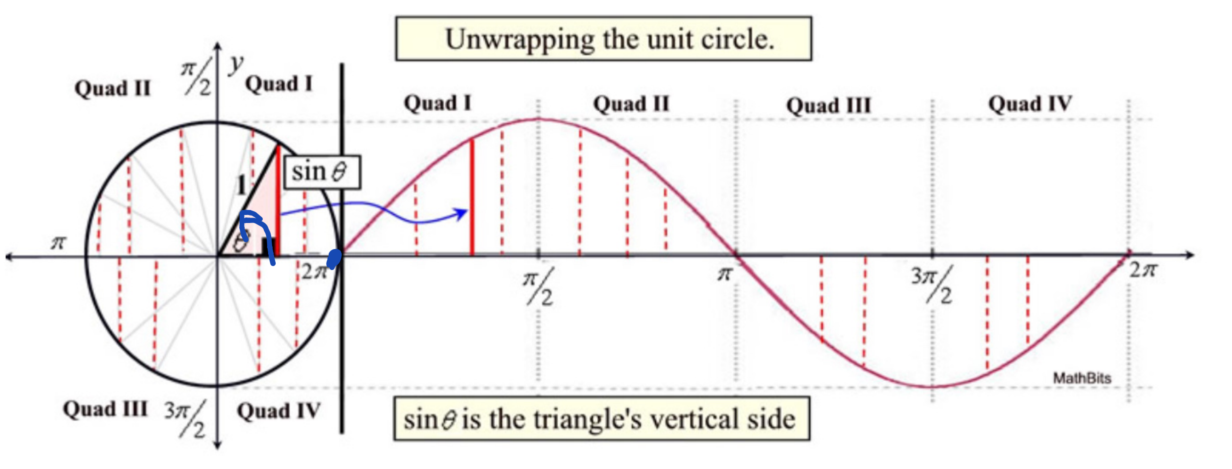
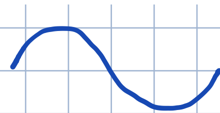
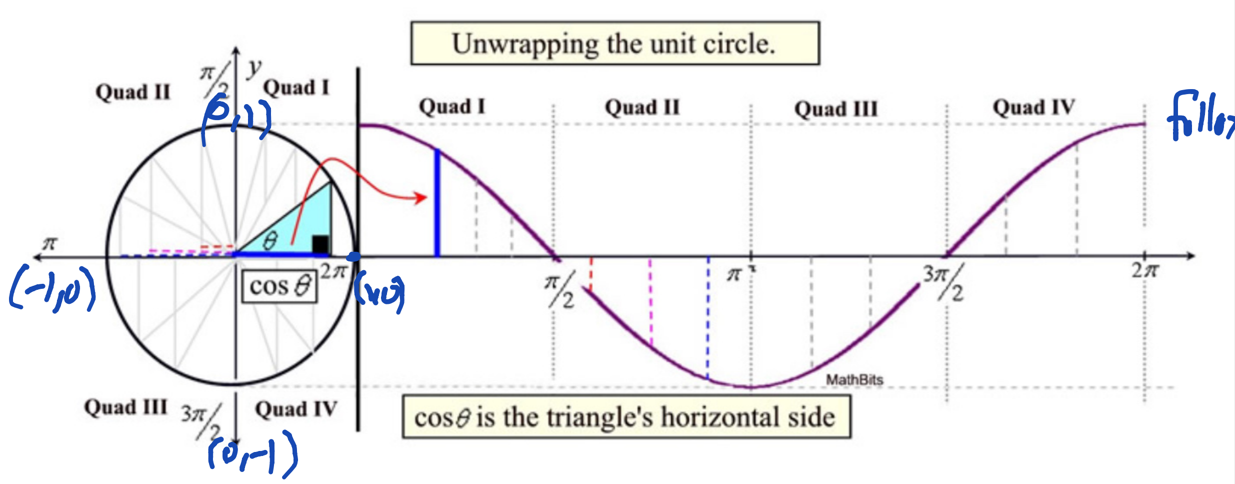


$y = \sin\theta$





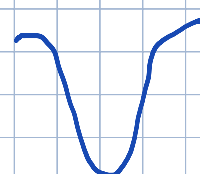
follow x

Like the sine graph, the cosine graph will "repeat", making it a periodic function.

The graph will repeat every period of 2π.

Remember that cosine is negative in Quadrants II and III (the x-coordinates are negative).

$\cos\theta$ graph looks like



$y = A\sin(Bx - C) + D$ or $y = A\cos(Bx - C) + D$

$|A|$ = amplitude

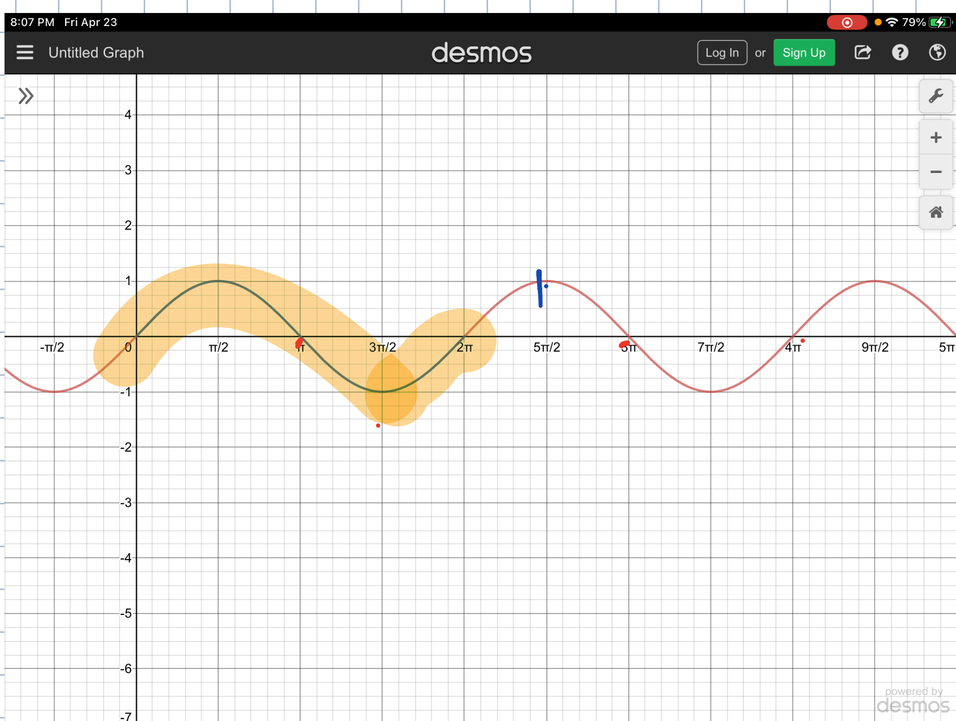
$y = D$ is mid line

* we don't work with midlines

* Note: normal period for $\sin\theta$, $\cos\theta$ is 2π

Period: $\dfrac{\text{normal period}}{|B|} = \dfrac{2\pi}{B}$

Phase shift: $\dfrac{C}{B}$ where graph starts



$y = \sin(x)$

Period: 2π

max: 1

@ $x = \frac{\pi}{2} + 2\pi n$,

$n \in \{0, \pm 1, \pm 2, \ldots\}$

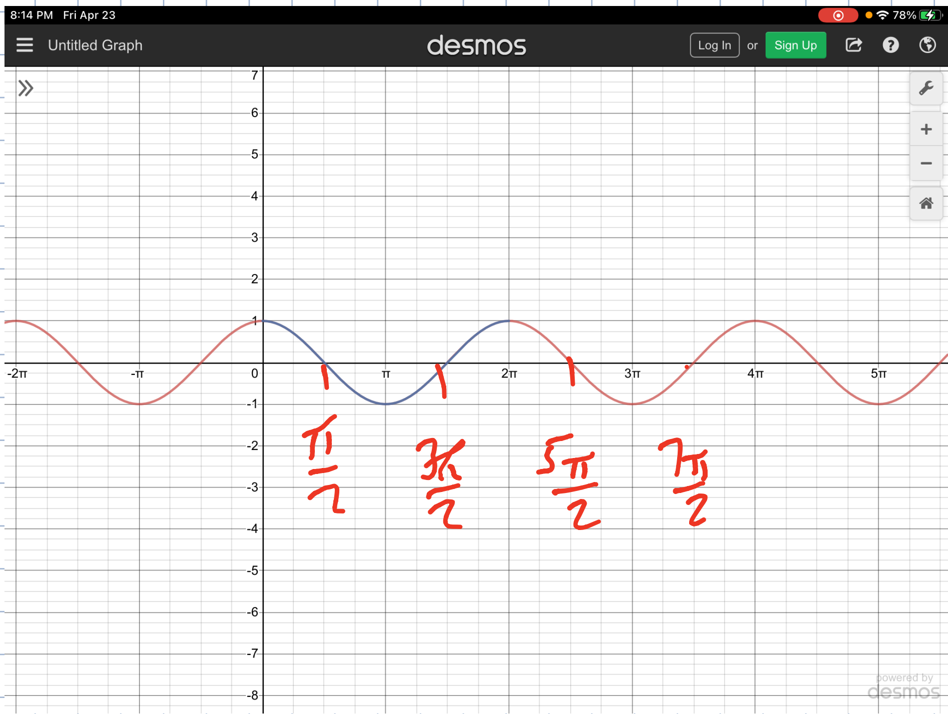
min: -1

@ $x = \frac{3\pi}{2} + 2\pi n$, $n \in \mathbb{Z}$

Zeroes: $0 + \pi n$, πn, $n \in \mathbb{Z}$

Pattern: 0, max, 0, min, 0

$y = \cos(x)$



max: 1

@ $x = 2n\pi$

"even multiples of π

min: -1

@ $x = (2n+1)\pi$

odd multiples of π

zeroes: $\frac{\pi}{2} + \pi n$. "any time $\frac{n\pi}{2}$" ← in this case n is odd

Pattern: max, 0, min, 0, max

* Noticed that in both graphs that over the course of the period of 2 pi, the zeros, Max, min are located on a quarter of the interval. This implies that there is the rule of 1/4s for sine and cosine the graphs will follow the pattern listed for each.

$y = A\sin(Bx)$ or $y = A\cos(Bx)$

In MAT 1375, you will go deeper

$y = A\sin(Bx - C) + D$ or $y = A\cos(Bx - C) + D$

Definitions

$y = D$ midline - horizontal centerline about which the function oscillates above and below.

- in our class $y = 0$ is midline.

amplitude : $|A|$ = $\frac{1}{2}$ positive distance between max & min.

= distance from max/min to the midline

= if A is negative, positive function is reflected over midline.

eg. $3\sin(x) \Rightarrow 0$, max, 0, min, 0

$-3\sin(x) \Rightarrow 0$, min, 0, max, 0

$2\cos(x) \Rightarrow$ max, 0, min, 0, max

$-2\cos(x) \Rightarrow$ min, 0, max, 0, min frequency (B) = number of cycles in a normal period length.

"2π" for sin, cos, csc, sec functions

π for tan, cot functions

Period $\left(\frac{\text{normal period length}}{|B|}\right)$ – the length of one cycle of periodic function

horizontal / phase shift – horizontal translation

Depends on form:

– $y = A\,f(Bx - C) + D$

f is trig function

$\rightarrow \frac{C}{B}$ → if $+$, $\rightarrow \frac{C}{B}$

→ if $-$, $\leftarrow \frac{C}{B}$

– the left most point of the cycle.

$x = \frac{C}{B}$ is the location of leftmost point of cycle

– if in $y = A\,f(B(x - C)) + D$

→ phase shift $= C$ → if $+$, $\rightarrow C$

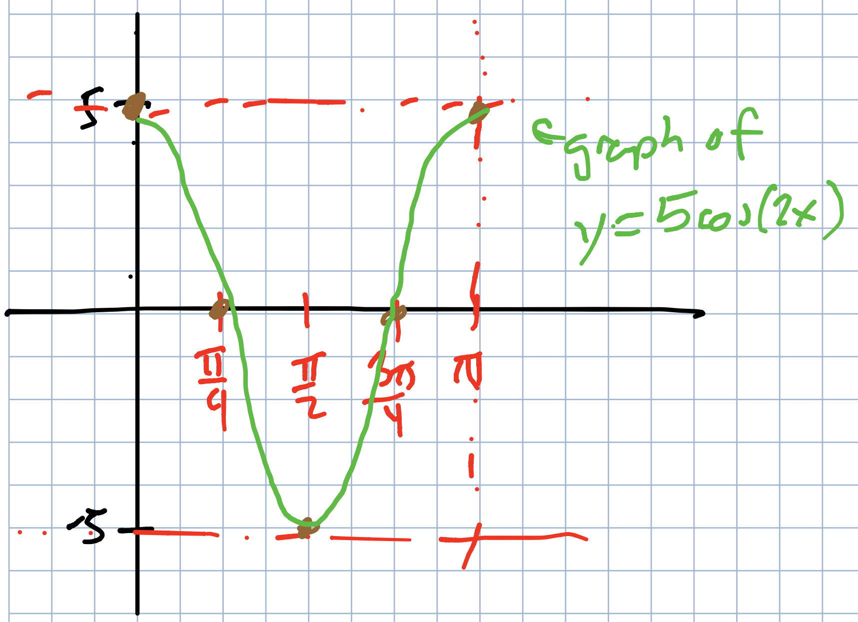
→ if $-$, $\leftarrow C$

e.g. $y = 5\cos(2x)$

$y = A\cos(Bx)$

$A = 5$ $\quad C = 0$

$B = 2$



period length: $\frac{2\pi}{|B|} = \frac{2\pi}{2} = \boxed{\pi}$

phase shift $= \frac{C}{B} = \frac{0}{2} = 0$

first point @ $x=0$

amplitude

$|A| = 5$

last end point: phase shift + period

$x = 0 + \pi$

@ $x = \pi$

Rule of 4ths

divide period by 4.

$\frac{\text{period}}{4} = \frac{\pi}{4}$

P $\quad x = 0 = 0$

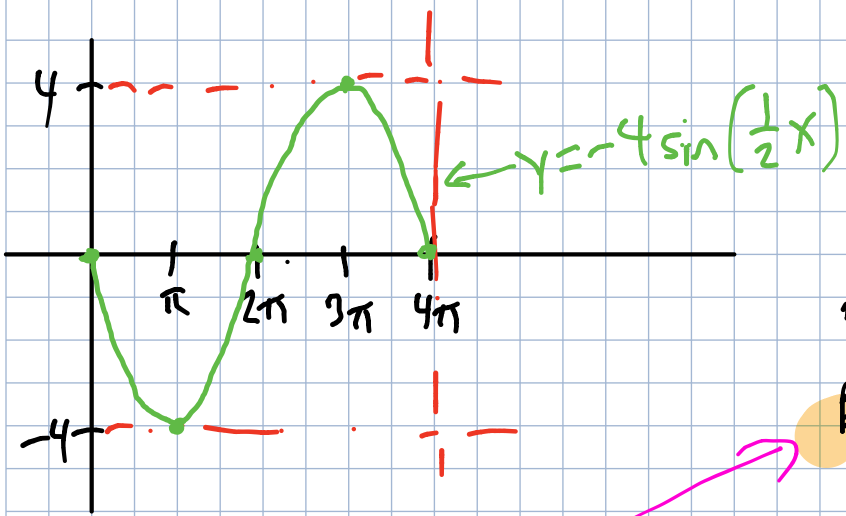
Q $\quad x = 0 + \frac{\pi}{4} = \frac{\pi}{4}$

R $\quad x = 0 + \frac{\pi}{4} + \frac{\pi}{4} = 0 + \frac{2\pi}{4} = \frac{2\pi}{4}, \frac{\pi}{2}$

S $\quad x = 0 + \frac{3\pi}{4} = \frac{3\pi}{4}$

T $\quad x = \pi$

$y = -4\sin\left(\frac{1}{2}x\right)$



$\times \quad \dfrac{2\pi}{\frac{1}{2}} = 2\pi \div \frac{1}{2}$

$= 2\pi \cdot \frac{2}{1} = 4\pi$

$y = A\sin(Bx)$

$A = -4$ ← pattern is reflected

$B = \frac{1}{2}$

$C = 0$

Amplitude $|A| = |-4| = 4$

Period: $\dfrac{2\pi}{|B|} = \dfrac{2\pi}{\frac{1}{2}} = 4\pi$

Phase shift: $\dfrac{C}{B} = \dfrac{0}{\frac{1}{2}} = 0$

→ 1st point @ $x=0$

Last point: phase shift + period

$0 + 4\pi = 4\pi$

Rule of 4ths: $\dfrac{\text{period}}{4} = \dfrac{4\pi}{4} = \pi$

P – $x = 0$

Q – $x = 0 + \pi = \pi$

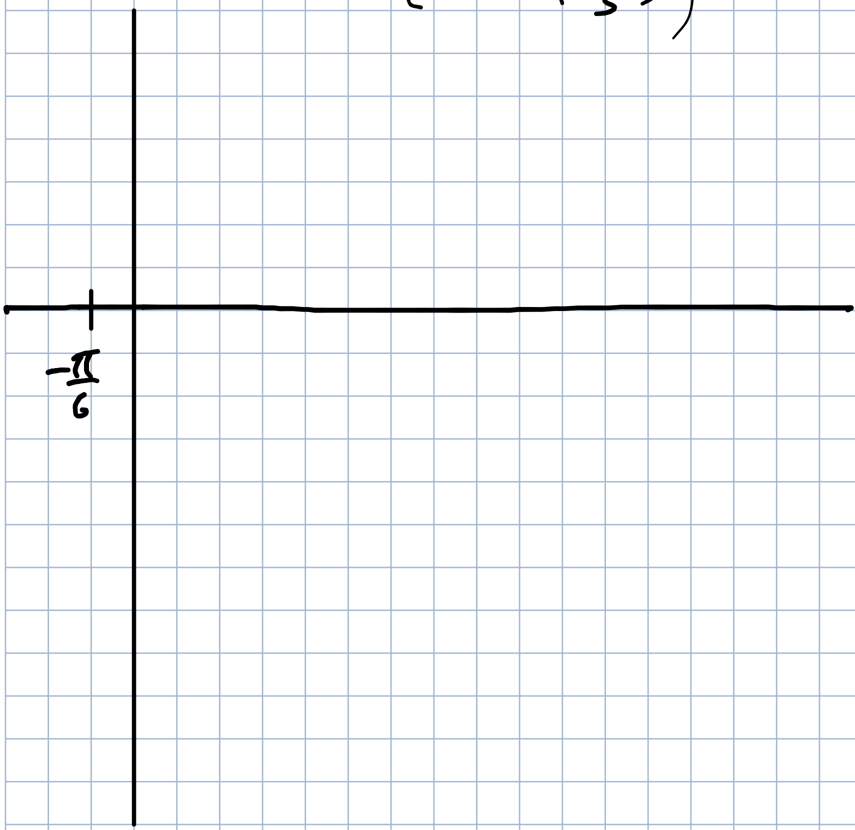
R – $x = 0 + 2\pi = 2\pi$

S – $x = 0 + 3\pi = 3\pi$

T – $x = 4\pi$

$y = -2\cos\left(2x + \frac{\pi}{3}\right)$

$= -2\cos\left(2x - \left(-\frac{\pi}{3}\right)\right)$



$y = A\cos(Bx - C)$

$A = -2$ → reflected pattern

$B = 2$

$C = -\frac{\pi}{3}$

Amplitude: $|A| = |-2| = 2$

phase shift: $\frac{C}{B} = \frac{-\frac{\pi}{3}}{2} = -\frac{\pi}{6}$

* $-\frac{\pi}{3} \div 2 = -\frac{\pi}{3}\left(\frac{1}{2}\right) = -\frac{\pi}{6}$

→ first point @ $x = -\frac{\pi}{6}$

period: $\frac{2\pi}{|B|} = \frac{\cancel{2}\pi}{\cancel{2}} = \pi$

→ last point

shift + period

$-\frac{\pi}{6} + \pi =$

$-\frac{\pi}{6} + \frac{6\pi}{6} = \frac{5\pi}{6}$

@ $x = \frac{5\pi}{6}$

Rule of 4ths $\frac{\text{period}}{4} = \frac{\pi}{4}$

P. $x = -\frac{\pi}{6} = -\frac{2\pi}{12}$

Q $x = -\frac{\pi}{6} + \frac{1\pi}{4} = -\frac{2\pi}{12} + \frac{3\pi}{12} = \frac{1\pi}{12}$

R $x = -\frac{\pi}{6} + \frac{2\pi}{4} = -\frac{2\pi}{12} + \frac{6\pi}{12} = \frac{4\pi}{12} = \frac{\pi}{3}$

S $x = -\frac{\pi}{6} + \frac{3\pi}{4} = -\frac{2\pi}{12} + \frac{9\pi}{12} = \frac{7\pi}{12}$

T $x = -\frac{\pi}{6} + \frac{4\pi}{4} = \frac{5\pi}{6} = \frac{10\pi}{12}$