

Module 6 #3.

$$a^2 + b^2 = c^2 \quad (\text{Pythagorean Theorem})$$

$$(x)^2 + (x)^2 = (8)^2$$

hypotenuse

legs,

$$x^2 + x^2 = 64$$

$$2x^2 = 64$$

$$x^2 = \frac{64}{2}$$

$$x^2 = 32$$

$$x = \pm\sqrt{32}$$

$$x = \sqrt{16}\sqrt{2}$$

$$\boxed{x = 4\sqrt{2} \text{ inches}}$$

(reject $-$, no negative measurement)

$$4. \quad w = 9h$$

$$V = Lwh$$

$$L = 3h$$

$$64 \text{ cm}^3 = (9h)(3h)h$$

$$V = 64 \text{ cm}^3$$

$$64 \text{ cm}^3 = 27h^3$$

$$\frac{64 \text{ cm}^3}{27} = h^3$$

$$\sqrt[3]{\frac{64 \text{ cm}^3}{27}} = \sqrt[3]{h^3}$$

$$\boxed{\frac{4}{3} \text{ cm} = h}$$

$$\begin{aligned} w &= 9h \\ &= 9\left(\frac{4}{3}\right) \text{ cm} \end{aligned}$$

$$\boxed{w = 12 \text{ cm}}$$

$$L = 3h$$

$$= 3\left(\frac{4}{3}\text{ cm}\right)$$

$$\boxed{L = 4 \text{ cm}}$$

5.

$$a^2 + b^2 = c^2$$

$$a^2 + (2a+1)^2 = (8)^2$$

$$a^2 + 4a^2 + 4a + 1 = 64$$

$$5a^2 + 4a + 1 = 64$$

$$5a^2 + 4a - 63 = 0$$

$$a = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(4) \pm \sqrt{(4)^2 - 4(5)(-63)}}{2(5)}$$

$$= \frac{-4 \pm \sqrt{16 + 1260}}{10}$$

$$= \frac{-4 \pm \sqrt{1276}}{10}$$

$$= \frac{-4 \pm \sqrt{4 \cdot 319}}{10}$$

$$= \frac{-4 \pm 2\sqrt{319}}{10}$$

$$a = \frac{-2 \pm \sqrt{319}}{5} = \boxed{\frac{-2 + \sqrt{319}}{5}, m = a.} \rightarrow (b = 2a + 1)$$

factor 2 from
numerical & denominator.
so from here

$$a = a$$

$$b = 2a + 1$$

$$c = 8m$$

$$(2a+1)^2 = (2a+1)(2a+1) \\ = 4a^2 + 2a + 2a + 1$$

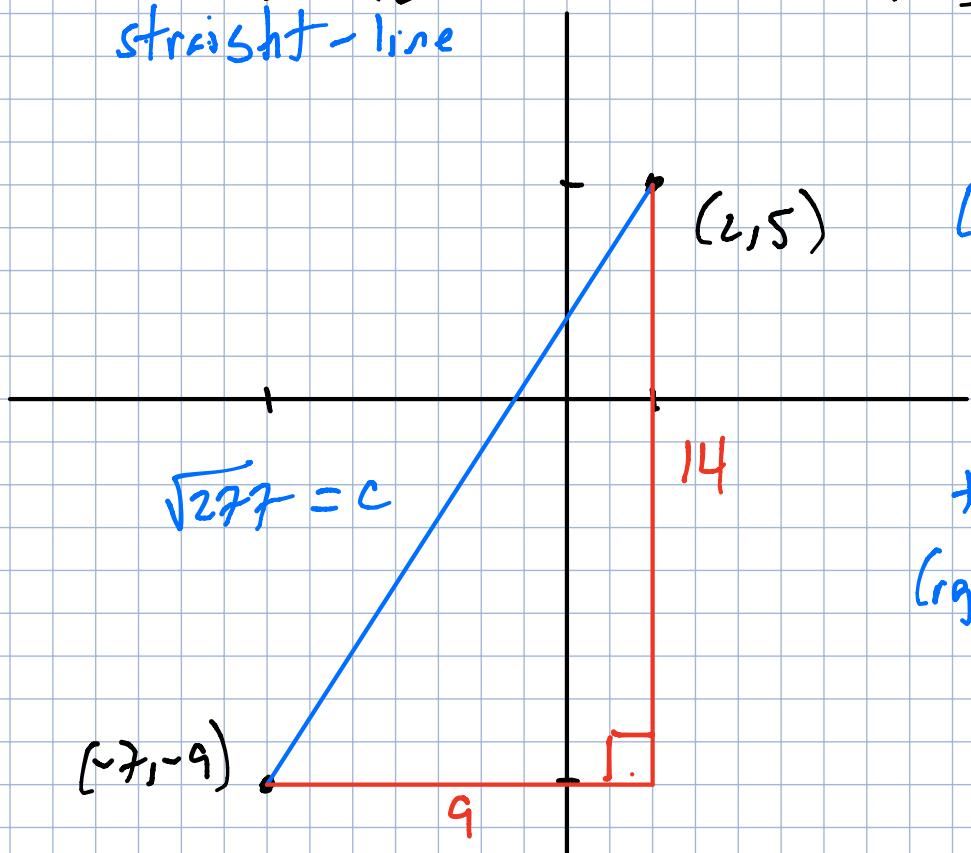
$$a = 5$$

$$b = 4$$

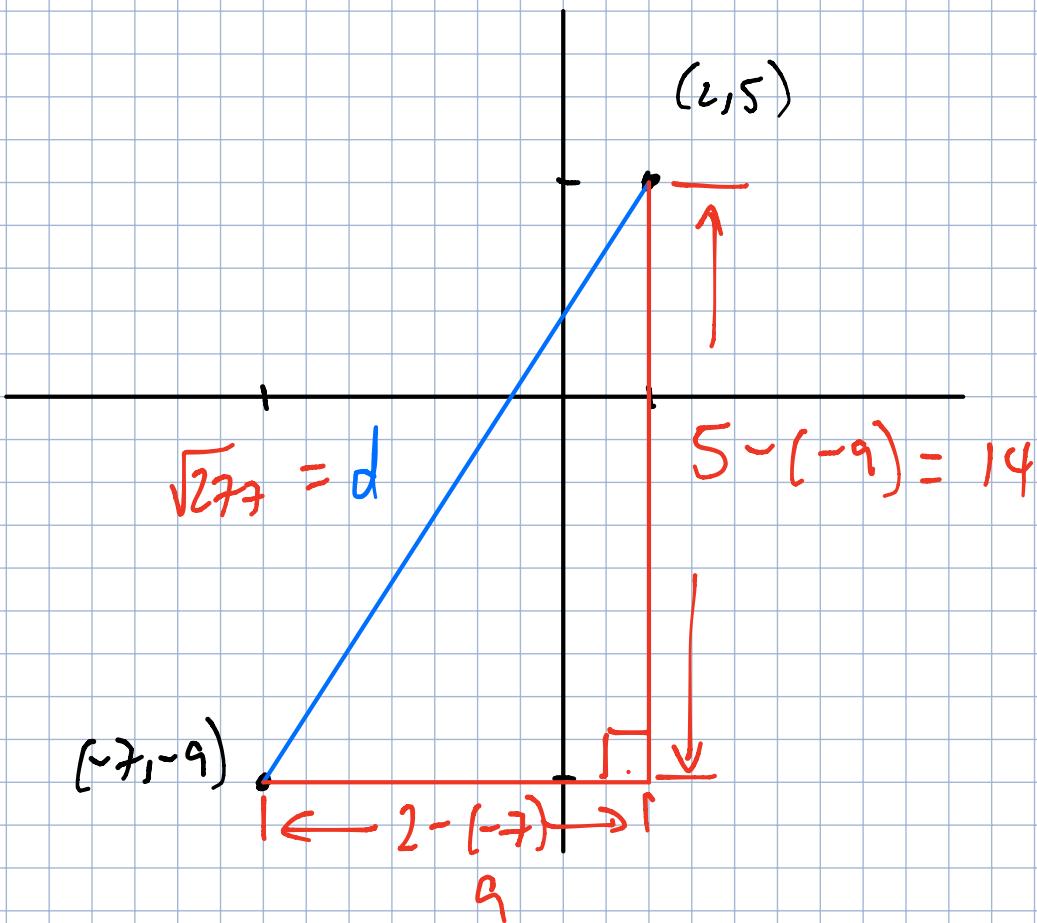
$$c = -63$$

$$\begin{array}{r} 1276 \\ 3 \cancel{1} \cancel{9} \end{array} \begin{array}{l} \diagdown \\ \diagup \\ 4 \\ \cancel{2} \cancel{2} \end{array}$$

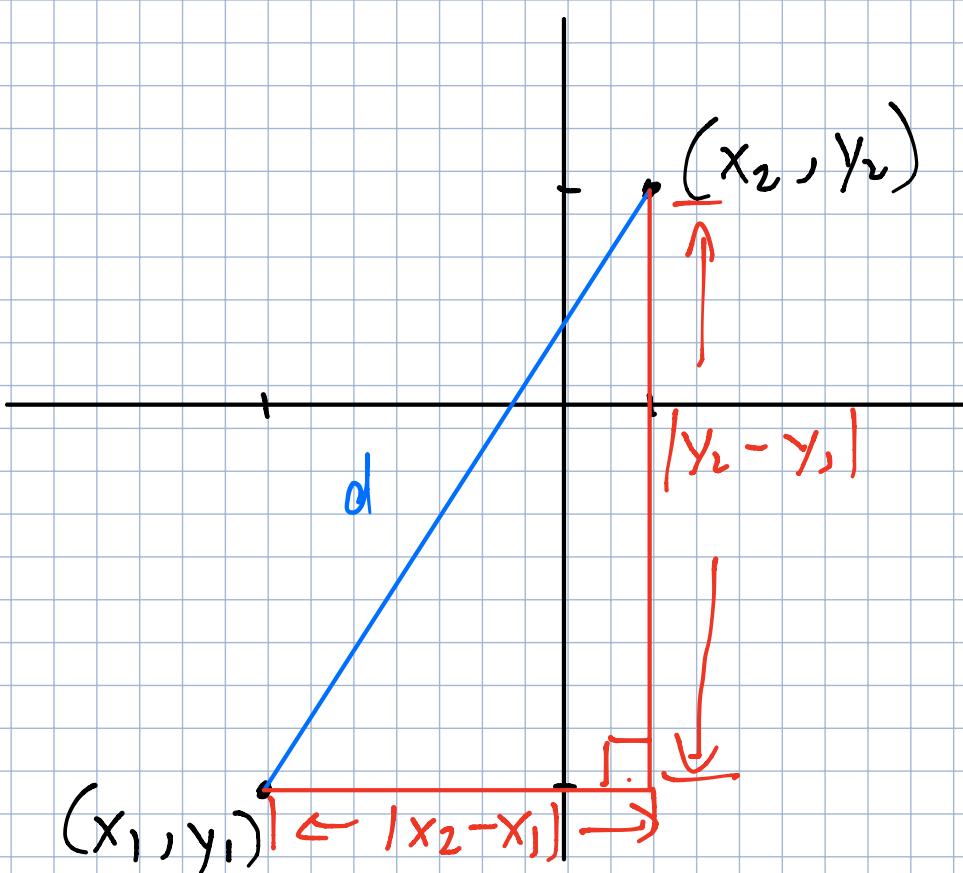
Find the distance between straight-line $(2, 5)$ and $(-7, -9)$



$$\begin{aligned}a^2 + b^2 &= c^2 \\(14)^2 + (5)^2 &= c^2 \\196 + 25 &= c^2 \\277 &= c^2 \\+ \sqrt{277} &= c \\(\text{right negative})\end{aligned}$$



$$a^2 + b^2 = c^2$$



Pythagorean Theorem $c^2 = a^2 + b^2$

$$d^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2$$

because absolute value & square are both positive

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

distance formula $\rightarrow d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Given (x_1, y_1)

(x_2, y_2)

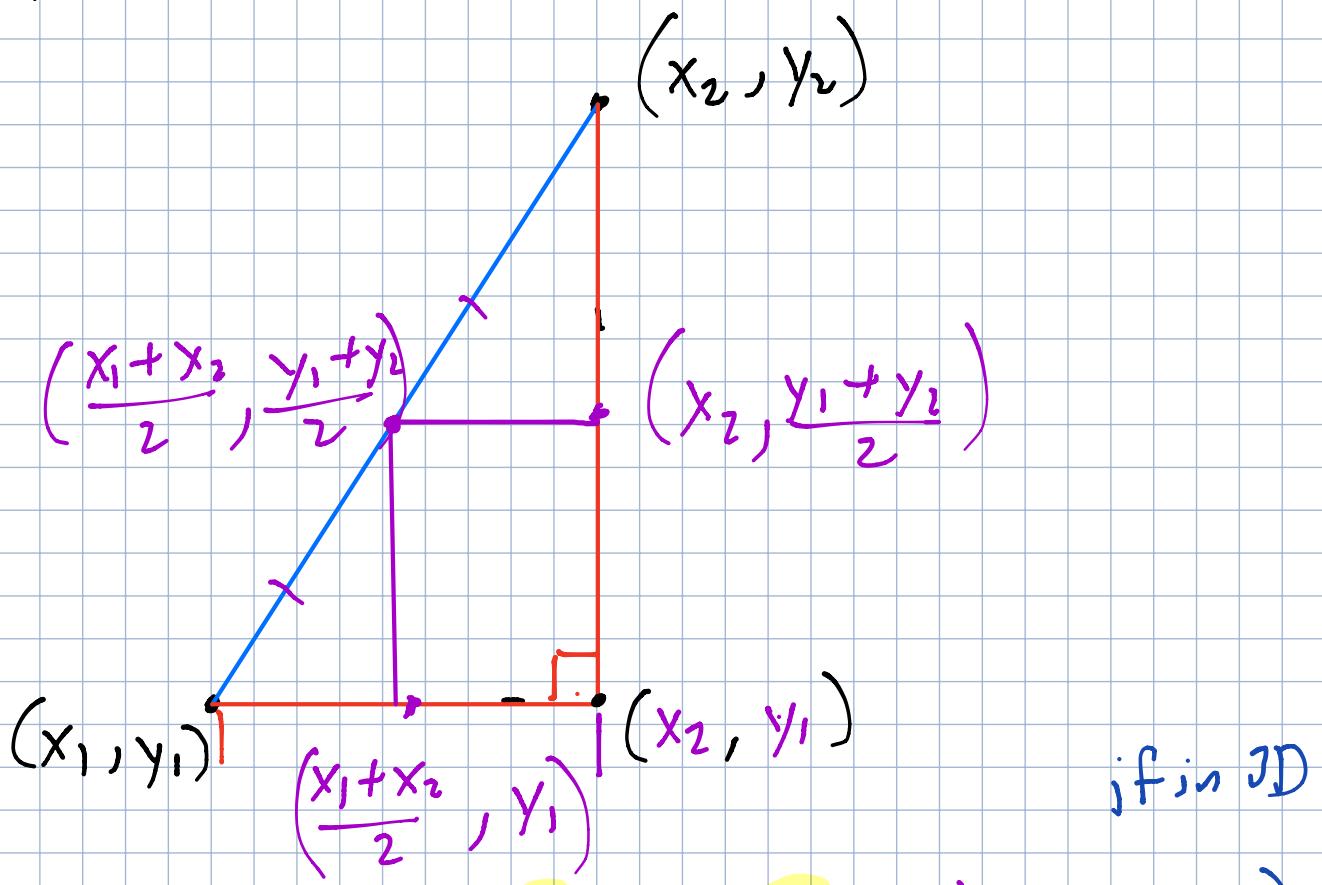
$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

↑ ↓
 change in x change in y

Distance between $(1, -8)$ and $(-6, -4)$
 (x_1, y_1) (x_2, y_2)

$$\begin{aligned}
 d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(-6 - 1)^2 + (-4 - (-8))^2} \\
 &= \sqrt{(-7)^2 + (-4)^2} \\
 &= \sqrt{49 + 16} = \sqrt{65}
 \end{aligned}$$

Midpoint



if in JD
 Midpoint formula $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right) \dots \frac{z_1+z_2}{2}$

Midpoint $(1, -8)$ and $(-6, -4)$
 (x_1, y_1) (x_2, y_2)

$$= \left(\frac{(1)+(-6)}{2}, \frac{(-8)+(-4)}{2} \right) = \left(\frac{-5}{2}, \frac{-12}{2} \right) = \left(-\frac{5}{2}, -6 \right)$$

Pythagorean Triple $a^2 + b^2 = c^2$

$a, b, c \in \mathbb{Z}^+$
positive integers

should form a \triangle .

Other solutions e.g.

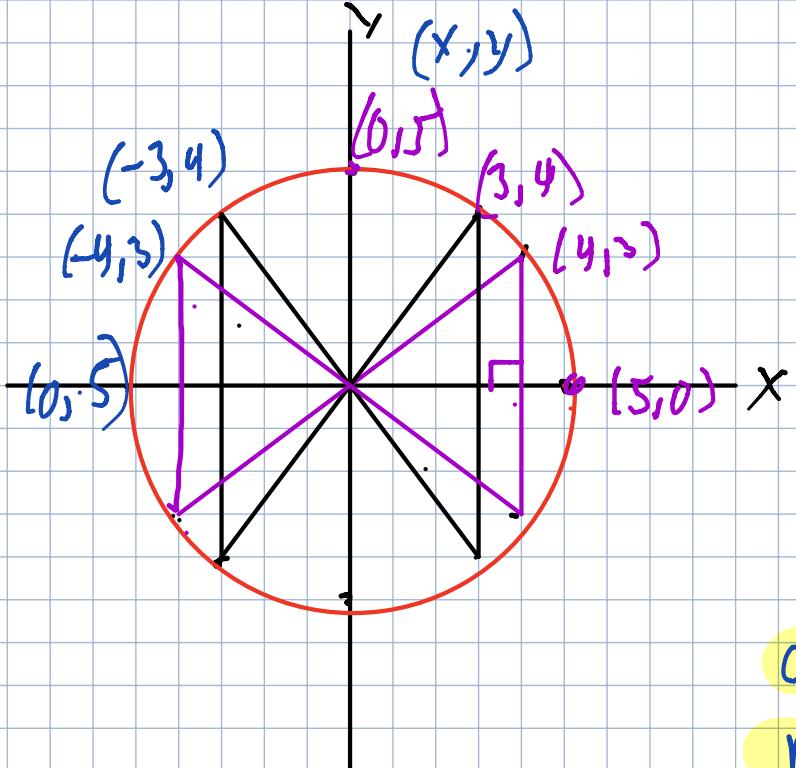
$$\text{to } a^2 + b^2 = c^2$$

$$a^2 + b^2 = (5)^2$$

$a, b \in \mathbb{Z}$
integers

$$\begin{array}{ccc} x & y & \\ 3, 4, 5 & & \leftarrow \text{hypotenuse} \\ 4, 3, 5 & & \\ -3, 4, 5 & & \\ -4, 3, 5 & & \\ -3, -4, 5 & & \\ -4, -3, 5 & & \\ 3, -4, 5 & & \\ 4, -3, 5 & & \end{array}$$

$$\begin{array}{ccc} x & y & \\ 5, 0, 5 & & \\ -5, 0, 5 & & \\ 0, 5, 5 & & \\ 0, -5, 5 & & \end{array}$$



$$x^2 + y^2 = 5^2$$

hypotenuse
 distance to $(0,0)$
 radius

$$x^2 + y^2 = r^2$$

Circle centered @ $(0,0)$

radius r .

(x,y) is a point on the
 circle that is
 distance r from $(0,0)$

e.g. $x^2 + y^2 = 36 \Leftrightarrow x^2 + y^2 = 6^2$

center: $(0,0)$

radius: $\sqrt{36} = 6$

Standard form of a circle

$$(x-h)^2 + (y-k)^2 = r^2$$

* Recall (h, k)

vertex of parabola
parabola shift

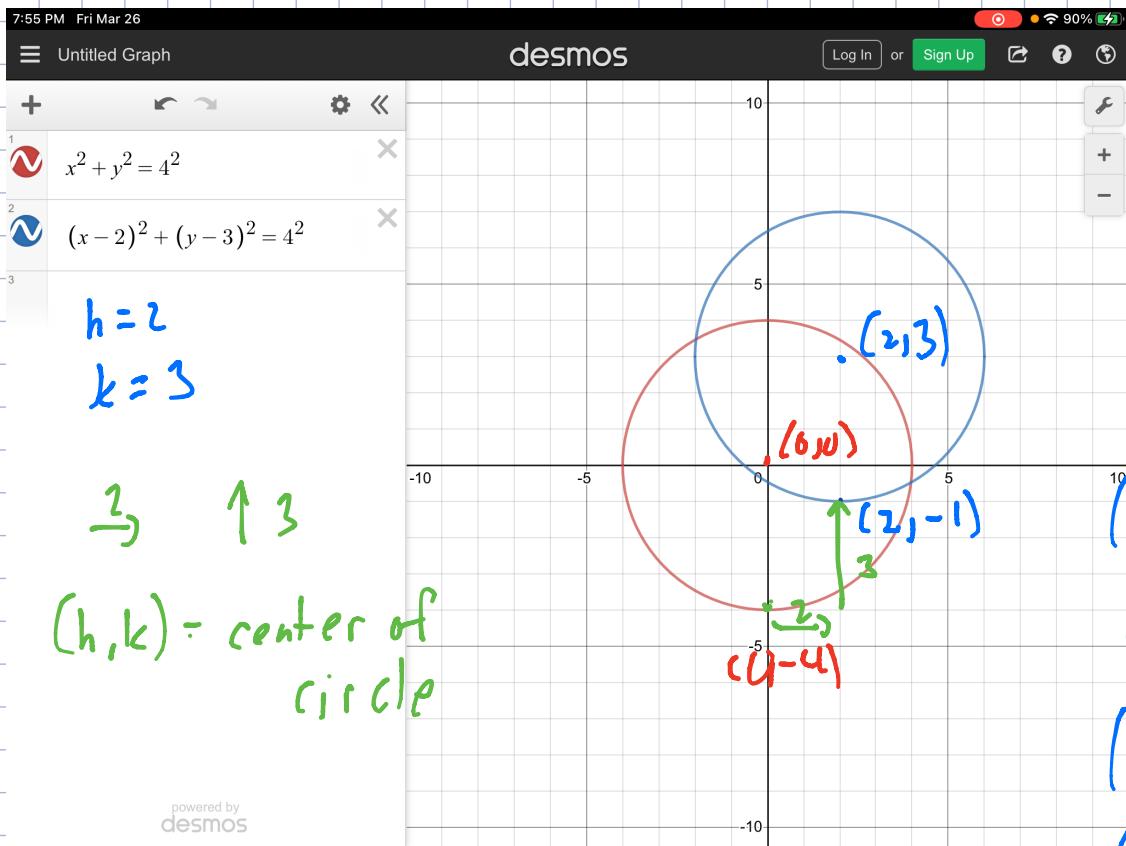
$$(h, k) \quad y = a(x-h)^2 + k$$

$$(+, +) \rightarrow h \quad \uparrow k$$

$$(+, -) \rightarrow h \quad \downarrow k$$

$$(-, +) \leftarrow h \quad \uparrow k$$

$$(-, -) \leftarrow h \quad \downarrow k$$



$$(x-3)^2 + (y-1)^2 = 16$$

center: $(2, 3)$

radius: $\sqrt{16} = 4$

$$(2, 3-4) = (2, -1)$$

$$(2, 3+4) = (2, 7)$$

$$(2+4, 3) = (6, 3)$$

$$(2-4, 3) = (-2, 3)$$

In general, to find 4 points, Given (h, k) and r .

$$(h+r, k)$$

$$(h-r, k)$$

$$(h, k+r)$$

$$(h, k-r)$$

$$(x+6)^2 + (y-8)^2 = 37$$

$$(x-(-6))^2 + (y-8)^2 = (\sqrt{37})^2$$

center: $(-6, 8)$

4 points.

radius: $\sqrt{37}$

$$(-6+\sqrt{37}, 8)$$

$$(-6-\sqrt{37}, 8)$$

$$(-6, 8+\sqrt{37})$$

$$(-6, 8-\sqrt{37})$$

Rewrite in standard form.

$$(x-h)^2 + (y-k)^2 = r^2$$

↑
need perfect squares

Complete the square
for x and y

$$x^2 + 12x + y^2 - 8y + 24 = 0$$

$$x^2 + 12x + y^2 - 8y = -24$$

$$x^2 + 12x + \left(\frac{12}{2}\right)^2 + y^2 - 8y + \left(-\frac{8}{2}\right)^2 = -24 + \left(\frac{12}{2}\right)^2 + \left(-\frac{8}{2}\right)^2$$

$$(x+6)^2 + (y-4)^2 = -24 + 36 + 16$$

$$(x+6)^2 + (y-4)^2 = 28$$

$$(x-(-6))^2 + (y-4)^2 = (\sqrt{28})^2$$

center: $(-6, 4)$

$$\text{radius: } \sqrt{28} : \sqrt{4} \sqrt{7} = 2\sqrt{7}$$

$$4 \text{ pts: } (-6, 4-2\sqrt{7}), (-6-2\sqrt{7}, 4),$$

$$(-6, 4+2\sqrt{7}), (-6+2\sqrt{7}, 4)$$