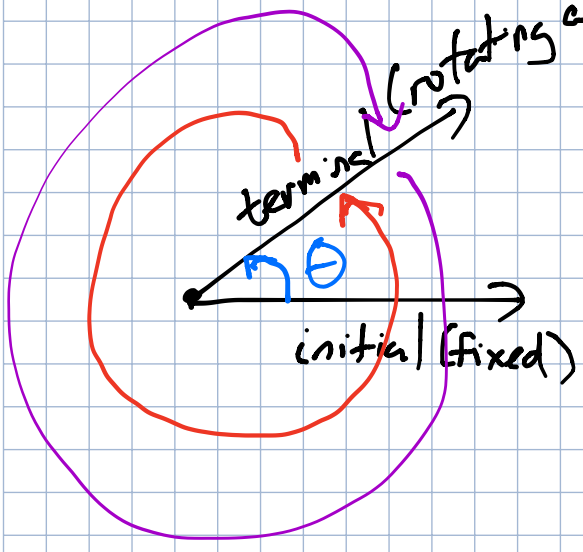
coterminal angle – an angle whose measure $m(\angle\theta) \pm 360°n$ is coterminal with $\angle\theta$



$n \in \{\pm 1, \pm 2, \pm 3, \ldots\}$

$n \in \mathbb{Z}$

they have same sides and vertex – terminal side does more full revolutions

e.g. 30 degree angle

positive coterminal angle (counter clockwise)

$30° + 360° = 390° + 360° = 750° + 360° \ldots$

negative coterminal angle (clockwise)

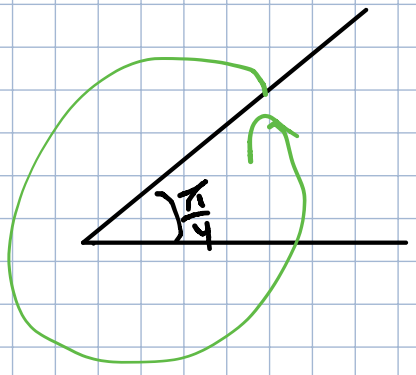
$30 - 360° = -330° - 360° = -690° - 360 \ldots$

$\ldots -690°, -330°, 30°, 390°, 750°, \ldots$

all are coterminal

e.g. $\frac{\pi}{4}$

$\frac{\pi}{4}$ rad $+ 360° \left(\frac{\pi}{180}\right)$



$\frac{\pi}{4} + \boxed{2\pi}$

↖ one rotation in radians

$$\frac{\pi}{4} + 2\pi = \frac{\pi}{4} + \frac{8\pi}{4} = \frac{9\pi}{4}$$

$\frac{9\pi}{4}$ is coterminal to $\frac{\pi}{4}$

$\theta \pm 2\pi n$, $n \in \mathbb{Z}$ is coterminal

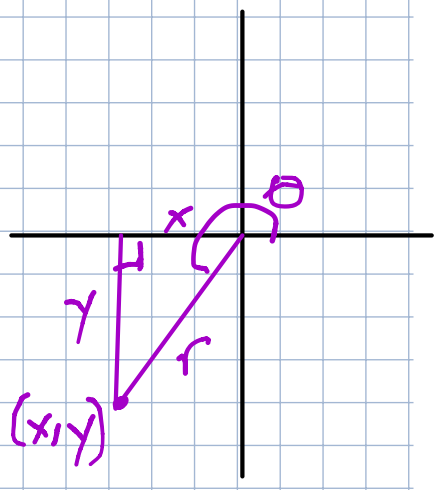
$n \in \{0, \pm 1, \pm 2, \ldots\}$

Recall last week Given (x,y), $r=\sqrt{x^2+y^2}$

$\sin\theta = \frac{y}{r}$ $\quad \csc\theta = \frac{r}{y}$

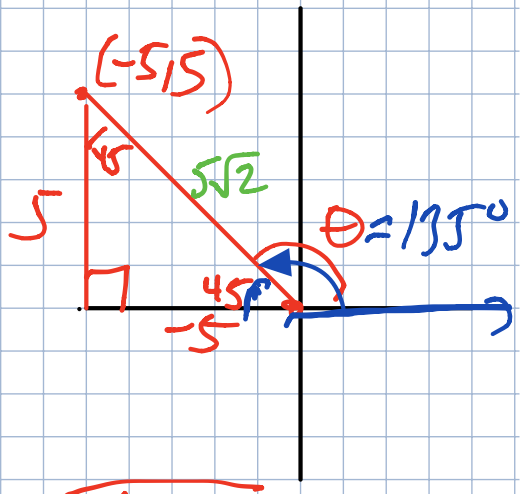
$\cos\theta = \frac{x}{r}$ $\quad \sec\theta = \frac{r}{x}$

$\tan\theta = \frac{y}{x}$ $\quad \cot\theta = \frac{x}{y}$



Given $P(-5,5)$ find six trig functions

$(-,+) \rightarrow$ QII



$\sin\theta = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\quad \csc\theta = \frac{5\sqrt{2}}{5} = \sqrt{2}$

$\cos\theta = \frac{-5}{5\sqrt{2}} = -\frac{\sqrt{2}}{2}$ $\quad \sec\theta = -\sqrt{2}$

$\tan\theta = \frac{5}{-5} = -1$ $\quad \cot\theta = -1$

$\theta = 135°$

$r = \sqrt{x^2+y^2} = 5\sqrt{2}$

$= \sqrt{(-5)^2+(5)^2}$

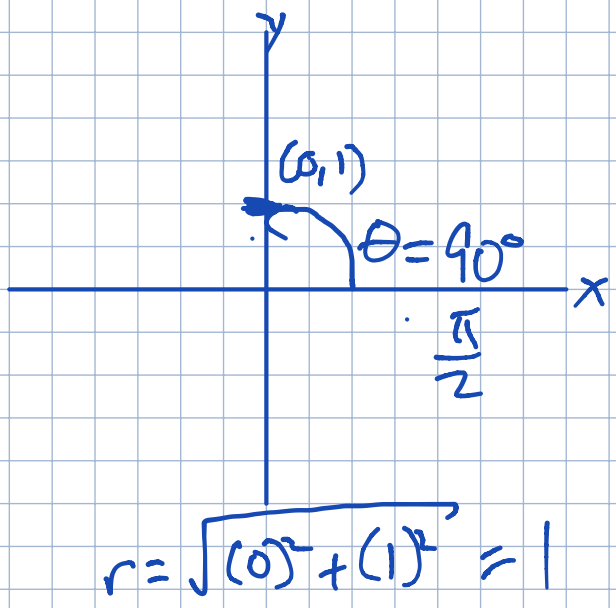
$= \sqrt{25+25}$

$= \sqrt{50} = 5\sqrt{2}$

$= \sqrt{25}\cdot\sqrt{2}$

* $\frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{\sqrt{2}}{2}$

Test $(0,1)$



$r = \sqrt{(0)^2 + (1)^2} = 1$

$\sin(90°) = 1$ $\quad \csc(90°) = 1$

$\cos(90°) = \frac{0}{1} = 0$ $\quad \sec(90°) = \text{undef}$

$\tan(90°) = \frac{1}{0} = \text{undefined}$ $\quad \cot(90°) = 0$

* Look up trig ratios for $0, 180, 270°$

$0, \pi, \frac{3\pi}{2}$

Recall

$\sin\theta = \frac{y}{r}$ $\quad \csc\theta = \frac{r}{y}$

$\cos\theta = \frac{x}{r}$ $\quad \sec\theta = \frac{r}{x}$

$\tan\theta = \frac{y}{x}$ $\quad \cot\theta = \frac{x}{y}$

unit circle — circle of radius 1.

center $(0,0)$

Equation of unit circle $(x-0)^2+(y-0)^2 = 1^2$

$x^2+y^2 = 1^2$

θ	$\frac{\pi}{180}$ rad	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	0	1	0
30	$\frac{\pi}{6}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{1}}{2}$	$\sqrt{3}$
90	$\frac{\pi}{2}$	$\frac{\sqrt{4}}{2} = 1$	0	undefined
180	π	0	-1	0
270	$\frac{3\pi}{2}$	-1	0	undefined
360	2π	0	1	0

inc

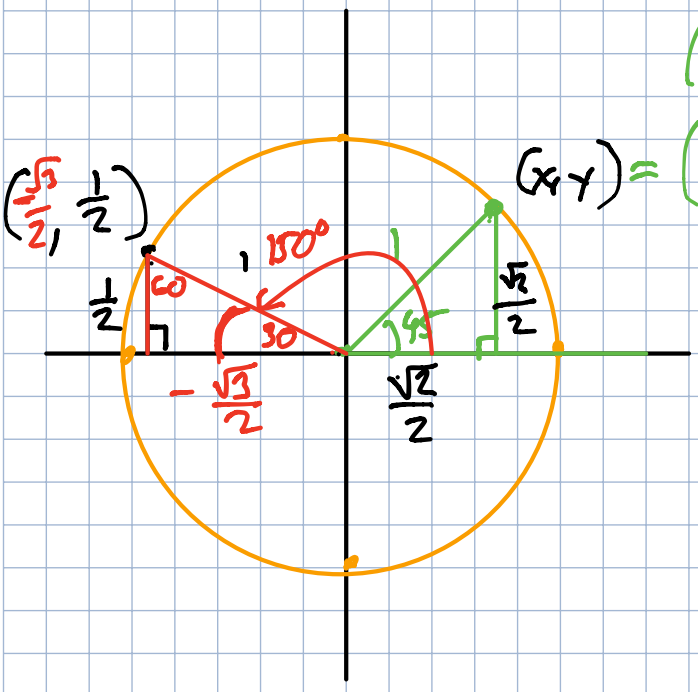
dec

four Three-finger rule

1. Memorize special angles

2. Each number of fingers represent $\frac{\sqrt{\text{fingers}}}{2}$

for special angle



(adj, opp)

$(x,y) = (\cos(45), \sin(45)) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

on unit circle

Note 150° in QII → cosθ negative
→ sinθ positive

$(x,y) = (\cos(150), \sin(150))$

* I don't expect you to remember cos(150) or sin(150

QII $\longrightarrow = (-\cos(30), +\sin(30))$

* 180 − 150 = 30°

30° is our reference angle

- acute angle between terminal side and x-axis

$= \left(-\frac{\frac{\sqrt{3}}{2}}{1}, \frac{\frac{1}{2}}{1}\right) = \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

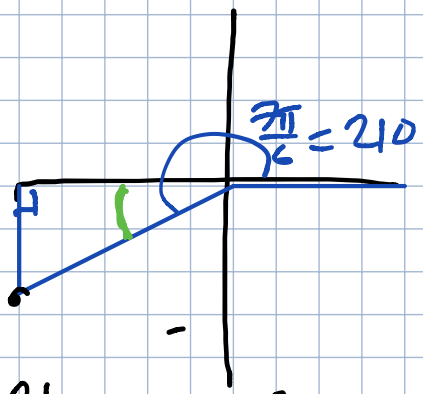
(adj, opposite)

Find points on a unit circle associated with rotation θ

$\theta = \frac{7\pi}{6}$ → to degrees $\frac{7\pi}{6} \cdot \frac{180}{\pi} = 210°$

QIII $\in (180, 270°)$ → $\cos\theta < 0$, $\sin\theta < 0$

$(x, y) = (-, -)$



$210 - 180 = 30$ degrees ↑ reference angle

$\frac{7\pi}{6} - \pi =$ ↓

$\frac{7\pi}{6} - \frac{6\pi}{6} = \frac{\pi}{6}$ radians

$(x, y) = (\cos\theta, \sin\theta)$

$= (\cos(210), \sin(210))$

$\underset{QIII}{=} (-\cos(30), -\sin(30°))$

$= \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

$(x, y) = (\cos\theta, \sin\theta)$

$= \left(\cos\left(\frac{7\pi}{6}\right), \sin\left(\frac{7\pi}{6}\right)\right)$

$\underset{QIII}{=} \left(-\cos\left(\frac{\pi}{6}\right), \sin\left(\frac{\pi}{6}\right)\right)$

$= \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

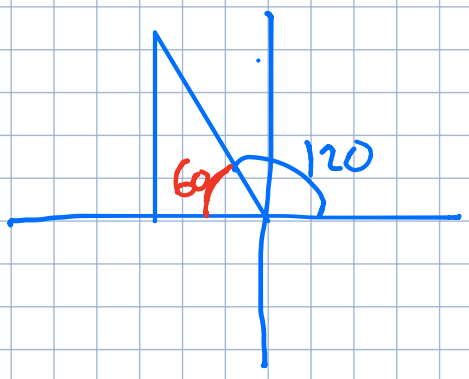
---

* $\frac{2\pi}{3}, \frac{7\pi}{4}$

Find point (x, y)

$\theta = \frac{2\pi}{3} \rightarrow \frac{2\pi}{3}\left(\frac{180}{\pi}\right) = 120°$ Ex $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

in QII



$180 - 120 = 60°$

$\pi - \frac{2\pi}{3} =$

$\frac{3\pi}{3} - \frac{2\pi}{3} = \frac{\pi}{3}$

$(x,y) = (\cos(120), \sin(120))$

$\overset{QII}{=} (-\cos(60), \sin(60))$

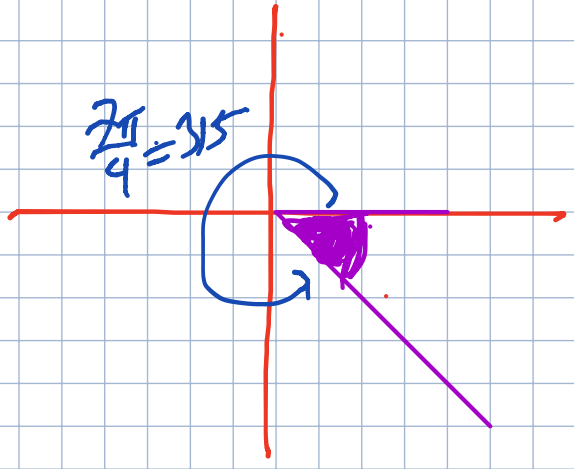
$= \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$(x,y) = \left(\cos\left(\frac{2\pi}{3}\right), \sin\left(\frac{2\pi}{3}\right)\right)$

$\overset{QII}{=} \left(-\cos\left(\frac{\pi}{3}\right), \sin\left(\frac{\pi}{3}\right)\right)$

$= \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$\theta = \frac{7\pi}{4}$ $\frac{7\pi}{4}\left(\frac{180}{\pi}\right) = 315° \rightarrow$ QIV



$360 - 315 = 45$

$2\pi - \frac{7\pi}{4} = \ldots = \frac{\pi}{4}$

$(x,y) = (\cos(315), \sin(315))$

$\overset{QIV}{=} (+\cos(45), -\sin(45))$

$= \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$$(x,y) = \left(\cos\left(\frac{7\pi}{4}\right), \sin\left(\frac{7\pi}{4}\right)\right)$$

$$\overset{}{=}_{QIV} \left(+\cos\left(\frac{\pi}{4}\right), -\sin\left(\frac{\pi}{4}\right)\right)$$

$$= \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$$

* Advantage to working in radians

If denominator is the same as that of a special angle in radians, the reference angle is $\frac{\pi}{\text{denominator}}$

e.g. $-\frac{25\pi}{6} \rightarrow \text{ref}\angle = \frac{\pi}{6}$ $\qquad -\frac{223\pi}{4} \rightarrow \text{ref}\angle = \frac{\pi}{4}$

$\frac{17252\pi}{3} \rightarrow \text{ref}\angle = \frac{\pi}{3}$