$f(x) = 2 - 3x \qquad g(x) = 2x^2 - 2x - 11$

$f(6) = 2 - 3(6) = 2 - 18 = -16$

$f(-5) = 2 - 3(-5) = 2 + 15 = 17$

$$g(8) = 2(8)^2 - 2(8) - 11$$
$$= 128 - 16 - 11 = 101$$

$$g(-5) = 2(-5)^2 - 2(-5) - 11$$
$$= 50 + 10 - 11 = 49$$

$g(a) = 2a^2 - 2a - 11$

$f(x) = 3x + 4$

$f(2) = 3(2) + 4 = 10$

$f(-3) = 3(-3) + 4 = -5$

$f(-1) = 3(-1) + 4 = 1$

$f(x) = \sqrt{x^2 - 3}$

$f(2) = \sqrt{(2)^2 - 3} = \sqrt{1} = 1$

$f(-3) = \sqrt{(-3)^2 - 3} = \sqrt{9 - 3} = \sqrt{6}$

$f(-1) = \sqrt{(-1)^2 - 3} = \sqrt{-2} = \sqrt{2}\, i$

$\rightarrow$ undefined in $\mathbb{R}$

$f(x) = \dfrac{x+2}{x+3}$

$f(2) = \dfrac{(2)+2}{(2)+3} = \dfrac{4}{5} \qquad f(-1) = \dfrac{(-1)+2}{(-1)+3} = \dfrac{1}{2}$

$f(-3) = \dfrac{(-3)+2}{(-3)+3} = -\dfrac{1}{0} \rightarrow$ undefined

$$y = \begin{cases} -\frac{4}{3}x - 7, & x < -5 \\ x + 6, & -4 < x \leq 0 \\ 2x + 7, & 5 \leq x \end{cases}$$

$f(9) = 2(9) + 7 = 25$

$f(-5)$ is undefined

$f(-6) = -\frac{4}{3}(-6) - 7 = 1$

$f(-1) = (-1) + 6 = 5$

$$y = \begin{cases} \frac{1}{5}x + 5, & x \leq -5 \\ \frac{3}{4}x + 4, & -3 \leq x < -1 \\ -1x + 5, & 5 \leq x \end{cases}$$

Domain: Look at the interval cases

$(-\infty, -5] \cup [-3, -1) \cup [5, \infty)$

$f(-3) = \frac{3}{4}(-3) + 4 = -\frac{9}{4} + \frac{16}{4} = \frac{7}{4}$

$f(-8) = \frac{1}{5}(-8) + 5 = -\frac{8}{5} + \frac{25}{5} = \frac{17}{5}$

$f(10) = -1(10) + 5 = -5$

Difference Quotient (variations on slope formula)

$$\frac{f(x+h)-f(x)}{h} \quad \text{or} \quad \frac{f(x)-f(a)}{x-a}$$

Let $f(x) = x^2+2x+3$

Find difference quotient $\dfrac{f(x+h)-f(x)}{h}$

$$f(x+h) = (x+h)^2 + 2(x+h)+3$$

$$= x^2+2hx+h^2+2x+2h+3$$

$$-\ f(x) = -x^2 \qquad -2x \qquad -3$$

$$f(x+h)-f(x) = 2hx+h^2 \qquad +2h$$

*Note

	x	$+y$
x	x^2	xy
$+y$	xy	y^2

$x^2+2xy+y^2$

$$\to \frac{f(x+h)-f(x)}{h} = \frac{2hx+h^2+2h}{h}$$

$$= \frac{h(2x+h+2)}{h}$$

$$= 2x+h+2$$

$$\frac{2hx}{h}+\frac{h^2}{h}+\frac{2h}{h}$$

$$= 2x+h+2$$

Practice

$f(x) = 2x^2 + 2x - 1$ Find $\dfrac{f(x+h) - f(x)}{h}$

$f(x+h) = 2(x+h)^2 + 2(x+h) - 1$

$= 2(x^2 + 2hx + h^2) + 2(x+h) - 1$

$= 2x^2 + 4xh + 2h^2 + 2x + 2h - 1$

$-f(x) = -2x^2 \qquad -2x \qquad +1$

$f(x+h) - f(x) = 4xh + 2h^2 + 2h$

$\dfrac{f(x+h) - f(x)}{h} = \dfrac{4xh + 2h^2 + 2h}{h} = 4x + 2h + 2$

$f(x) = \frac{1}{x}$

Find difference quotient

$$\frac{f(x+h) - f(x)}{h}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

← Complex fraction

→ need LCD for all fractions

$x, x+h, 1$

LCD: $x(x+h)$

$$= \frac{\left(\frac{1}{x+h} - \frac{1}{x}\right)}{h} \cdot \frac{x(x+h)}{x(x+h)}$$

$$= \frac{\frac{x\cancel{(x+h)}}{\cancel{x+h}} - \frac{\cancel{x}(x+h)}{\cancel{x}}}{h\,x(x+h)}$$

$$= \frac{x - (x+h)}{h\,x(x+h)}$$

$$= \frac{\cancel{x} - \cancel{x} - h}{h\,x(x+h)}$$

$$= \frac{-\cancel{h}\ 1}{\cancel{h}\,x(x+h)}$$

$$= -\frac{1}{x(x+h)}$$

$f(x) = \sqrt{x}$ Find $\dfrac{f(x+h) - f(x)}{h}$

$$\frac{f(x+h) - f(x)}{h} = \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

Should rationalize numerator

use conjugate

$$= \frac{\left(\sqrt{x+h} - \sqrt{x}\right)}{h} \cdot \frac{\left(\sqrt{x+h} + \sqrt{x}\right)}{\left(\sqrt{x+h} + \sqrt{x}\right)}$$

$*(a-b)(a+b) = a^2 + b^2$

$$= \frac{x + h - x}{h\left(\sqrt{x+h} + \sqrt{x}\right)}$$

$$= \frac{h}{h\left(\sqrt{x+h} + \sqrt{x}\right)}$$

$$= \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$f(x)=\frac{1}{x^2}$

Find $\frac{f(x+h)-f(x)}{h}$

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{\frac{h}{1}}$$

Complex fraction
LCD of $x^2, (x+h)^2, 1$
is $x^2(x+h)^2$

$$= \frac{\left(\frac{1}{(x+h)^2}-\frac{1}{x^2}\right)}{h}\cdot\frac{x^2(x+h)^2}{x^2(x+h)^2}$$

$$= \frac{x^2-(x+h)^2}{hx^2(x+h)^2}$$

$$= \frac{x^2-(x^2+2xh+h^2)}{hx^2(x+h)^2}$$

$$= \frac{\cancel{x^2-x^2}+2xh+h^2}{hx^2(x+h)^2}$$

$$= \frac{2x+h}{x^2(x+h)^2}$$

$$f(x) = x^2 + 2x + 3 \qquad \frac{f(x) - f(a)}{x-a}$$

$$\frac{f(x)-f(a)}{x-a} = \frac{x^2+2x+3-a^2-2a-3}{x-a}$$

$$= \frac{x^2-a^2+2x-2a}{x-a}$$

$$= \frac{(x^2-a^2)+(2x-2a)}{x-a}$$

$$= \frac{(x+a)(x-a)+2(x-a)}{x-a}$$

$$= \frac{(x+a+2)\cancel{(x-a)}}{\cancel{x-a}}$$

$$= x+a+2$$