

$$y = (x-4)^2 - 5 \quad \leftarrow \text{quadratic "parabolic"}$$

$$y = -7x + 21 \quad \leftarrow \text{line}$$

substitution → both are y
 \rightarrow they are equal to each other

$$(x-4)^2 + 5 = -7x + 21$$

$$x^2 - 8x + 16 + 5 = -7x + 21$$

$$+7x - 21 \quad +7x - 21$$

$$\underline{x^2 - x} = 0$$

$$x(x-1) = 0$$

$$x=0 \quad | \quad x-1=0$$

$$x=1$$

$$x \in \{0, 1\}$$

still need to find y

Let $x=0$

$$y = -7x + 21$$

$$y = -7(0) + 21$$

$$y = 21$$

$$(0, 21)$$

Let $x=1$

$$y = -7x + 21$$

$$y = -7(1) + 21$$

$$y = -7 + 21$$

$$y = 14$$

$$(1, 14)$$

$$(x, y) \in \{(0, 21), (1, 14)\}$$

$$y = \sqrt{x}$$

← radical

$$x^2 + y^2 = 132 \leftarrow \text{circle} \quad \text{center}(0,0)$$

radius: $\sqrt{132} = \sqrt{4 \cdot 33} = 2\sqrt{33}$

$$x^2 + (\sqrt{x})^2 = 132$$

$$x^2 + x = 132$$

$$x^2 + x - 132 = 0$$

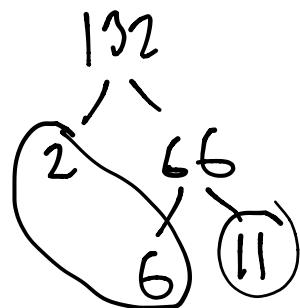
$$(x+12)(x-11) = 0$$

$$x+12=0$$

$$x-11=0$$

$$x = -12$$

$$x = 11$$



Need to find y

$$\text{Let } x = -12$$

$y = \sqrt{x} \leftarrow \text{do not use circles or other conics unless necessary}$

$$y = \sqrt{-12} = \sqrt{4 \cdot 3}i = 2\sqrt{3}i$$

~~$(-12, 2\sqrt{3}i)$~~
cannot graph $2\sqrt{3}i$
reject

$$\text{Let } x = 11$$

$$y = \sqrt{11}$$

$$(11, \sqrt{11})$$

only solution.

$$x - 2y = 10 \quad \leftarrow \text{line}$$

$$x^2 + y^2 = 25 \quad \leftarrow \text{circle}$$

$$x = 2y + 10$$

$$(2y+10)^2 + y^2 = 25$$

$$4y^2 + 40y + 100 + y^2 = 25$$

$$5y^2 + 40y + 75 = 0$$

$$5(y^2 + 8y + 15) = 0$$

$$5(y + 5)(y + 3) = 0$$

$$5 \neq 0 \quad y + 5 = 0 \quad \text{or} \quad y + 3 = 0 \\ y = -5 \quad \quad \quad \quad \quad y = -3$$

Find x . Always choose linear, conics give extraneous solutions

$$\text{Let } y = -5$$

$$x = 2y + 10$$

$$x = 2(-5) + 10$$

$$x = -10 + 10$$

$$x = 0$$

$$(0, -5)$$

$$y = -3$$

$$x = 2y + 10$$

$$x = 2(-3) + 10$$

$$x = -6 + 10$$

$$x = 4$$

$$(4, -3)$$

solutions unless

2 conics

$$4x^2 + 3y^2 = 43 \quad \leftarrow \text{ellipse}$$

$$3x^2 - 2y^2 = -6 \quad \leftarrow \text{hyperbola}$$

$$ax^2 \pm bx^2 = c$$

conic equation

* Since two variables are the same,
we can use elimination

$$4x^2 + 3y^2 = 43$$

$$3x^2 - 2y^2 = -6$$

* Choose to
eliminate
 y .

$$\underline{2(4x^2 + 3y^2 = 43)}$$

$$\underline{3(3x^2 - 2y^2 = -6)}$$

$$8x^2 + \cancel{6y^2} = 86$$

$$9x^2 - \cancel{6y^2} = -18$$

$$\underline{17x^2} = 68$$

$$x^2 = \frac{68}{17} = 4$$

$$x^2 = 4$$

$$x = \pm 2$$

* Forced to use conic equation to solve for y.

$$\text{Let } x = 2$$

$$3x^2 - 2y^2 = -6$$

$$3(2)^2 - 2y^2 = -6$$

$$\begin{array}{r} 12 - 2y^2 = -6 \\ -12 \hline -2y^2 = -18 \end{array}$$

$$y^2 = 9$$

$$y = \pm 3$$

$$(2, 3), (2, -3)$$

$$\text{Let } x = -2$$

$$3x^2 - 2y^2 = -6$$

$$3(-2)^2 - 2y^2 = -6$$

$$\begin{array}{r} 12 - 2y^2 = -6 \\ -12 \hline -2y^2 = -18 \end{array}$$

$$y^2 = 9$$

$$y = \pm 3$$

$$(-2, 3), (-2, -3)$$

$$\begin{array}{r} 3x + y = 4 \\ x^2 - 5y = -32 \\ \hline 3(3x + y = 4) \end{array}$$

*Note elimination doesn't work on x.
 x^2 , x are not like terms

$$\begin{array}{r} x^2 - 5y = -32 \\ \hline \end{array}$$

$$\begin{array}{r} 9x + 3y = 12 \\ x^2 - 3y = -32 \\ \hline x^2 + 9x = -20 \end{array}$$

$$\begin{array}{r} x^2 + 9x + 20 = 0 \\ (x+4)(x+5) = 0 \end{array}$$

$x = -4$ $3x + y = 4$ $3(-4) + y = 4$ $-12 + y = 4$ $+12 \quad \quad \quad +12$ \hline $y = 16$	$x = -5$ $3x + y = 4$ $3(-5) + y = 4$ $-15 + y = 4$ $+15 \quad \quad \quad +15$ \hline $y = 19$
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$(-4, 16)$ $(-5, 19)$