \\ \underline{-6x+12} \\ 0$$

$$\dots x^3 - 4x^2 + 3x + 2 - 6 = \boxed{x^3 - 4x^2 + 3x - 4}$$

2. Find all real roots of the polynomial. Express the irrational roots in simplest radical form.

$$f(x) = 3x^3 - 11x^2 + 2x + 2$$

$$\text{Roots} = -\frac{1}{3}, 2+\sqrt{2}, 2-\sqrt{2}$$

$$p = \pm 1, \pm 2$$

$$q = \pm 1, \pm 3$$

$$\textcircled{p} \frac{p}{q} = \pm 1, \pm \frac{1}{3}, \pm 2, \pm \frac{2}{3}$$

first root

$$0 = 3\left(\frac{-1}{3}\right)^3 - 11\left(\frac{-1}{3}\right)^2 + 2\left(\frac{-1}{3}\right) + 2$$

$$0 = -\frac{1}{9} - \frac{11}{9} - \frac{2}{3} + 2$$

$$0 = 0$$

Then divide to get it in a standard form

$$\begin{array}{r} x^2 - 4x + 2 \\ 3x+1 \overline{) 3x^2 - 11x^2 - 2x + 2} \\ \underline{-3x^3 + x^2} \\ -12x^2 + 2x \\ \underline{-12x^2 - 4x} \\ 6x + 2 \\ \underline{-6x + 2} \\ 0 \end{array}$$

$$\begin{array}{r} 3x+1=0 \\ -1-1 \\ \hline 3x = -1 \\ x = -\frac{1}{3} \end{array}$$

$x = -\frac{1}{3} \leftarrow \text{root}$

Last roots

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{4 \pm \sqrt{16-8}}{2}$$

$$x = \frac{4 \pm \sqrt{8}}{2} = \frac{4 \pm 2\sqrt{2}}{2}$$

$$\frac{4 + 2\sqrt{2}}{2} = 2 + \sqrt{2}$$

$$\frac{4 - 2\sqrt{2}}{2} = 2 - \sqrt{2}$$

root

Root

3. Draw the complete graph of the function. State the domain, horizontal and vertical asymptotes, and the x and y intercepts.

$$f(x) = \frac{4x+8}{x^2+2x-3} = \frac{4(x+2)}{(x+3)(x-1)}$$

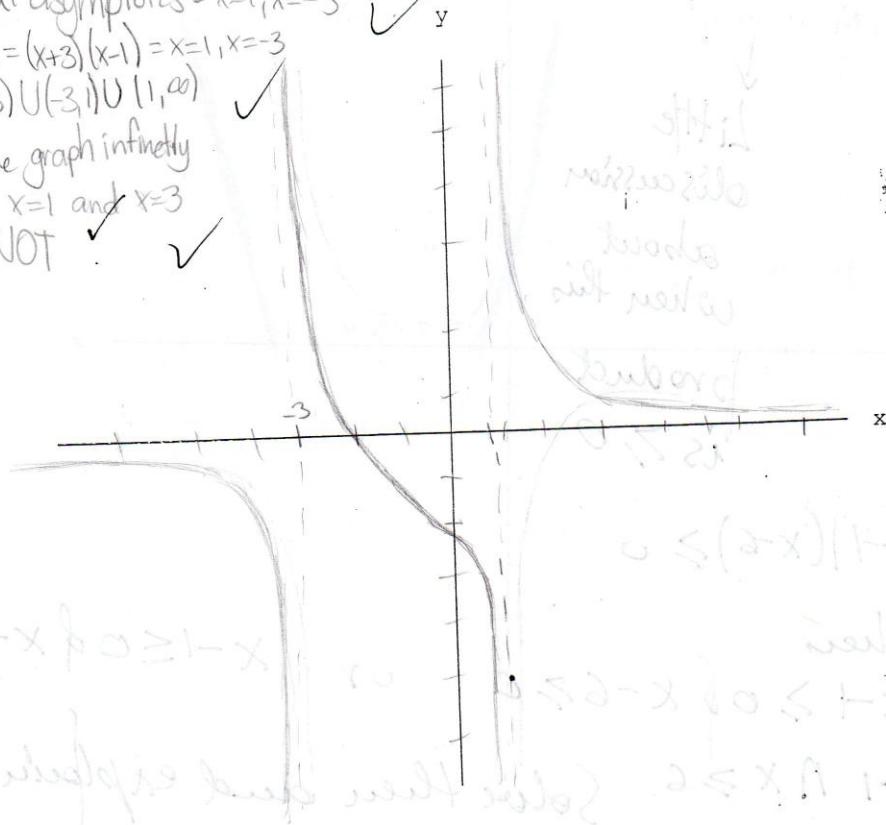
→ To find the vertical asymptote, set the denominator = 0

Vertical asymptotes = $x=1, x=-3$

Domain = $(x+3)(x-1) \neq 0 \Rightarrow x \neq -3, x \neq 1$

$D = (-\infty, -3) \cup (-3, 1) \cup (1, \infty)$

- Because the graph infinitely approaches $x=1$ and $x=-3$ but does NOT touch it.



9

Horizontal asymptote $y=0$ because the degree of p < degree of q

x and y intercepts:

(x) numerator equal to zero:

$$4x+8=0 \Rightarrow 4x=-8 \Rightarrow x=-2$$

plug in 0 to find the y intercept

$$\frac{4(0)+8}{(0)^2+2(0)-3} = \frac{0+8}{-3} = \frac{8}{-3} = -\frac{8}{3}$$

$$y = -\frac{8}{3} \quad (-2, 0)$$

$$y = -2\frac{2}{3}$$

4. Solve the inequality using the graphing method. Express your answer in interval notation.

$$x^2 \geq 7x - 6$$

First, bring everything to one side.

$$x^2 - 7x + 6 \geq 0$$

$$(x-1)(x-6) \geq 0$$

$$x-1=0 \quad x-6=0$$

$$x=1 \quad x=6$$

Roots, 1 and 6

Little discussion about when this product is ≥ 0

$$S = (-\infty, 1] \cup [6, \infty)$$

$$(x-1)(x-6) \geq 0$$

When

$$x-1 \geq 0 \quad \text{or} \quad x-6 \geq 0$$

$$x \geq 1 \quad \text{or} \quad x \geq 6$$

$$S = \{x \in \mathbb{R} / x \geq 6\}$$

$$\text{or } S = [6, \infty)$$

$$x \leq 1 \quad \text{or} \quad x \leq 6$$

$$S = \{x \in \mathbb{R} / x \leq 1\}$$

$$\text{or } S = (-\infty, 1]$$

$$S = (-\infty, 1] \cup [6, \infty)$$

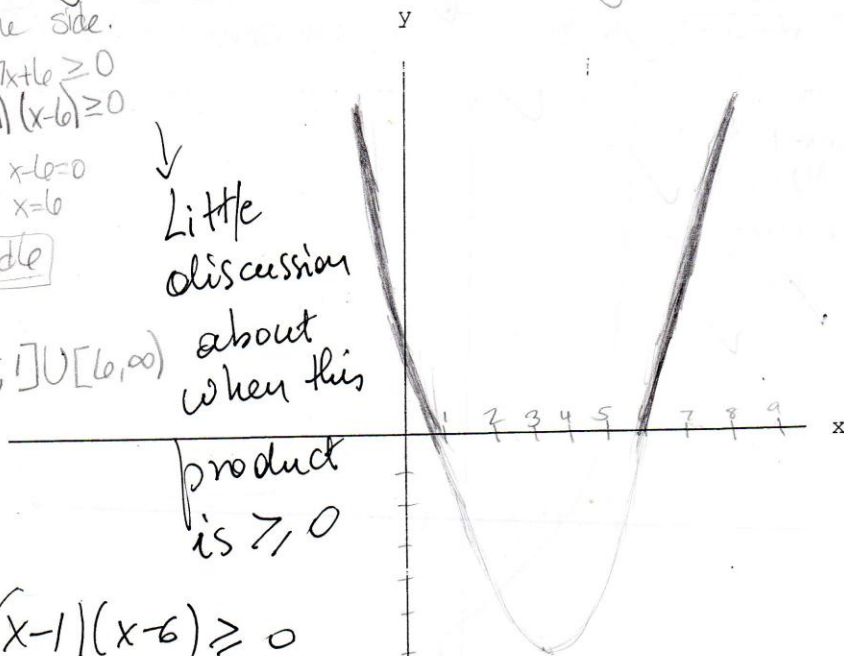
Y-intercept
plug zero in

$$(0)^2 - 7(0) + 6 = f(0)$$

$$0 + 0 + 6 = f(0)$$

$$6 = f(0)$$

6



8

origin is
bump

8

5. Solve the inequality using the graphing method. Express your answer in interval notation.

$$\frac{4}{x-2} \geq 4$$

Bring everything to one side.

$$\frac{4}{x-2} - 4 \geq 0$$

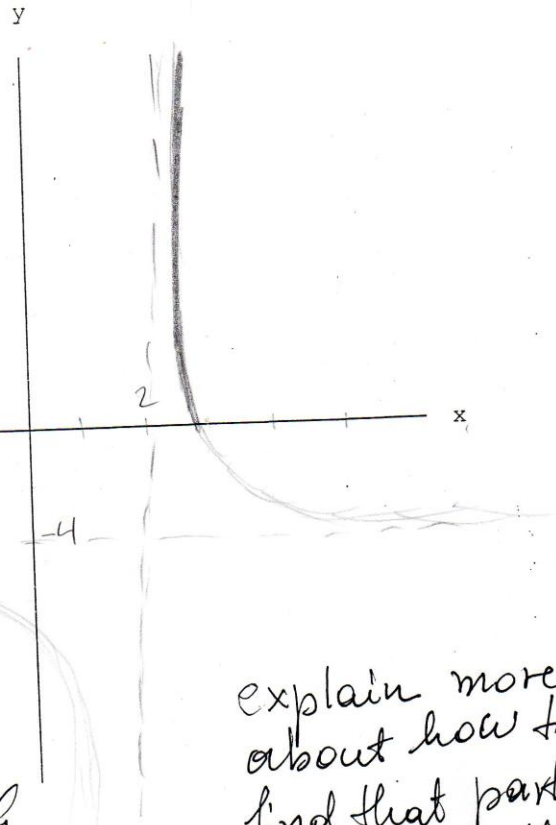
then i turned (-4) into $\frac{-4x-8}{x-2}$ to have a common denominator.

$$\frac{4-4x-8}{x-2} \geq 0$$

$$S = (2, 3]$$

$$\frac{12-4x}{x-2} \geq 0$$

Think of how could you sketch the graph if you didn't have a calculator



$$4 \geq 4x-8$$

$$4x-12 \leq 0$$

explain more about how to find that part of the graph that our inequality states. what are the asymptotes (why) the intercepts ect.



Math 1375

Exit Ticket: Inverse Functions Mat 1175

Instructor: L. Mingla

Student Name: Diamonique Johnson

Date: 4/16/15

For each question answered correctly and explained there are 10 points available.

State whether the given functions are inverse or not. Explain the reason why they are or they are not.

* on the back

The inverse of $f(n)$ is $f^{-1}(n) = -\sqrt[3]{n} - 1$. So $g(n)$ is not the inverse.

1) $f(n) = -(n+1)^3$

$g(n) = 3+n^3$

first I replaced $f(n)$ with $y = -(n+1)^3$

Then switch y and n , and solve for y .

$n = -\frac{(y+1)^3}{-1}$

10 $g(n)$ is the inverse of $f(n)$ because

when you plug $g(n)$ as n in $f(n)$, you get $f(n)$

2) $f(n) = 2(n-2)^3$

$g(n) = \frac{4+\sqrt[3]{4n}}{2}$

$2\left(\frac{4+\sqrt[3]{4n}}{2}-2\right)^3 = 2\left(\frac{4+\sqrt[3]{4n}}{2}-\frac{4}{2}\right)^3$
 $2\left(\frac{\sqrt[3]{4n}}{2}\right)^3 = 2\left(\frac{\sqrt[3]{4n}}{2}\right)^3 = 2\left(\frac{\sqrt[3]{n}}{2}\right)^3 = r$

Find the inverse of each function. Write down the procedure for finding the inverse, explain reasoning and check your answer applying the property of inverse functions.

10

3) $g(x) = -4x+1$

first replace $g(x)$ with (y) : $y = -4x+1$

Then switch the (x) and the (y) : $x = -4y+1$

Then solve by bringing (y) to one side isolated to find the inverse.

subtract 1 on both sides (subtraction property of equality)

$x-1 = -4y+1-1 \rightarrow x-1 = -4y$

divide both sides by -4 (division property of equality)

$\frac{(x-1)}{-4} = \frac{(-4y)}{-4} \rightarrow \frac{x-1}{-4} = y$

$g^{-1}(x) = \frac{x-1}{-4}$

check $g(g^{-1}(x)) = -4\left(\frac{x-1}{-4}\right)+1$
 $g(g^{-1}(x)) = x-1+1$
 $g(g^{-1}(x)) = x$

10

4) $h(x) = 2x^3+3$

$y = 2x^3+3$

$x = \frac{y-3}{2}$

$\frac{x-3}{2} = \frac{2y^3}{2}$

$\frac{x-3}{2} = y^3$

$\sqrt[3]{\frac{x-3}{2}} = \sqrt[3]{y^3}$

$y = \sqrt[3]{\frac{x-3}{2}}$

$h^{-1}(x) = \sqrt[3]{\frac{x-3}{2}}$

check:
 $h(x) = 2\left(\sqrt[3]{\frac{x-3}{2}}\right)^3+3$
 $h(x) = 2\left(\frac{x-3}{2}\right)+3$
 $h(x) = x-3+3$
 $h(x) = x$

$$f(n) = -(n+1)^3$$

$$g(n) = 3+n^3$$

If $g(n)$ is the inverse of $f(n)$ then $n = -(y+1)^3$ will be the same as $y = 3+n^3$ (if you replace $f(n)$ and $g(n)$ with y).

$n = -(y+1)^3 \rightarrow$ switch the variables then solve ✓

$\frac{n}{-1} = \frac{-(y+1)^3}{-1} \Rightarrow n = (y+1)^3 \rightarrow$ divide both sides by -1 , to isolate the y . Division property of equality.

$\sqrt[3]{n} = \sqrt[3]{(y+1)^3} \Rightarrow \sqrt[3]{n} = y+1$ then to remove the exponent (3) find the cubic root of both sides

$$y+1 = \sqrt[3]{n}$$

$$y+1 = -\sqrt[3]{n}$$

$$y+1 = -\sqrt[3]{n} \rightarrow y = -\sqrt[3]{n} - 1$$

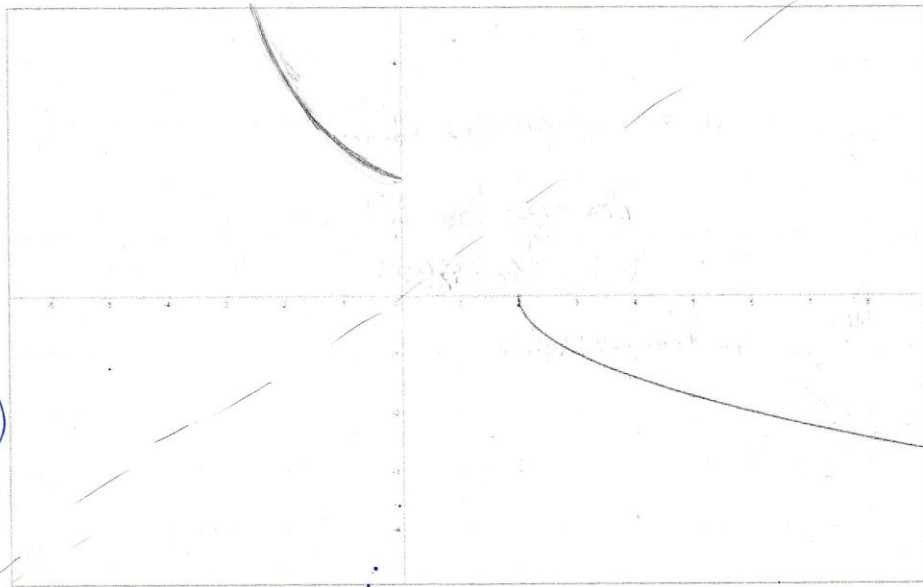
(switch around the equation to get y on the left side) ✓
The cubic root of -1 is -1 , so you can take it out of the radical.

Subtract 1 from both sides so that y can be by itself. Subtraction property of equality. ✓

$y = -\sqrt[3]{n} - 1$ is the inverse of $f(n)$. Since it isn't equivalent to $g(n)$, $g(n)$ is not the inverse of $f(n)$ ✓

5) Find the inverse of function $f(x) = -\sqrt{x-2}$, $x \geq 2$. Determine whether the inverse is also a function, and find the domain and the range of the inverse. Draw the inverse function if it exists. Explain the procedure and reasoning.

10



To find the inverse, replace $f(x)$ with y , then switch the places of x and y , then solve for y .

$$f(x) = -\sqrt{x-2} \rightarrow y = -\sqrt{x-2} \rightarrow x = -\sqrt{y-2}$$

$$-x = \sqrt{y-2}$$

$$(-x)^2 = (\sqrt{y-2})^2$$

$$x^2 = y-2$$

$$x^2 + 2 = y$$

$$y = x^2 + 2$$

divide both sides by -1 to remove (-1) from the right side (the side with the y) Division property of equal

Then to get rid of the radical sign, square both sides.

Then add 2 to both sides so that y can be by itself. Addition property of equalit

$y = x^2 + 2$ is a parabola, so $y = x^2 + 2$ is only the inverse when x is less than or equal to zero because the inverse should be a reflection over $x=y$ (the dotted line on the graph)

Therefore,

$$f^{-1}(x) = x^2 + 2, x \leq 0$$

91

100

1.1)

$$|3x - 9| = 6$$

Drop the absolute value signs and set up two equations, one equal to 6 and -6

$$3x - 9 = 6$$

$$3x - 9 = -6$$

You set these two equations up because the equation without the absolute value signs can equal either 6 or -6

Solve both equations for x

$$3x - 9 = 6 \quad \checkmark$$

$$3x = 15 \quad \checkmark$$

$$x = 5 \quad \checkmark$$

What x equals is now your solution set.

$$S = \{5, 1\} \quad \checkmark$$

An interpretation of absolute value is needed here!

$$3x - 9 = -6 \quad \checkmark$$

$$3x = 3 \quad \checkmark$$

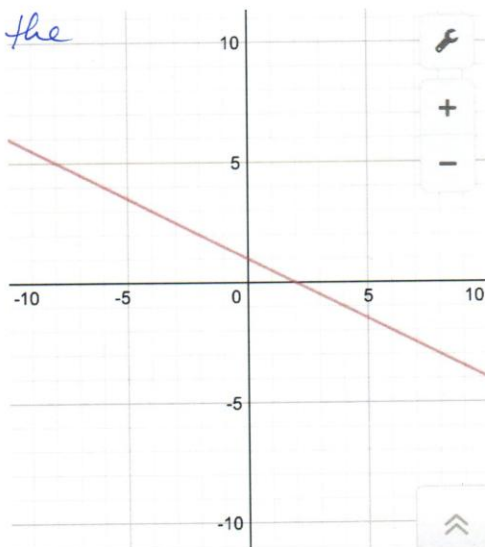
$$x = 1 \quad \checkmark$$

9

1.2)

Please write the question!

8



This graph is $y = x$ with transformations applied to it. The line intercepts the y-axis at 1, and continues on a slope of $-\frac{1}{2}$, as seen in the graph. This means that the formula for this graph is $y = -\frac{1}{2}x + 1$

How do you see this?

Please explain!

1.5)

Domain of f : $[2, 7]$ The domain of a function is all of its inputs

Range of f : $(1, 4]$ The range of a function is all of its outputs

$f(3) = 2$

When your input is 3 for function f , your output will be 2.

$f(5) = 2$

When your input is 5 for function f , your output will be 2.

$f(7) = 4$

When your input is 7 for function f , your output will be 4.

$f(9) = \text{Undefined}$

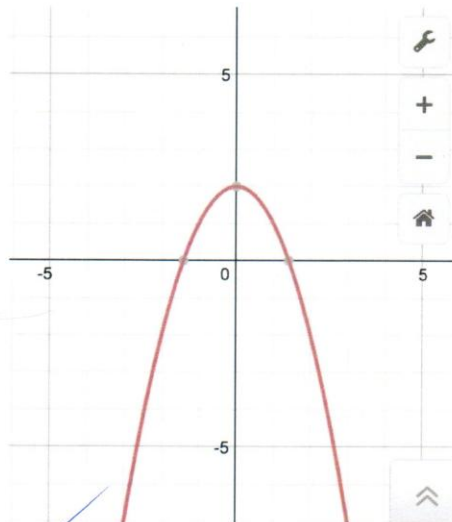
When your input is 9 for function f , your output will be undefined.

I would prefer to see the graph here so your interpretation is clear.

Always write the question and given information (for all)

9

1.6)



10

This graph is $y = x^2$ with transformations applied onto it. The parabola is reflected over the x-axis and has been shifted up 2 units, as seen in the graph. This means that the formula is $y = -x^2 + 2$

1.7)

$$f(x) = 5x + 4$$

$$g(x) = x^2 + 8x + 7$$

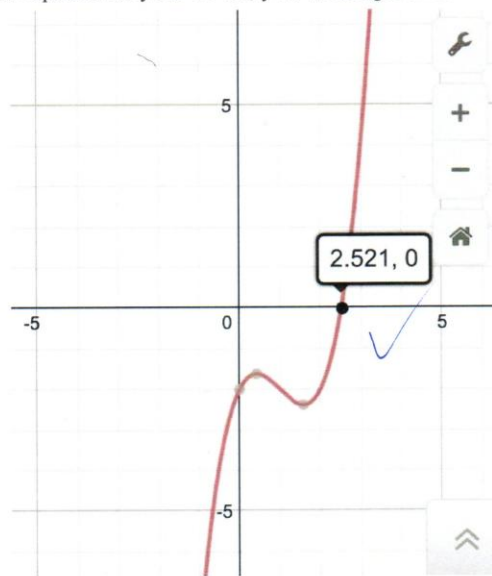
To find $(\frac{f}{g})(x)$, you simply create a fraction in which $f(x)$ is the numerator and $g(x)$ is the denominator,

1.3)

$$x^3 - 3x^2 + 2x - 2 = 0$$

If you were to graph this equation in your TI 84, you would get this:

10



Now to find the solutions of the equation, what you have to do is press **2ND** and **TRACE** on your calculator. Hit **2** to find solutions, or zeros. To get the exact point, what you would do is use the left and right keys to specify a point on the graph that a bit left and a bit right of the zero we seek. Press **ENTER** once you have gotten your point, press **ENTER** on "Guess?" and you will be presented with your solution, which in this case is $x = 2.521$.

1.4)

$$f(x) = x^2 - 2x + 5$$

To simplify the difference quotient, $\frac{f(x+h) - f(x)}{h}$, for $f(x+h)$, substitute all x 's with $(x+h)$, and plug in $x^2 - 2x + 5$ for $f(x)$

10

$$\frac{(x+h)^2 - 2(x+h) + 5 - (x^2 - 2x + 5)}{h}$$

Now begin to simplify

$$\frac{x^2 + 2xh + h^2 - 2x - 2h + 5 - x^2 + 2x - 5}{h}$$

$$\frac{2xh + h^2 - 2h}{h}$$

You should end up with

$$2x + h - 2$$

$$\begin{aligned} \left(\frac{f}{g}\right)(x) &= (5x+4)/(x^2+8x+7) \\ &= (5x+4)/[(x+7)(x+1)] \end{aligned}$$

✓ You can use the "Insert" → "Equations" "Insert new equation" in word doc to write fractions

The denominator of a fraction cannot equal zero. As such, x cannot equal 7 or 1. This means that the domain of $\left(\frac{f}{g}\right)(x)$ is all real numbers, except for 7 or 1.

(7)

$$D = \mathbb{R} - \{7, 1\}$$

-7 and -1 are not acceptable. $x+7=0$
 $x=-7 \dots$

1.8)

$$\begin{aligned} f(x) &= x^2 + \sqrt{x-3} \\ g(x) &= 2x-3 \end{aligned}$$

Please write the complete question than solve it!

To find $(f \circ g)(x)$, you simply substitute all x 's in $f(x)$ with $g(x)$.

(8)

$$\begin{aligned} (f \circ g)(x) &= (2x-3)^2 + \sqrt{(2x-3)-3} \\ &= 4x^2 - 12x + 9 + \sqrt{2x-6} \end{aligned}$$

✓ Write the condition! $2x-6 \geq 0$ Solve it!

The inside of the square root cannot be negative. As such, x cannot equal anything less than 3. This means that the domain of $(f \circ g)(x)$ is all real numbers, except for those less than 3.

$$D = [3, \infty)$$

$$D = [3, \infty)$$

Write in symbols, justify the answers.

1.9)

x	2	3	4	5	6
$f(x)$	5	0	2	4	2
$g(x)$	6	2	3	4	1
$(f \circ g)(x)$	2 ✓	5 ✓	0 ✓	2 ✓	Undefined ✓

(10)

In order to get $(f \circ g)(x)$, you simply substitute $g(x)$ for all x 's in $f(x)$. For example, if I were to find the output for $(f \circ g)(2)$, I'd find $g(2)$, which is 6. Then, I'd substitute 6 for x in $f(x)$, resulting in $f(6)$ which is 2.

Very good!

1.10)

$$f(x) = \frac{1}{2x+5}$$

To find the inverse of a function, first replace $f(x)$ with y

$$y = \frac{1}{2x+5}$$

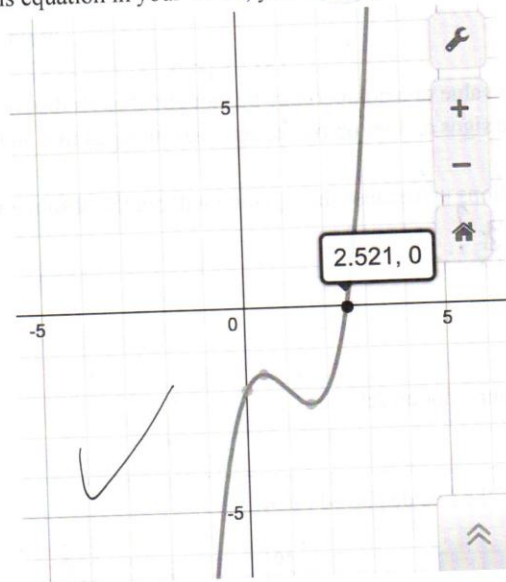
(110)

Now switch all the x 's and the y 's

1.3)

$$x^3 - 3x^2 + 2x - 2 = 0$$

If you were to graph this equation in your TI 84, you would get this:



(10)

Now to find the solutions of the equation, what you have to do is press **2ND** and **TRACE** on your calculator. Hit **2** to find solutions, or zeros. To get the exact point, what you would do is use the left and right keys to specify a point on the graph that a bit left and a bit right of the zero we seek. Press **ENTER** once you have gotten your point, press **ENTER** on "Guess?" and you will be presented with your solution, which in this case is $x = 2.521$.

1.4)

$$f(x) = x^2 - 2x + 5$$

To simplify the difference quotient, $\frac{f(x+h) - f(x)}{h}$, for $f(x+h)$, substitute all x 's with $(x+h)$, and plug in $x^2 - 2x + 5$ for $f(x)$

$$\frac{(x+h)^2 - 2(x+h) + 5 - (x^2 - 2x + 5)}{h}$$

Now begin to simplify

$$\frac{x^2 + 2xh + h^2 - 2x - 2h + 5 - x^2 + 2x - 5}{h}$$

$$\frac{2xh + h^2 - 2h}{h}$$

You should end up with

$$2x + h - 2$$

(how?)

(10)

1.5)

Domain of f : $[2, 7]$ The domain of a function is all of its inputs

Range of f : $(1, 4]$ The range of a function is all of its outputs

$$f(3) = 2$$

When your input is 3 for function f , your output will be 2.

$$f(5) = 2$$

When your input is 5 for function f , your output will be 2.

$$f(7) = 4$$

When your input is 7 for function f , your output will be 4.

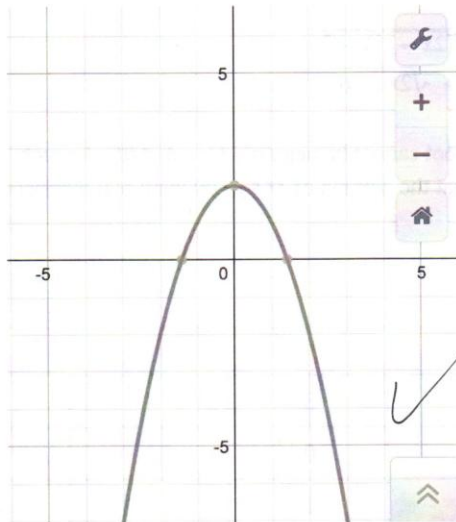
$$f(9) = \text{Undefined}$$

When your input is 9 for function f , your output will be undefined.

I asked you to display the graph here!

8

1.6)



10

This graph is $y = x^2$ with transformations applied onto it. The parabola is reflected over the x -axis and has been shifted up 2 units, as seen in the graph. This means that the formula is $y = -x^2 + 2$

1.7)

$$f(x) = 5x + 4$$

$$g(x) = x^2 + 8x + 7$$

10

To find $(\frac{f}{g})(x)$, you simply create a fraction in which $f(x)$ is the numerator and $g(x)$ is the denominator,

$$\begin{aligned} \left(\frac{f}{g}\right)(x) &= \frac{5x+4}{x^2+8x+7} \\ &= \frac{5x+4}{(x+7)(x+1)} \end{aligned}$$

The denominator of a fraction cannot equal zero. As such, x cannot equal 7 or -1. This means that the domain of $\left(\frac{f}{g}\right)(x)$ is all real numbers, except for -7 or -1.

$$D = \mathbb{R} - \{-7, -1\}$$

10

$$\begin{aligned} 1.8) \quad f(x) &= x^2 + \sqrt{x-3} \\ g(x) &= 2x - 3 \end{aligned}$$

Find the composition $(f \circ g)(x)$ and state its domain.

To find $(f \circ g)(x)$, you simply substitute all x 's in $f(x)$ with $g(x)$.

$$\begin{aligned} (f \circ g)(x) &= (2x-3)^2 + \sqrt{(2x-3)-3} \\ &= 4x^2 - 12x + 9 + \sqrt{2x-6} \end{aligned}$$

The inside of the square root cannot be negative ($2x-6 \geq 0$). As such, x cannot equal anything less than 3. This means that the domain of $(f \circ g)(x)$ is all real numbers, except for those less than 3.

$$D = [3, \infty)$$

10

1.9)

x	2	3	4	5	6
$f(x)$	5	0	2	4	2
$g(x)$	6	2	3	4	1
$(f \circ g)(x)$	2 ✓	5 ✓	0 ✓	2 ✓	Undefined ✓

In order to get $(f \circ g)(x)$, you simply substitute $g(x)$ for all x 's in $f(x)$. For example, if I were to find the output for $(f \circ g)(2)$, I'd find $g(2)$, which is 6. Then, I'd substitute 6 for x in $f(x)$, resulting in $f(6)$ which is 2.

$$1.10) \quad f(x) = \frac{1}{2x+5}$$

To find the inverse of a function, first replace $f(x)$ with y

$$y = \frac{1}{2x+5}$$

Now switch all the x's and the y's

$$x = \frac{1}{2y+5}$$

Now solve for y

$$(2y+5)x = \frac{1}{2y+5}(2y+5)$$

$$2yx+5x = 1$$

$$2yx = 1 - 5x$$

$$y = \frac{1}{2x} - \frac{5}{2}$$

Now finally, replace y with $f^{-1}(x)$

$$f^{-1}(x) = \frac{1}{2x} - \frac{5}{2}$$

A checking of your answer
would be good to have after you
find $f^{-1}(x)$.

$$f(f^{-1}(x)) = x \text{ or } f^{-1}(f(x)) = x$$

Shu Yi Dey

Exercise 1.4

Exercise 1.4. Let $f(x) = x^2 - 2x + 5$. Simplify the difference quotient $\frac{f(x+h)-f(x)}{h}$ as much as possible.

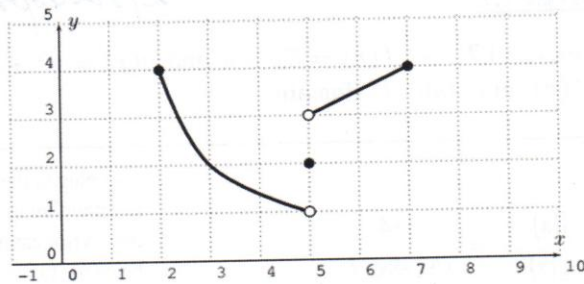
(10)

$$\begin{aligned} & \frac{[(x+h)^2 - 2(x+h) + 5] - (x^2 - 2x + 5)}{h} \\ &= \frac{x^2 + 2xh + h^2 - 2x - 2h + 5 - x^2 + 2x - 5}{h} \\ &= \frac{2xh + h^2 - 2h}{h} \\ &= 2x + h - 2 \end{aligned}$$

On the left is my work shown to my answer. The first equation is what I have after I plugged in the equations. I then distribute it, and simplify it by dividing it by h. This left me with my simplified answer, $2x+h-2$.

Exercise 1.5

Exercise 1.5. Consider the following graph of a function f .



Find: domain of f , range of f , $f(3)$, $f(5)$, $f(7)$, $f(9)$.

(10)

Domain of f : $D = [2, 7]$ ✓ The domain of the function shown above is $[2, 7]$, it includes the numbers 2 and 7, and all numbers in between because the line and the dots that pass these numbers on the x-axis are dark and shaded.

Range of f : $R = (1, 4]$ ✓ The range of the function is $(1, 4]$ because dot is hollow when the lines pass by the number 1 on the y-axis, therefore 1 is not included.

$f(3) = 2$ ✓ For $f(3)$, it means if x is 3 then what y would be, which is 2 on this graph; resulting me $f(3) = 2$. the same method apply for $f(5)$ and $f(7)$.

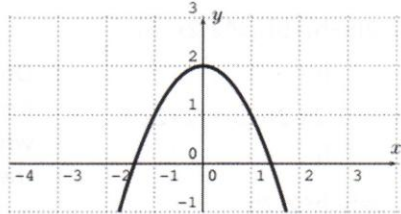
$f(5) = 2$ ✓

$f(7) = 4$ ✓

$f(9) = \text{Undefined}$ ✓ On the other hand, $f(9)$ is quite different, since $f(9)$ is out of the range of this graph's domain, the answer is undefined.

Exercise 1.6

Exercise 1.6. Find the formula of the graph displayed below.



8

The formula of the graph is $y = -x^2 + 2$. This is because it is the same as $y = x^2$ but an upside-down one, which was $y = -x^2$. This is also known as the transformation graph of $f(x) = x^2$, $-f(x) = -x^2$ can rewrite as $f(x) = -x^2$. The y-intersect of this graph was shown on the graph crossing the y-axis, which is 2. Therefore, it resulted me with the formula $y = -x^2 + 2$.

*relate this to the shift of the graph 2 units up
(2 transformations here from the basic graph $y = x^2$)*

Exercise 1.7

Exercise 1.7. Let $f(x) = 5x + 4$ and $g(x) = x^2 + 8x + 7$. Find the quotient $(\frac{f}{g})(x)$ and state its domain.

10

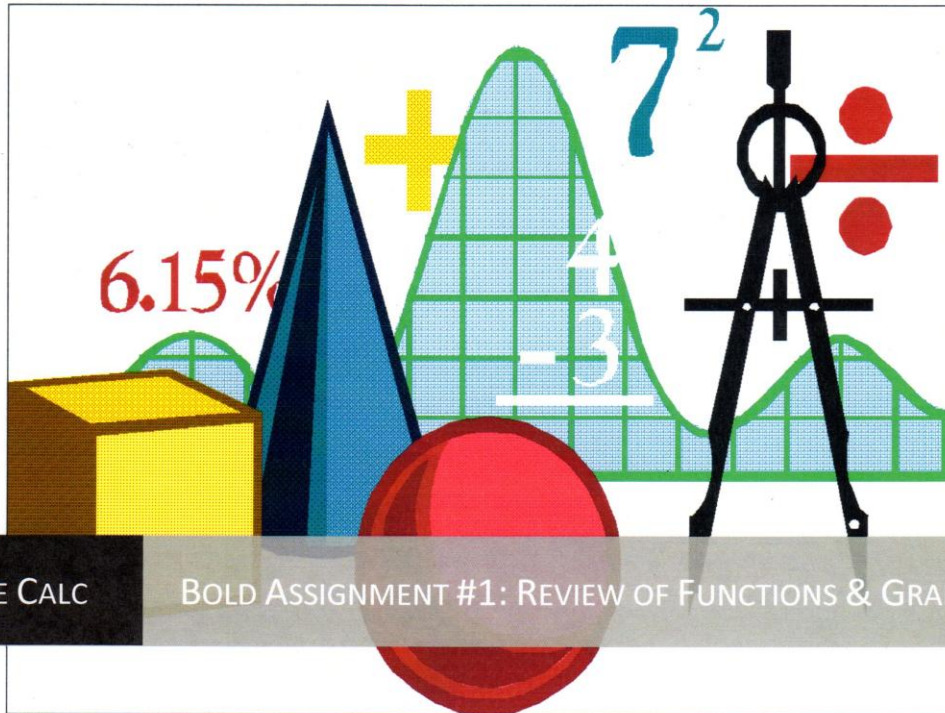
$$\frac{f(x)}{g(x)} = \frac{5x+4}{x^2+8x+7}$$

$$\frac{f(x)}{g(x)} = \frac{5x+4}{(x+7)(x+1)}$$

Domain: $D = (-\infty, -7) \cup (-7, -1) \cup (-1, \infty)$

As I plug in the equation, the dividend is not as important as the divisor. This is because the divisor cannot be equal to zero, and this helps me to identify the domain. I can rewrite the trinomial $x^2 + 8x + 7$ as $(x+7)(x+1)$, and resulted that x cannot equal to -7 or -1 because it is the denominator of the fraction and a denominator cannot be zero. Therefore, my domain for this equation is $D = (-\infty, -7) \cup (-7, -1) \cup (-1, \infty)$.

3/15/2015



PRE CALC

BOLD ASSIGNMENT #1: REVIEW OF FUNCTIONS & GRAPHS

$$\frac{87}{100}$$

+ 3 extra credit
for a nice
cover and
correct typing
of equations
functions etc

Good Job!

$$\frac{90}{100}$$

Radeesha Stroy | Prof. Mingla

Exercise 1.1

Find all solutions of the equation $|3x - 9| = 6$

$$3x - 9 = 6$$

$$3x = 15$$

$$\frac{3x}{3} = \frac{15}{3} \quad \checkmark$$

$$X = 5$$

OR

$$-(3x - 9) = 6$$

$$-3x + 9 = 6$$

$$\frac{-3x}{-3} = \frac{-3}{-3}$$

$$X = 1 \quad \checkmark$$

10

Check

$$|3x - 9| = 6$$

$$|3(5) - 9| = 6$$

$$|15 - 9| = 6$$

$$|6| = 6 \quad \checkmark$$

$$|3x - 9| = 6$$

$$|3(1) - 9| = 6$$

$$|3 - 9| = 6$$

$$|-6| = 6 \quad \checkmark$$

AND

Explanation

The reason why we create two different equations when solving an absolute value equation is because, the number inside of an absolute value notation can also be that number's opposite, being that an absolute value can not be negative. An example of this is in the equation above when $|x| = 6$. X can either be 6 or -6. Due to the fact that absolute values can only produce numbers greater than zero and -6 is the same distance away from zero as 6; the numbers are opposite.

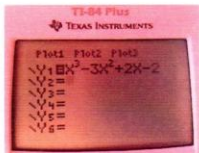



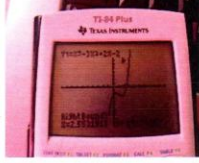
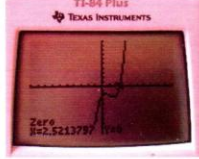
Exercise 1.3

Solve for x

$$x^3 - 3x^2 + 2x - 2 = 0$$

In order to find the solution, I used the TI-84 Plus graphing calculator. The steps are listed below.

(10)

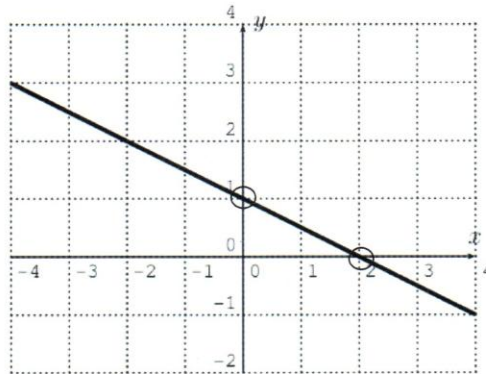
-  → Press the button on the calculator that says "y=" and then insert the equation next to y_1
-  → After entering the equation, press the "graph button", a graph of the equation should pop up.
-  → By pressing the "2nd" key and the "Trace" key, you activate Calc. In calc, you move down to "zero" and press enter.
-  → After getting to the zero command, you find the point that crosses the x axis at $y = 0$ by first finding the left bound (point below the x axis)
-  → After selecting the left bound, you select the right bound (the point above the x axis). After pressing enter, you enter guess.
-  → After entering a "guess", the calculator will give you the x intercept. In this case, the point is 2.5213797, which is the solution.

Very good!



Exercise 1.2

Find the equation of the line



10

Answer: $y = -\frac{1}{2}x + 1$ ✓

Explanation:

By using the formula $y = mx + b$, I was able to find the formula of the line above. In the given formula, you can use two points on the line, $(0, 1)$ and $(2, 0)$ and substitute it into the equation $y = mx + b$; where y and x will be the x and y in the chosen point $(0, 1)$ or $(2, 0)$; m will be the slope of the line, and b will be the y intercept of the line. In this case, the line intercepts the equation $x = 0$ at $y = 1$; therefore "1" is the y intercept. After studying the two points $(0, 1)$ and $(2, 0)$, you can find the slope by using the slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ and substituting the Y s and X s with the corresponding points in $(0, 1)$ and $(2, 0)$. With the substitution, $m = \frac{0 - 1}{2 - 0}$ the slope now becomes $-\frac{1}{2}$. With both the slope and the y intercept, we are now able to complete the equation as a value of y ; making the equation of the line

$$y = -\frac{1}{2}x + 1.$$

Check

Point $(2, 0)$

$$y = mx + b$$

$$y = -\frac{1}{2}x + b$$

$$0 = -\frac{1}{2}(2) + b$$

$$0 = -1 + b$$

$$0 = 0$$

Exercise 1.8

Given the functions $f(x) = x^2\sqrt{x-3}$ and $g(x) = 2x-3$, find $(f \circ g)(x)$

$$(f \circ g)(x) = f(g(x)) = (2x-3)^2 + \sqrt{2x-6} = 0$$

$$\sqrt{2(0)-6} = \sqrt{-6} \quad ?$$

you must write
the condition

$$\sqrt{2(3)-6} = 0$$

6

$$2x-6 \geq 0 \quad \text{Domain: } [3, \infty) \quad \checkmark$$

solve for x!

Explanation:

When studying the domain of a function with a square root, we must beware of negatives. Being that the square root of zero is zero, that number is acceptable. However, numbers less than zero could not work because they produce negative numbers. An example of this is the number two. If you have $\sqrt{2(1)-6}$ you would end up with $\sqrt{-4}$. The problem with negative numbers in the square root of the domain is, they produce imaginary numbers. When there is an imaginary domain (input) then there is not a domain and the function does not exist.

? Not clear

you cannot count only in
tries!

Exercise 1.9:

Fill in the chart for $(f \circ g)(x)$. Is f a function?

F is a function because for every ^{input} output, there is an ^{only one output} input. ?
However, $f(x)$ is **not one to one** because for the inputs "4" and "6", there is a similar output; "2"

x	2	3	4	5	6
f(x)	5	0	2	4	2
g(x)	6	2	3	4	1
$(f \circ g)(x)$	2 ✓	5 ✓	0 ✓	2 ✓	Und ✓

8

Explanation:

To find out $(f \circ g)(x)$, you must take the values of $g(x)$ and place them into the values of $f(x)$. An example of this is when $g(2)$ is 6. You take that output and place it into the input for $f(x)$. With this, $f(x)$ now becomes $f(6)$ which is equal to two. ✓
This is why $(f \circ g)(2)$ is two. With this same theory, $(f \circ g)(6)$ is undefined because $g(6)$ is one and "one" is not included on the chart; making it not an input. ✓

Exercise I.4. Let $f(x) = x^2 - 2x + 5$. Simplify the difference quotient $\frac{f(x+h) - f(x)}{h}$ as much as possible.

9

Do not separate the rational expression!

First I found $f(x+h)$ by replacing all the "x"s in $x^2 - 2x + 5$ with " $x+h$ "

$$f(x+h) = x^2 + 2xh + h^2 - 2x - 2h + 5$$

Then I did $f(x+h) - f(x)$

$$(x^2 + 2xh + h^2 - 2x - 2h + 5) - (x^2 - 2x + 5) = \checkmark$$

$$x^2 + 2xh + h^2 - 2x - 2h + 5 - x^2 + 2x - 5 = 2xh + h^2 - 2h$$

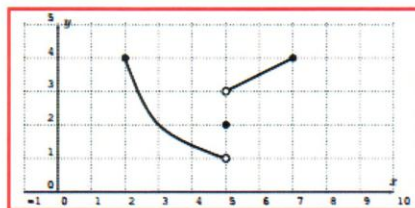
Then I divided that by h and got the answer

$$(2xh + h^2 - 2h)/h = 2x + h - 2$$

Keep the ratio throughout the whole solution!
 $\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - 2(x+h) + 5 - (x^2 - 2x + 5)}{h}$

Exercise I.5. Consider the following graph of a function f

8



The domain would be $D = [2, 7]$ because there is a line from two to five not including five, and from five to seven not including seven. The point at $(5, 2)$ replaces the points in both lines that don't

include five.

The range of the function is $(1, 4]$ because the line at the bottom does not include one, which is indicated at point $(5, 1)$.

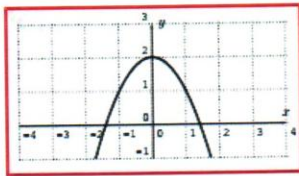
$$f(3) = 2 \quad f(5) = 2 \quad f(7) = 4 \quad f(9) = \text{undefined}$$

← why?
 Explain!
 →

To find these values, I used the number in the parenthesis and used x from "f(x)" and plugged it in to the graph to find what value would be appropriate for y. There was no y value when x equaled 9, therefore, f(9) is undefined.

Exercise I.6. Find the formula of the graph displayed below.

10



This graph shows a parabola $y = -x^2$. Since the graph is flipped upside down, the parabola is $y = -x^2$. Also, the parabola is shifted up 2 units so the formula is $y = -x^2 + 2$.

Exercise I.7. Let $f(x) = 5x + 4$ and $g(x) = x^2 + 8x + 7$. Find the quotient $(f/g)(x)$ and state its domain.

First I set up $(f/g)(x)$ and simplified it:

10

$$\frac{f(x)}{g(x)} = \frac{5x+4}{x^2+8x+7} = \frac{5x+4}{(x+1)(x+7)}$$

The denominator of a fraction cannot equal zero. Therefore, the domain would be any number except those that would make the denominator equal zero.

$$x + 1 = 0$$

$$x = -1$$

$$x + 7 = 0$$

$$x = -7$$

$$D = \mathbb{R} - \{-1, -7\}$$

Exercise I.8. Let $f(x) = x^2 + \sqrt[2]{x-3}$ and $g(x) = 2x - 3$. Find the composition $(f \circ g)(x)$ and state its domain.

Substitute:

Simplify:

$$(f \circ g)(x) = f(g(x)) = (2x-3)^2 + \sqrt[2]{2x-3-3} = 4x^2 - 12x + 9 + \sqrt[2]{2x-6}$$

The domain of the function will be any value that the numbers inside the square root does not equal to a negative number. I had to find what value of x would make $2x - 6$ equal zero.

$$2x - 6 \geq 0 \Rightarrow 2x \geq 6 \Rightarrow x \geq 6/2 \Rightarrow x \geq 3$$

$$D = [3, \infty)$$

Exercise I.9. Consider the assignments for f and g given by the table below.

x	2	3	4	5	6
$f(x)$	5	0	2	4	2
$g(x)$	6	2	3	4	1

Is f a function? Is g a function? Write the composed assignment for $(f \circ g)(x)$ as a table.

Both f and g are functions because each value for x has exactly one value for $f(x)$ and one value for $g(x)$.

x	2	3	4	5	6
$f(x)$	5	0	2	4	2
$g(x)$	6	2	3	4	1
$(f \circ g)(x)$	2	5	0	2	undefined

10

Diamonique Johnson

March 12th 2015

Mat 1375 L. Mingla

+3

Research Paper: Inverse Functions

The concept of inverse functions can be difficult to interpret. Getting a thorough understanding of what they are and the way they work can be made easier with detailed explanations. In order to understand inverse functions, you need to know what they are, how to find them, and finding the domain and range.

Inverse functions reverse other functions. They switch the roles of the inputs and outputs. For instance, $f(x) = y$ is the same as having $f^{-1}(y) = x$. The relationship between these two values are reflected over the $x=y$ line on a graph. The inverse of a function won't always be a function as well. Having a one-to-one function guarantees that the inverse will definitely be a function.

When solving for the inverse of a function, there is more than one approach. It can be solved by switching the x and the y ($f(x)$), or finding the values and then switching it at the end. The first step to finding the inverse of a function is replacing $f(x)$ with " y ". Once you have " y " alone on one side of the equal sign, switch the positions of the " x " and the " y ", solve for " y ". Once " y " is completely isolated, that is the value of the inverse of the function.

Sometimes, questions will ask about the domain and range of the inverse of a function. The domain of the function is all suitable values for " x ". Therefore, the range of the function is all suitable values for " y ". However, it can get tricky because if the function is a fraction, the denominator cannot equal zero, because it would lead to an undefined function. In addition,

Function is a relationship!

values are specific as for example input value or output value

Exercise II.2. Find the remainder when dividing $x^3 + 3x^2 - 5x + 7$ by $x + 2$

$$\begin{aligned} & \frac{x^3+3x^2-5x+7}{x+2} \\ = & \frac{(x+2)(x^2+x-7)+21}{x+2} \\ = & (x^2+x-7)R\ 21 \\ & \text{Remainder : } 7 \end{aligned}$$

Checking:

$$\begin{aligned} & (x+2)(x^2+x-7)+21 \checkmark \\ = & (x^3+3x^2-5x-14)+21 \checkmark \\ = & x^3+3x^2-5x+7 \checkmark \end{aligned}$$

$$\begin{array}{r} x^2+x-7\ R\ 21 \\ x+2 \overline{) x^3+3x^2-5x+7} \\ \underline{-(x^3+2x^2)} \checkmark \\ x^2-5x \\ \underline{-(x^2+2x)} \\ -7x+7 \\ \underline{-(-7x-14)} \\ 21 \end{array}$$

To check for a divided polynomial with a remainder you multiply the divisor by the quotient (the answer of the division) then the answer should equal the dividend because by doing that you are reversing the division recreating the original problem. Also, a remainder is left behind when the first term of divisor can't divide into the dividend anymore.

(10)

86
100

Shu Yi Deng 4-12-15

College Pre Calculus: Bold Assignment

Exercise II.1. Divide the polynomials: $\frac{2x^3+x^2-9x-8}{2x+3}$

$$\begin{aligned} & \frac{2x^3+x^2-9x-8}{2x+3} \\ = & (x^2-x-3)R1 \\ = & x^2-x-3+\frac{1}{2x+3} \end{aligned}$$

Checking:

$$\begin{aligned} & (2x+3)\left[(x^2-x-3)+\left(\frac{1}{2x+3}\right)\right] \\ = & (2x+3)(x^2-x-3)+\cancel{(2x+3)}\left(\frac{1}{\cancel{2x+3}}\right) \\ = & (2x^3+x^2-9x-9)+1 \\ = & 2x^3+x^2-9x-8 \end{aligned}$$

10

$$\begin{array}{r} x^2-x-3 \text{ R1} \\ 2x+3 \overline{) 2x^3+x^2-9x-8} \\ \underline{-(2x^3+3x^2)} \\ -2x^2-9x \\ \underline{-(-2x^2-3x)} \\ -6x-8 \\ \underline{-(-6x-9)} \\ +1 \end{array}$$

To divide polynomials, I first set the problem up in long division form instead of fraction form. Then I put the divisor (the number that is doing the dividing) on the outside and the dividend (the number that is being divided) on the inside. Then the first term of the divisor divides the first term of the dividend. Then I put the result at top where the answer goes. Afterwards I multiply the answer by the divisor then subtract it from the equation creating a whole new polynomial. I repeat these steps until there's no new polynomial to be divided or there's a number that can't be divided which is called the remainder. However, I can rewrite the remainder in a fraction form, which is $1/(2x+3)$.

Exercise II.3. Which of the following is a factor of $x^{400} - 2x^{99} + 1$

$$x - 1, \quad x + 1, \quad x - 0$$

$$X^{400} - 2X^{99} + 1$$

	$x+1; x=-1$	$x-0; x=0$
$x-1; x=1$	$(-1)^{400} - 2(-1)^{99} + 1$	$0^{400} - 2(0)^{99} + 1$
$1^{400} - 2(1)^{99} + 1$	$= 1 - (-2) + 1$	$= 0 - 2(0) + 1$
$= 1 - 2 + 1$	$= 1 + 2 + 1$	$= 1$
$= 0$	$= 4$	

Answer: $x-1; x=1$. Positive 1 is a factor. ?

↓
root
 $x-1$ is the factor.

⑧

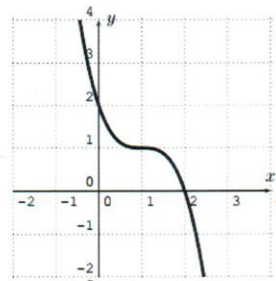
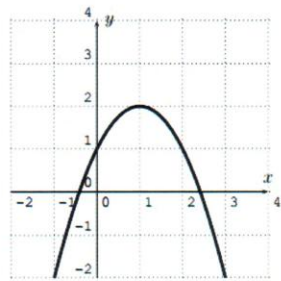
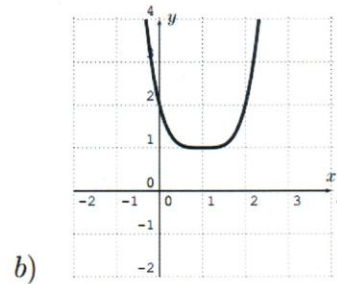
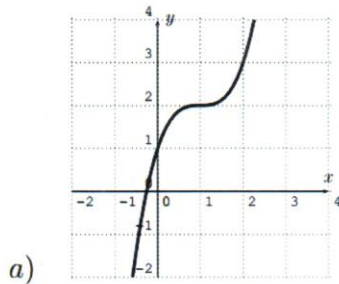
To find which one of these factors is the actual factor for the given equation, I found the roots of each factor by isolating the "x". Then I plug the roots into the equation to see which one makes the equation equals zero.

To find the roots you must equal to zero.

$$X^{400} - 2X^{99} + 1 = 0$$

If $x=c$ makes this equation a true statement, then the $x-c$ is its factor.

Exercise II.4. Identify the polynomial with its graph.



(10)

- | | |
|--|-----------------|
| i) $f(x) = -x^2 + 2x + 1,$ | graph: _____ |
| ii) $f(x) = -x^3 + 3x^2 - 3x + 2,$ | graph: <u>d</u> |
| iii) $f(x) = x^3 - 3x^2 + 3x + 1,$ | graph: <u>a</u> |
| iv) $f(x) = x^4 - 4x^3 + 6x^2 - 4x + 2,$ | graph: <u>b</u> |

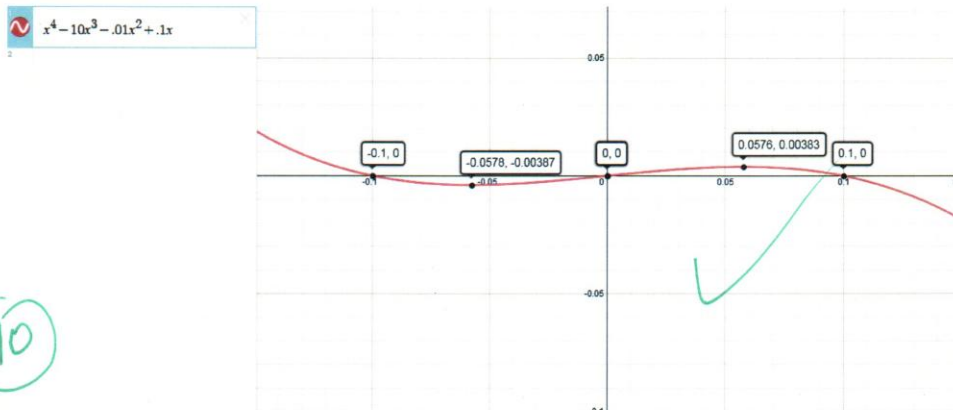
ANSWER and EXPLANATION:

- I) **Graph C**, because although $-x^2$ takes the similar shape of x^2 it is flipped upside-down, resulted to look like something similar to an upside-down U.
- II) **Graph D**, because the function is negative (which flips the graph), y-intercept is 2 and largest degree is odd (which makes the function go into the positive and negative y infinity).
- III) **Graph A**, because the function is positive, y-intercept is 1 and largest degree is odd (which makes the function go into the positive and negative y infinity).
- IV) **Graph B**, because the function isn't negative, y-intercept is 2 and the largest degree is even (it looks like a parabola only going in one direction.)

Exercise II.5. Sketch the graph of the function:

$$f(x) = x^4 - 10x^3 - 0.01x^2 + 0.1x$$

- What is your viewing window?
- Find all roots, all maxima and all minima of the graph with the calculator.



10

Roots: -0.1, 0, 0.1

Relative Maxima: (0.0576, 0.00383)

Relative Minima: (-0.0578, -0.00387)

The roots are -0.1, 0, and 0.1 because they are touching the x-axis where the output is 0. The roots are also called the zeros. Also, there is no maxima or minima because the graph goes on to infinity, therefore this two points on the graph are the relative maxima and minima, which is that the relative maxima is (0.0576, 0.00383) and the relative minima is (-0.0578, -0.00387).

absolute

Exercise II.6. Find all roots of $f(x) = x^3 + 6x^2 + 5x - 12$.

Use this information to factor $f(x)$ completely.

p- Possible factors of 12 so $p = \pm(4, 3, 2, 1, 12)$

q- Possible factor of 1 so $q = \pm 1$

Possible roots = $-4, -3, -2, -1, 1, 12, 2, 3, 4$ ± 6

$$x^3 + 6x^2 + 5x - 12 = 0$$

$$x = 1$$

$$1^3 + 6(1)^2 + 5(1) - 12 = 0$$

$$1 + 6 + 5 - 12 = 0$$

$$0 = 0$$

$$x = -3$$

$$(-3)^3 + 6(-3)^2 + 5(-3) - 12 = 0$$

$$-27 + 54 - 15 - 12 = 0$$

$$0 = 0$$

$$x = -4$$

$$(-4)^3 + 6(-4)^2 + 5(-4) - 12 = 0$$

$$-64 + 96 - 20 - 12 = 0$$

$$0 = 0$$

First I find all the possible factors, and then use the formula p/q for defining the possible roots. I resulted with $-4, -3, -2, -1, 0, 1, 2, 3,$ and 4 as my possible roots. Then I plugged them in to see which ones are equal to 0. I resulted with $x=1, x=-3,$ and $x=-4$ as my roots for this equation $x^3+6x^2+5x-12=0$.

$f(x) = (x-c_1)(x-c_2)(x-c_3) \dots$
 $f(x) = (x-1)(x+3)(x+4)$ Check by multiplying
factor the $f(x)$ completely.

Exercise 11.7. Find a polynomial of degree 3 whose roots are 0, 1, and 3, and so that $f(2) = 10$.

$0=x$	$1=x$	$3=x$	$f(2) = 10$
$x-0=0$	$x-1=0$	$x-3=0$	$(x)(x-1)(x-3) = 10$
$0-0=0$	$1-1=0$	$3-3=0$	$(2)(2-1)(2-3) = 10$
(x) ✓	$(x-1)$ ✓	$(x-3)$ ✓	$(-2 \div -2) = 10(\div (-2))$
	$(x)(x-1)(x-3)$		$0 \neq -5$

$$\leftarrow (-5)(x)(x-1)(x-3)$$

$$f(2) = 10$$

$$(-5)(2)(2-1)(2-3) = 10$$

$$10 = 10$$

Answer: $f(x) = (-5x)(x-1)(x-3)$

↓
simplify: $f(x) = \dots$

9

To solve this question I need to do the reverse of solving a polynomial because instead of trying to get factors to get roots I am trying to use factors to get roots. As shown above, I resulted with $(x)(x-1)(x-3)$. Then I plugged in 2 to see if it equals to 10. However, it didn't so I multiply x by -5 because then both side would equal, and resulted the final equation as $f(x) = (-5x)(x-1)(x-3)$.

Please note that if c is a root than the polynomial is divisible by $x-c$.

So: $(x-0)(x-1)(x-3)$ are factors.

Another condition is that $f(2) = 10$
So you can find the $f(x) = -5x^3 + 20x^2 - 15x$.

Exercise 11.8. Find a polynomial of degree 4 with real coefficients, whose roots include -2 , 5 , and $3 - 2i$.

$$\begin{array}{lll}
 -2 = x & 5 = x & \\
 x+2 = 0 & x-5=0 & 3\pm 2i = x \\
 -2+2 = 0 & 5-5=0 & x-3-2i = 0 ; x-3+2i = 0 \\
 (x+2) & (x-5) & (x-3-2i) ; (x-3+2i)
 \end{array}$$

Please Explain!

Why you are getting \pm sign.

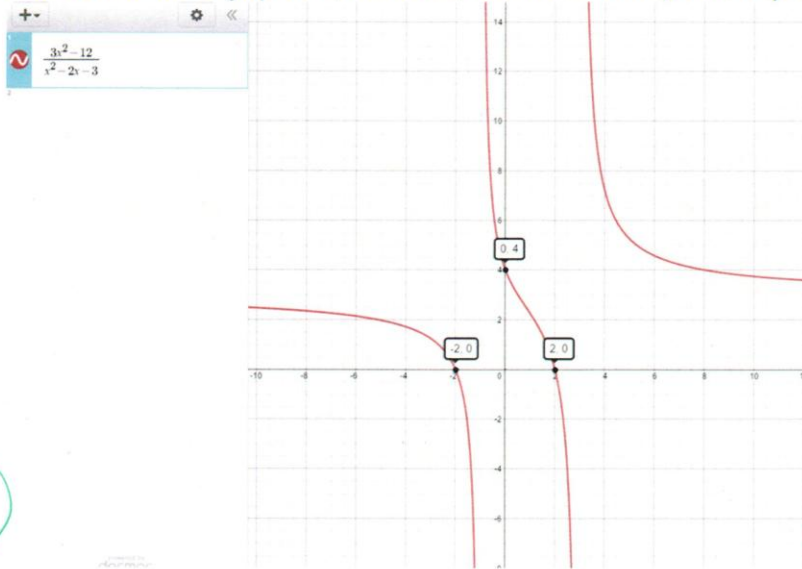
Answer: $f(x) = (x+2)(x-5)(x-3-2i)(x-3+2i)$

8

Simplify!

To solve this question I use the similar method in Exercise 11.7 for this question. I plugged in the given roots and resulted $(x+2)(x-5)(x-3-2i)(x-3+2i)$. Therefore my answer is $f(x) = (x+2)(x-5)(x-3-2i)(x-3+2i)$.

Exercise II.9. Let $f(x) = \frac{3x^2-12}{x^2-2x-3}$. Sketch the graph of f . Include all vertical and horizontal asymptotes, all holes, and all x - and y -intercepts.



y = 3 is a horizontal asymptote! you can see in the graph also!

⑦

X-intercept = -2, 2

Y-intercept = 4

$$f(x) = \frac{3x^2 - 12}{x^2 - 2x - 3}$$

$$f(x) = \frac{3(x-2)(x+2)}{(x-3)(x+1)}$$

$$(x-3)(x+1) \neq 0$$

Vertical asymptote = $x = -1$; $x = 3$

Deg(p) = Deg(q); then the horizontal asymptote is the line $y = a/b$; $2/2 = 1$.

Horizontal asymptote = $y = 1$

X-intercept is the point where the graph hits on the x-axis; Y-intercept is the point where the graph hits on the y-axis. This resulted me with the x-intercept of -2 and 2, and y-intercept of 4. For the vertical asymptote, I notice that the denominator cannot equal to zero, because if it does then the equation is undefined. Therefore, I did $x^2-2x-3 \neq 0$, and it resulted me with $x \neq -1$ and $x \neq 3$, which is also that my vertical asymptote is $x = -1$ and $x = 3$. For the horizontal asymptote, I first look at the greatest degree both the numerator and denominator, the rule if the degree of both is the same is that then the horizontal asymptote is the line $y = a/b$, which is that $2/2$ in this equation. This resulted me with a horizontal asymptote of $y = 1$.

*$\frac{3x^2}{1x^2} \Rightarrow$ horizontal asymptote is the ratio of leading coefficients
 $\frac{3}{1} = 3$*

Exercise II.10. Solve for x:

a) $x^4 + 2x < 2x^3 + x^2$, b) $x^2 + 3x \geq 7$, c) $\frac{x+1}{x+4} \leq 2$

$x^4 - 2x^3 - x^2 + 2x < 0$

A)

$x^4 + 2x < 2x^3 + x^2$

$x^4 - 2x^3 - x^2 + 2x < 0$

$x(x^3 - 2x^2 - x + 2) < 0$

$x(x-2)(x^2-x) < 0$

$x \neq 2, x \neq 1, x \neq 0$

$S = (-1, 0) \cup (1, 2)$

Checking:

$x = 3/2$

$(1/2)^4 + 2(1/2) < 2(1/2)^3 + (1/2)^2$ $(3/2)^4 + 2(3/2) < 2(3/2)^3 + (3/2)^2$

$(1/16) + 1 < (1/4) + (1/4)$ $(81/16) + 3 < 2(27/8) + 3$

$(17/16) < (1/2)$ $(129/16) < (156/16)$

False

Check

True

for example $x = -0.5$
Not true

You must graph

$f(x) = x^2 - 2x^3 - x^2 + 2x$

The part of the graph under x axes will be the solution.

Don't rely just on checking some values!

7

For this problem, I first move everything to one side and make another 0. Since I realized that it is dividable by x, for that reason I rewrote the inequality as $x(x^3 - 2x^2 - x + 2) = 0$. Then I use the method of FOIL, which resulted me with $x(x - 2)(x^2 - x) < 0$. Therefore, I resulted with $x \neq 2, x \neq 1, \text{ and } x \neq 0$. As shown in the checking, any numbers in between 0 and 1 makes this inequality false. This proves that x cannot be any numbers between 0 and 1. My final answer for this question is $S = (-1, 0) \cup (1, 2)$.

Please try to use the same type and size of writing!

$S = [1, 2.27]$

B)

$$x^2 + 3x \geq 7$$

$$x^2 + 3x - 7 \geq 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-3 \pm \sqrt{3^2 - 4(1)(-7)}}{2(1)}$$

$$= \frac{-3 \pm \sqrt{37}}{2}$$

$$S = \left(-\infty, \frac{-3 - \sqrt{37}}{2}\right] \cup \left[\frac{-3 + \sqrt{37}}{2}, \infty\right) \quad \checkmark$$

Checking:

$x=0$	$x=10$
$x^2 + 3x \geq 7$	$x^2 + 3x \geq 7$
$0^2 + 3(0) \geq 7$	$10^2 + 3(10) \geq 7$
$0 + 0 \geq 7$	$130 \geq 7$
False	True

*↓
again
do not rely only
in checking*

Similar to Exercise 11.10A), I moved everything to one side leaving another equal to 0. For this problem, I then use the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. As I plugged in the numbers, I got $x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-7)}}{2}$. This resulted me with $x = \frac{-3 \pm \sqrt{37}}{2}$. My final answer for this problem is $S = \left(-\infty, \frac{-3 - \sqrt{37}}{2}\right) \cup \left(\frac{-3 + \sqrt{37}}{2}, \infty\right)$. As shown in my checking, $x=0$ and it makes the inequality false, this proves that x cannot equal to any numbers in between $\frac{-3 - \sqrt{37}}{2}$ and $\frac{-3 + \sqrt{37}}{2}$.

*justify
your
solution.*

c)

$$\frac{x+1}{x+4} \leq 2$$

$$x + 1 \leq 2x + 8$$

$$\frac{-x - 8}{-7} \geq x$$

$$S = (-\infty, -7] \cup (-4, \infty)$$

Checking:

$x = -7$	$x = -6$
$\frac{-7 + 1}{-7 + 4} \leq 2$	$\frac{-6 + 1}{-6 + 4} \leq 2$
$\frac{-6}{-3} \leq 2$	$\frac{-5}{-2} \leq 2$
$2 \leq 2$	$5/2 \leq 2$
True	False

Due to the fact that a denominator cannot equal zero because if the denominator is zero then the inequality is undefined. Therefore, $x \neq -4$, because $-4+4=0$ and this leaves the inequality undefined.

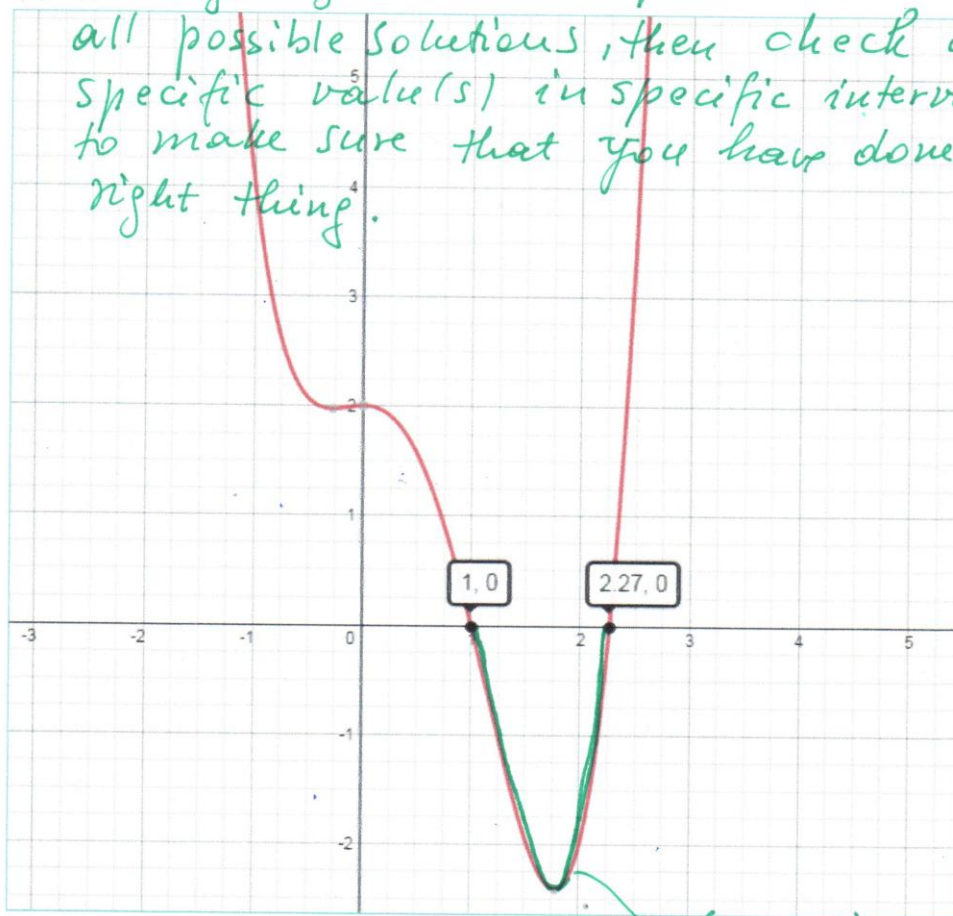
$x = -4$ is a vertical asymptote.

For this problem, I first multiply both side by $(x+4)$ to cancel the denominator, which resulted me with $x + 1 \leq 2x + 8$. I then subtract x and 8 from both side to cancel some of the numbers, which resulted me with $-7 \geq x$. As I took notice that $(x+4)$ is a denominator, x cannot equal to -4 because $-4+4=0$ and it makes the denominator equals to 0 . The denominator cannot equal to zero because if the denominator equals to 0 then the inequality is undefined. For this reason, $x \neq -4$. This resulted me with my final answer that $S = (-\infty, -7] \cup (-4, \infty)$. As shown in the checking, when $x=-6$, this makes the inequality false. This can also proves that x cannot equal to any numbers between -7 and -4 .

$y = -1$ is horizontal asymptote.

For a future reference:

1) Always try to solve the problem including all possible solutions, then check any specific value(s) in specific intervals to make sure that you have done the right thing.



I graphed it for you: The solution
 $S = [1, 2.27]$
Please focus a little more in polynomial inequalities!

Exercise II.10. Solve for x :

a) $x^4 + 2x < 2x^3 + x^2$,

b) $x^2 + 3x \geq 7$,

c) $\frac{x+1}{x+4} \leq 2$

$x^4 - 2x^3 - x^2 + 2x < 0$

A)

$x^4 + 2x < 2x^3 + x^2$

$x^4 - 2x^3 - x^2 + 2x < 0$

$x(x^3 - 2x^2 - x + 2) < 0$

$x(x-2)(x^2 - x) < 0$

$x \neq 2, x \neq 1, x \neq 0$

$S = (-1, 0) \cup (1, 2)$

Checking:

$x = 3/2$

$(3/2)^4 + 2(3/2) < 2(3/2)^3 + (3/2)^2$

$(81/16) + 3 < 2(27/8) + 3$

$(129/16) < (156/16)$

True

$x = 1/2$
 $(1/2)^4 + 2(1/2) < 2(1/2)^3 + (1/2)^2$

$(1/16) + 1 < (1/4) + (1/4)$

$(17/16) < (1/2)$

False

*you must graph
 $f(x) = x^4 - 2x^3 - x^2 + 2x$
 The part of the graph under x axes will be the solution.*

Do not rely just on checking some values!

7

For this problem, I first move everything to one side and make another 0. Since I realized that it is dividable by x, for that reason I rewrote the inequality as $x(x^3 - 2x^2 - x + 2) = 0$. Then I use the method of FOIL, which resulted me with $x(x - 2)(x^2 - x) < 0$. Therefore, I resulted with $x \neq 2, x \neq 1, \text{ and } x \neq 0$. As shown in the checking, any numbers in between 0 and 1 makes this inequality false. This proves that x cannot be any numbers between 0 and 1. My final answer for this question is $S = (-1, 0) \cup (1, 2)$.

Jayson Anderson

4/9/2015

Pre-Cal 1375

Review of polynomials and rational Functions

Exercise II.1. Divide the polynomials $:(2x^3+x^2-9x-8)/2x+3$

$$\begin{array}{r}
 \underline{X^2-x-3+(1/2x+3)} \\
 2x+3 \overline{) 2x^3+x^2-9x-8} \\
 \underline{-2x^3-3x^2} \\
 6x-8 \\
 \underline{2x^2+3x} \\
 6x-8 \\
 \underline{-6x+9} \\
 1
 \end{array}$$

(a)

$$X^2-x-3 + (1/2x+3)$$

To do long division with polynomials I Divide the first term of the numerator by the first term of the denominator, and put that in the answer, at the top. Afterwards I multiply the denominator by that answer, put that below the numerator. Subtract to create a new polynomial. Then repeat until the result becomes zero or there's a number that can't be divided into.

Check

$$\begin{aligned}
 (2x+3)(X^2-x-3)+1 &= 2x^3+x^2-9x-8 \\
 2x^3-2x^2-6x+3x^2-3x-9+1 &= 2x^3+x^2-9x-8 \\
 2x^3+x^2-9x-8 &= 2x^3+x^2-9x-8z+u \quad ?
 \end{aligned}$$

To check long division you multiply the divisor by the quotient and if the problem was done right the dividend should be the answer because the opposite of division is multiplication.

Exercise II.2. Find the remainder when dividing $x^3 + 3x^2 - 5x + 7$ by $x + 2$.

$$\begin{array}{r}
 \underline{X^2+x-7+(21)/(X+2)} \\
 X+2 \overline{) x^3 + 3x^2 - 5x + 7} \\
 \underline{-x^3 - 2x^2} \\
 5x^2-5x+7 \\
 \underline{-5x^2-10x} \\
 7x+7 \\
 \underline{7x+14} \\
 21
 \end{array}$$

$$\text{Remainder } 21$$

Jayson Anderson

4/9/2015

Pre-Cal 1375

87
100

Review of polynomials and rational Functions

Exercise II.1. Divide the polynomials $:(2x^3+x^2-9x-8)/2x+3$

(9)

$$\begin{array}{r} \underline{X^2-x-3+(1/2x+3)} \\ 2x+3 \overline{) 2x^3+x^2-9x-8} \\ \underline{-2x^3-3x^2} \\ 3x^2-9x-8 \\ \underline{ 3x^2+3x} \\ 6x-8 \\ \underline{ 6x+9} \\ 17 \end{array}$$

To do long division with polynomials I Divide the first term of the numerator by the first term of the denominator, and put that in the answer, at the top. Afterwards I multiply the denominator by that answer, put that below the numerator. Subtract to create a new polynomial. Then repeat until the result becomes zero or there's a number that can't be divided into.

1

$$\boxed{X^2-x-3+(1/2x+3)}$$

Check

$$\begin{aligned} (2x+3)(X^2-x-3)+1 &= 2x^3+x^2-9x-8 \\ 2x^3-2x^2-6x+3x^2-3x-9+1 &= 2x^3+x^2-9x-8 \\ 2x^3+x^2-9x-8 &= 2x^3+x^2-9x-8z+u \end{aligned}$$

To check long division you multiply the divisor by the quotient and if the problem was done right the dividend should be the answer because the opposite of division is multiplication.

Exercise II.2. Find the remainder when dividing $x^3 + 3x^2 - 5x + 7$ by $x + 2$.

(10)

$$\begin{array}{r} \underline{X^2+x-7+(21/X+2)} \\ X+2 \overline{) x^3 + 3x^2 - 5x + 7} \\ \underline{-x^3 - 2x^2} \\ 5x^2-5x+7 \\ \underline{ 5x^2+10x} \\ -7x+7 \\ \underline{ 7x+14} \\ 21 \end{array}$$

$$\boxed{\text{Remainder 21}}$$

If $a > 0$, the parabola opens upwards

$$ax^2 + bx + c$$

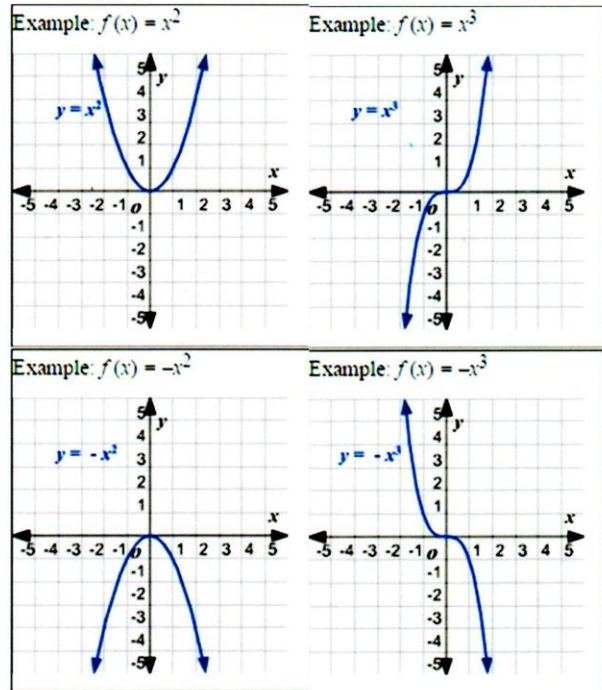
if $a < 0$, it opens downwards.

$$-ax^2 + bx + c$$

If the highest degree is odd then
the graph will stretch to both

– and + y infinity .

If the highest degree is even then
the graph will only stretch towards
only one direction .



The function isn't negative

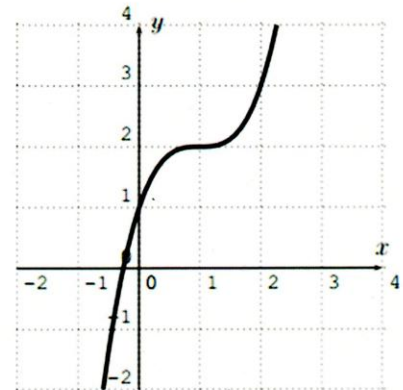
y-intercept = 1

Largest degree = odd

iii) $f(x) = x^3 - 3x^2 + 3x + 1$



a)



The function isn't negative

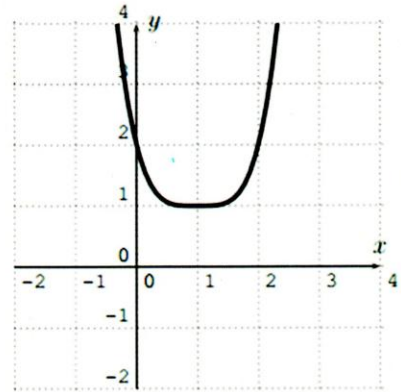
y-intercept=2

Largest degree = even

$$\text{iv) } f(x) = x^4 - 4x^3 + 6x^2 - 4x + 2$$



b)



The function is negative

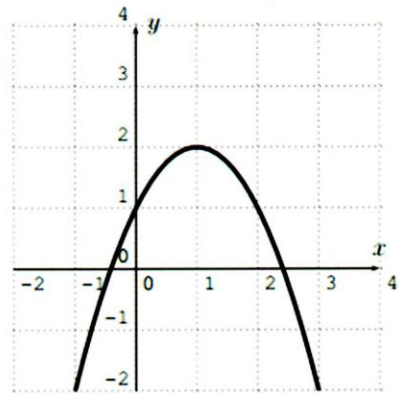
y-intercept=2

Largest degree=even

$$\text{i) } f(x) = -x^2 + 2x + 1$$



c)



The function is negative

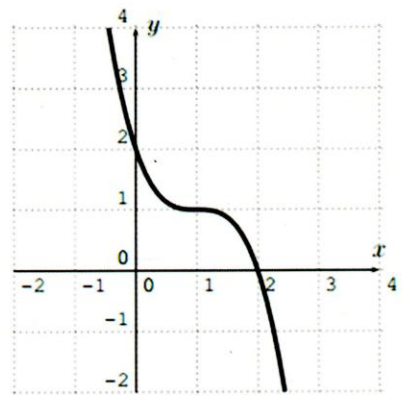
y-intercept=2

largest degree=odd

$$\text{ii) } f(x) = -x^3 + 3x^2 - 3x + 2$$



d)



Exercise II.5. Sketch the graph of the function:

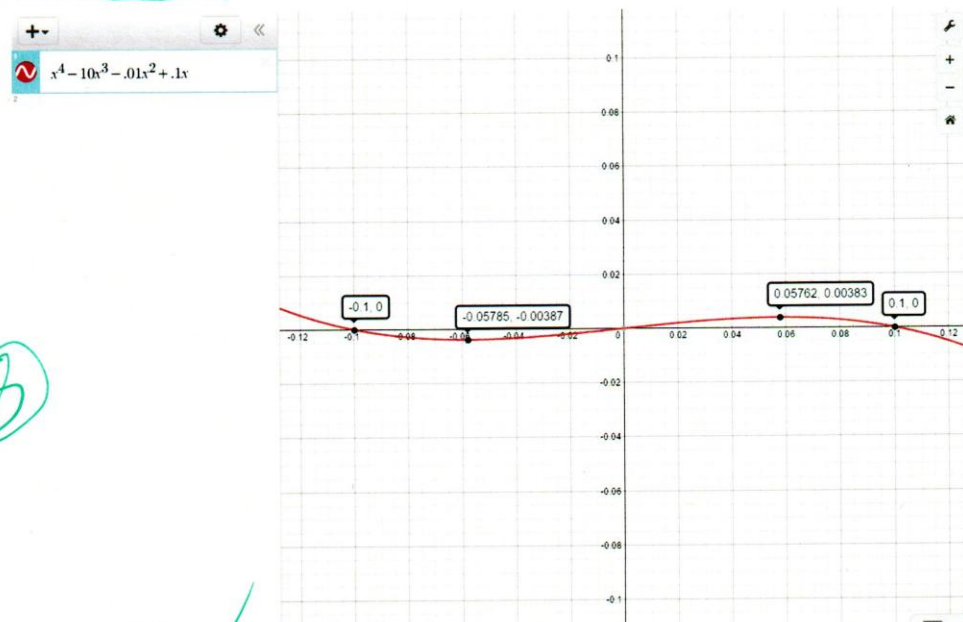
$$f(x) = x^4 - 10x^3 - 0.01x^2 + 0.1x$$

- Find all roots, all maxima and all minima of the graph with the calculator.

Roots- a solution that satisfies the equation. Also called the zeros because it's all the point where the graph hits the x-axis.

Maxima -the highest point of graph and relative maxima is the highest point on graph. In this graph there's no maxima because graph never ended and went onto infinity.

Minima- the lowest point of graph and relative minima is the lowest point on graph. In this graph there's no minima because graph never ended and went onto infinity.



Roots=(-.1,0,.1)
Maxima=(.05762,.00383)
Minima=(-0.05785,-0.00387)

Please note that there are absolute extremas and relative extremas

This graph is unlimited in both directions so there are not absolute extremas but yes there are relative or local extremas

Exercise II.6. Find all roots of $f(x) = x^3 + 6x^2 + 5x - 12$.

Use this information to factor $f(x)$ completely.

(a0) p- possible factors of 12 so $p = \pm(4, 3, 2, 1, 12)$

(a3) q- possible factor of 1 so $q = \pm 1, -1$

Possible roots $= -4, -3, -2, -1, 1, 12, 2, 3, 4$

$$x^3 + 6x^2 + 5x - 12 = 0$$

$$X = 1$$

$$(1)^3 + 6(1)^2 + 5(1) - 12 = 0$$

$$1 + 6 + 5 - 12 = 0$$

$$0 = 0$$

substitute possible roots to find real roots.

$$X = -3$$

$$(-3)^3 + 6(-3)^2 + 5(-3) - 12 = 0$$

$$-27 + 54 - 15 - 12 = 0$$

$$0 = 0$$

$$X = -4$$

$$(-4)^3 + 6(-4)^2 + 5(-4) - 12 = 0$$

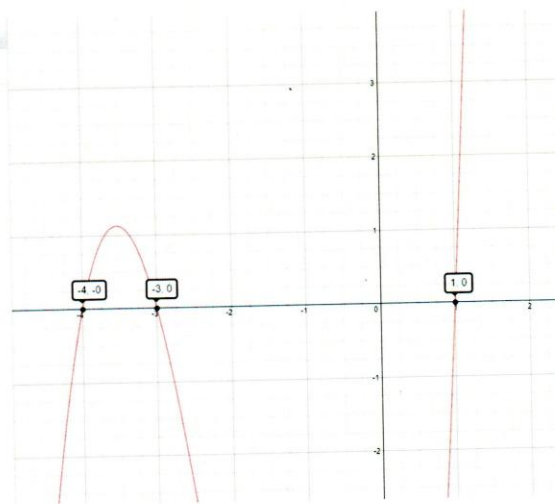
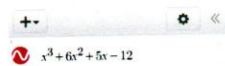
$$-64 + 96 - 20 - 12 = 0$$

$$0 = 0$$

$$\text{Roots} = 1, -3, -4$$

$$f(x) = (x - 1)(x + 3)(x + 4)$$

Please check by multiplying!



Exercise II.7. Find a polynomial of degree 3 whose roots are 0, 1, and 3, and so that $f(2) = 10$.

Please Explain this procedure!

0	1	3
$0=x$	$x=1$	$x=3$
$x-0=0$	$x-1=0$	$x-3=0$
(x) ✓	$(x-1)$ ✓	$(x-3)$ ✓

(7)

$$F(x)=(x)(x-1)(x-3)$$

$$F(2)=10$$

$$(2)(2-1)(2-3)=10$$

$$(2)(1)(-1)=10$$

$$-2 \neq 10$$

0 doesn't equal -5

$$-5(2)(2-1)(2-3)=10$$

$$-5(2)(1)(-1)=10$$

$$(-5)(-2)=10$$

$$10=10$$

$F(x)=(-5x)(x-1)(x-3)$

Multiply back to see if the product gives you the polynomial

with roots that are given to you!

Basically you need to write the polynomial as answer.

Exercise II.8. Find a polynomial of degree 4 with real coefficients, whose roots include -2 , 5 , and $3 - 2i$.

$$-2$$

$$X = -2$$

$$X + 2 = 0$$

$$-2 + 2 = 0$$

$$(X + 2)$$

$$-5$$

$$x = 5$$

$$x - 5 = 0$$

$$5 - 5 = 0$$

$$(x - 5)$$

$$3 - 2i$$

$$3 + 2i = x$$

$$x - 3 - 2i = 0 ; x - 3 + 2i = 0$$

$$(x - 3 - 2i) ; (x - 3 + 2i)$$

①

$$F(x) = (x+2)(x-5)(x-3-2i)(x-3+2i)$$

→ multiply and simplify!

→ You have to explain that if c is the root then the polynomial is divisible by $x - c$ that is for all roots.

So if -2 , 5 , and $3 - 2i$ are roots of the polynomial then you know that it is divisible

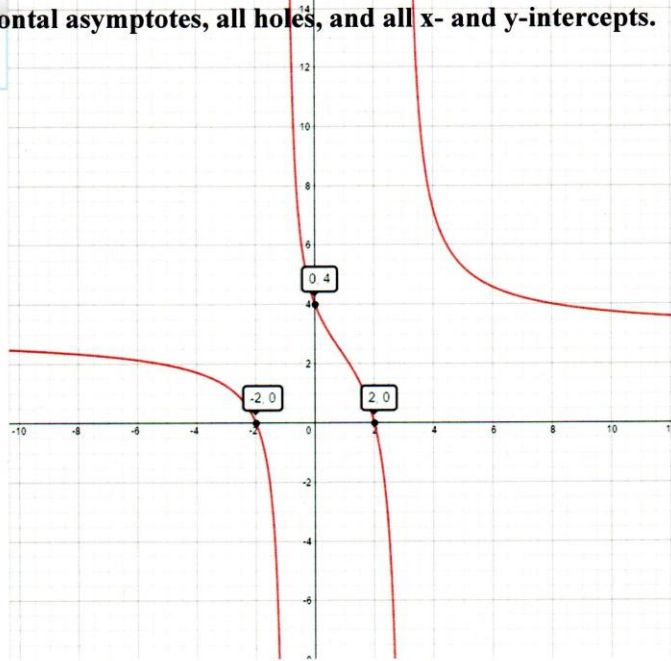
by $[x - (-2)] ; (x - 5) ; [x - (3 - 2i)]$

another condition is $f(2) = 10$

Exercise II.9. Let $f(x) = \frac{3x^2-12}{x^2-2x-3}$. Sketch the graph of f . Include all vertical and horizontal asymptotes, all holes, and all x- and y-intercepts.

$$\frac{3x^2-12}{x^2-2x-3}$$

9



x-intercept-or the points where the graph hits x-axis = -2, 2
 y-intercept-or the points where the graph hits the y-axis = 4

Horizontal asymptotes- since this graph as the rule $\deg(p) = \deg(q)$ then the vertical asymptote would be the highest coefficient of p /highest coefficient of q .

Which is $3/1 = 3$

horizontal asymptote = $y = 3$

X

Vertical asymptote- To get the vertical asymptote you need to factor out the whole equation, cancel out what you can then find the root for the denominator. The root would be the vertical asymptote.

$$(3x^2-12)/(x^2-2x-3)$$

$$(3(x-2)(x+2))/(x-3)(x+1)$$

$$(x-3)(x+1)=0$$

$$x=3, x=-1$$

Vertical asymptote $\Rightarrow x=-1, x=3$

Exercise II.10. Solve for x:

a) $x^4 + 2x < 2x^3 + x^2$

$$x^4 + 2x < 2x^3 + x^2$$

Subtract $2x^3 + x^2$ from both sides

$$x^4 + 2x - (2x^3 + x^2) < 2x^3 + x^2 - (2x^3 + x^2)$$

Refine

$$x^4 - 2x^3 - x^2 + 2x < 0$$

Factor the left hand side $x^4 - 2x^3 - x^2 + 2x$: $x(x+1)(x-1)(x-2)$

$$x^4 - 2x^3 - x^2 + 2x$$

Factor out x

$$= x(x^3 - 2x^2 - x + 2)$$

Factor $x^3 - 2x^2 - x + 2$: $(x+1)(x-1)(x-2)$

$$= x(x+1)(x-1)(x-2)$$

$$x(x+1)(x-1)(x-2) < 0$$

Solution: $-1 < x < 0$ or $1 < x < 2$
Interval Notation: $(-1, 0) \cup (1, 2)$