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Working on Exponents and Radicals

All operations in Mathematics have the inverse operation that undoes the previous one.

How the exponents and radicals are inverse operations?

Example 1:

1) $3^2 = 9$ while $\sqrt{9} = 3$ or $\sqrt{3^2} = 3$
2) $7^2 = 49$ while $\sqrt{49} = 7$ or $\sqrt{7^2} = 7$
3) $(\frac{2}{3})^2 = \frac{4}{9}$ while $\sqrt{\frac{4}{9}} = \frac{2}{3}$ or $\sqrt{(\frac{2}{3})^2} = \frac{2}{3}$

What if we had to find $\sqrt{x^2}$

As we can see from the examples above: squaring a number and finding the square root undue each other as operations. **That means that if you apply both operations one after the other the number doesn't change.**

For that reason the $\sqrt{x^2} = x$

What if we find the square root first and then we square it? Let's get an example:

$$\sqrt{16} = 4 \; while \;\; 4^2 = 16 \;\; or \; (\sqrt{16}\,)^2 = 16$$

We see that the same thing happens if we find the radical first and then square.

The example above is very simple because the 16 is a perfect square, so we can find the $\sqrt{16}$ as a whole number.

Look at the example below to create a better idea of what happens at any kind of number when we apply these two operations one after another:

$(\sqrt{3})^2$ in this case you should know that the best way to prove that the result will be 3 is using the properties of exponents.

The property that can be used in this example is the power of the power property. $(a^m)^n = a^{m \cdot n}$

Let's work on it: $1)\ (\sqrt{3})^2 = (3^{\frac{1}{2}})^2 = 3^{2\cdot\frac{1}{2}} = 3^1 = 3$

So as you can see the exponent can go under the $\sqrt{}$

So we could have done it in easier way: $2)\ (\sqrt{3})^2 = \sqrt{3^2} = 3$

Example 2: Use both ways to find using properties of exponents:

1) $(\sqrt{\frac{4}{7}})^2 = (\frac{4}{7})^{\frac{1}{2}\cdot 2} = \frac{4}{7}$

2) $(\sqrt{\frac{4}{7}})^2 = \sqrt{(\frac{4}{7})^2} = \sqrt{\frac{4^2}{7^2}} = \frac{4}{7}$

3) $\sqrt{2.5^2} = 2.5$

Similarly we can work in $\sqrt[3]{}$ and 3 rd exponent.

$Example\ 3: \left(\sqrt[3]{27}\right)^3 = 27$

How can we prove it? $Method\ 1)\left(\sqrt[3]{3^3}\right)^3 = 3^3 = 27$

$Method\ 2)\ (\sqrt[3]{27})^3 = \sqrt[3]{27^3} = 27$

(27 is a perfect cube of 3 so the number that is under $\sqrt[3]{}$ is a perfect cube so $\sqrt[3]{3^3} = 3$

Example 4 Prove that $\left(\sqrt[3]{13}\right)^3 = 13$

$Method\ 1)\ \left(\sqrt[3]{13}\right)^3 = 13^{\frac{1}{3}\cdot 3} = 13$ $Method\ 2: \left(\sqrt[3]{13}\right)^3 = \sqrt[3]{13^3} = 13$

Use these examples and properties of exponents and radicals to simplify expressions:

1) $\sqrt{16x^2z^7y^3}$

2) $\sqrt{162c^{12}k^{13}s^6}$

3) $\sqrt{24n^5\, m^3\, n^{\frac{2}{5}}}$

4) $\sqrt[3]{64x^3v^2q^6}$

5) $\sqrt[3]{250c^{10}d^{15}}$

6) $\sqrt[3]{16n^{\frac{2}{3}}m^{13}n^9}$

Write the exponents at the same base:

1) $16 \quad = 2$
2) $3 \quad = 27$
3) $25 \quad = 5$
4) $x^{12} = (x^3)$
5) $y^4 = (y^3)$

Write under the same exponent:

1) $6 \quad = 36$

2) $7 \quad = 49$

3) $x^9 = x^{18}$

4) $64c^6 = (\quad)$

5) $\left(\frac{1}{81}\right) \quad = (\frac{1}{3})^8$

$Evaluate\ expressions\ by\ using\ the\ properties$

$of\ radicals\ and\ exponents$:

1) $(8x^{3}y^{9})^{\frac{1}{3}}$

2) $\frac{(2c)^4 b^4}{2b^2c^4}$

3) $\sqrt[4]{\frac{4^3x^8}{4y^{20}}}$

5) $(\sqrt[7]{\frac{3}{8}})^{14}$

6) $\sqrt[6]{x^3}\ \sqrt[3]{x^{12}}$

7) $\sqrt[8]{(\frac{5}{9})^4} = \sqrt[16]{(\frac{5}{9})}$

8) $\frac{(\sqrt[3]{2^4})^{\frac{3}{4}}}{\sqrt[6]{4^3}}$