

$$x^2 = 81$$

$$\sqrt{x^2} = \pm \sqrt{81}$$

$$x = \pm 9$$

by factoring
 $x^2 - 81 = 0$
 quadratic equation

$$x^2 - 81 = 0$$

$$x^2 - a^2 = 0$$

$$ax^2 + bx + c = 0$$

$$a=1 \quad "x^2 + 0x - 81" \quad \text{difference of squares}$$

$$b=0 \quad ac = 1 \cdot -81 = -81 = -9 \cdot 9$$

$$c=-81 \quad b=0 \quad = -\underline{9+9}$$

$$(x^2 - 9^2) = 0$$

$$(x+9)(x-9) = 0$$

$$x = -9, x = 9$$

$$*\sqrt{81} = 9$$

$$-\sqrt{81} = -9$$

* Trap:

$$x^2 = 81$$

$$x = 9$$

Not necessarily true

$$b/c(-9)^2 = 81$$

$$\text{so } x = -9 \text{ also}$$

$$x^2 = 36$$

$$\sqrt{x^2} = \pm \sqrt{36}$$

$$x = \pm 6$$

$$\text{on WW: } x = -6, 6$$

$$\text{or } x = 6, -6$$

$$1. x^2 = 25$$

$$x = \pm \sqrt{25}$$

$$x = \pm 5$$

$$2. \frac{4x^2}{4} = \frac{9}{4} \rightarrow 4x^2 - 9 = 0$$

$$(2x)^2 - (3)^2 = 0$$

$$(2x+3)(2x-3) = 0$$

$$\sqrt{x^2} = \pm \sqrt{\frac{9}{4}}$$

$$x = \pm \frac{3}{2}$$

$$\begin{cases} 2x+3=0 \\ x=-\frac{3}{2} \end{cases} \quad \begin{cases} 2x-3=0 \\ x=\frac{3}{2} \end{cases}$$

$$3. \quad x^2 + 25 = 0$$
$$\underline{-25 \quad -25}$$
$$x^2 = -25$$

$$\sqrt{x^2} = \pm\sqrt{-25}$$

$$x = \pm \sqrt{25} \sqrt{-1}$$

$$x = \pm 5$$

$$x^2 + 25 = 0$$

$$x^2 + 5^2 = 0$$

$$(x+5i)(x-5i) = 0$$

$$x+5i=0 \quad | \quad x-5i=0$$

$$x = -5i \quad | \quad x = 5i$$

$$4. (t - 5)^2 = -18$$

Let $u = t^{-5}$

$$u^2 = -18$$

$$\sqrt{u^2} = \pm \sqrt{-18}$$

$$n = \pm \sqrt{18} \sqrt{-1}$$

$$y = \pm \sqrt{9} \sqrt{2} \sqrt{-1}$$

$$u = \pm 3\sqrt{2} i$$

$$t - 5 = \pm 3\sqrt{2}i$$

+ 5 + 5

$$t = 5 \pm 3\sqrt{2} i$$

$$4. \quad (t-5)^2 = -18$$

$$i = \sqrt{-1}$$

$$\sqrt{(t-5)^2} = \pm \sqrt{-18}$$

$$t-5 = \pm \sqrt{18} i$$

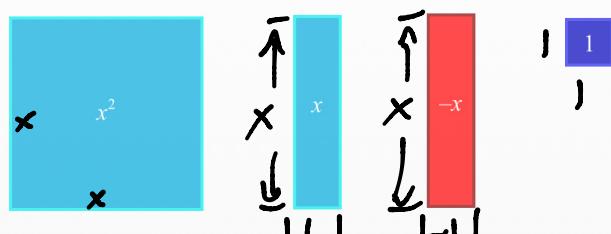
$$t-5 = \pm \sqrt{9} \sqrt{2} i$$

$$t-5 = \pm 3\sqrt{2} i$$

$$t = 5 \pm 3\sqrt{2} i$$

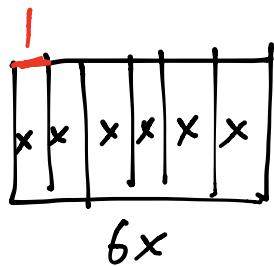
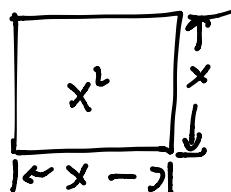
$$t = 5 - 3\sqrt{2} i, 5 + 3\sqrt{2} i$$

Solving quadratic equations using square root property.

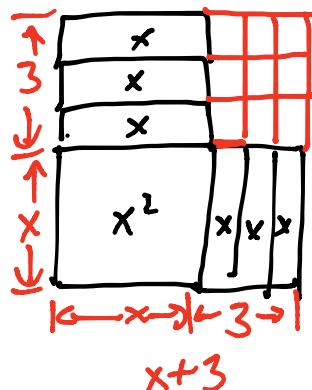


Complete the square

$$x^2 + 6x$$



$$x+3$$



9 units
to complete
the square

$$x^2 + 6x + \underline{9}$$

perfect square trinomial

factored

$$x^2 + 6x + 9 = (x+3)(x+3)$$

$$= (x+3)^2$$

* split the 6 in half,
square the result

$$\begin{aligned}
 & x^2 + 6x + \left(\frac{6}{2}\right)^2 \\
 &= x^2 + 6x + 9 = (x+3)(x+3) \\
 & \qquad\qquad\qquad (x+3)^2 \\
 &= (x+\frac{6}{2})(x+\frac{6}{2})
 \end{aligned}$$

Given $x^2 + bx$

To complete the square $x^2 + bx + \left(\frac{b}{2}\right)^2$
 Factored form $\therefore (x+\frac{b}{2})^2$
 $(x+\frac{b}{2})(x+\frac{b}{2})$

Given $x^2 - bx$

To complete the square $x^2 - bx + \left(\frac{b}{2}\right)^2$
 Factored form $\therefore (x-\frac{b}{2})^2$
 $(x-\frac{b}{2})(x-\frac{b}{2})$

$$\text{Given: } x^2 + 10x$$

$$\begin{aligned} &\rightarrow x^2 + 10x + \left(\frac{10}{2}\right)^2 \leftarrow \text{completed squares} \\ &= x^2 + 10x + 25 \leftarrow \\ &= (x+5)(x+5) \leftarrow \text{factored} \end{aligned}$$

Complete the square & factor to verify

$$b.) x^2 - 5x$$

$$\begin{aligned} &\rightarrow x^2 - 5x + \left(\frac{5}{2}\right)^2 \leftarrow \text{square root} \\ &= x^2 - 5x + \frac{25}{4} \leftarrow \text{take new c-values} \\ &= \left(x - \frac{5}{2}\right) \left(x - \frac{5}{2}\right) \leftarrow \text{make them constant terms} \\ &= \left(x - \frac{5}{2}\right)^2 \leftarrow \text{of the factors} \end{aligned}$$

$$x^2 + 6x = -13$$

use square root property
+ complete the square

$$x^2 + 6x + \left(\frac{6}{2}\right)^2 = -13 + \left(\frac{6}{2}\right)^2$$

$$x^2 + 6x + 9 = -13 + 9$$

$$(x+3)^2 = -4 \quad \leftarrow \text{need perfect square}$$

$$\sqrt{(x+3)^2} = \pm \sqrt{-4}$$

$$x+3 = \pm \sqrt{4} i$$

$$x+3 = \pm 2i$$

$$x = -3 \pm 2i$$

$$x^2 + 9x - 3 = 0$$

complete the square
use sqrt property

$$x^2 + 9x = 3$$

$$x^2 + 9x + \left(\frac{9}{2}\right)^2 = 3 + \left(\frac{9}{2}\right)^2$$

$$\left(x + \frac{9}{2}\right)^2 = 3 + \frac{81}{4}$$

$$\left(x + \frac{9}{2}\right)^2 = \frac{12}{4} + \frac{81}{4}$$

$$\left(x + \frac{9}{2}\right)^2 = \frac{93}{4}$$

$$x + \frac{9}{2} = \pm \sqrt{\frac{93}{4}}$$

$$x + \frac{9}{2} = \pm \frac{\sqrt{93}}{\sqrt{4}}$$

$$x + \frac{9}{2} = \pm \frac{\sqrt{93}}{2}$$

$$x = -\frac{9}{2} \pm \frac{\sqrt{93}}{2}$$

$$x = \frac{-9 \pm \sqrt{93}}{2}$$

$$\begin{array}{r} x^2 - 5x - 10 = 0 \\ +10 \quad +10 \\ \hline x^2 - 5x = 10 \end{array}$$

$$x^2 - 5x = 10$$

$$x^2 - 5x + \left(-\frac{5}{2}\right)^2 = 10 + \left(-\frac{5}{2}\right)^2 \leftarrow x^2 - bx + \left(\frac{b}{2}\right)^2$$

$$x^2 - 5x + \frac{25}{4} = 10 + \frac{25}{4} = \left(x - \frac{b}{2}\right)^2$$

$$\left(x - \frac{5}{2}\right)^2 = \frac{40}{4} + \frac{25}{4}$$

$$\left(x - \frac{5}{2}\right)^2 = \frac{65}{4}$$

$$\sqrt{\left(x - \frac{5}{2}\right)^2} = \pm \sqrt{\frac{65}{4}}$$

$$x - \frac{5}{2} = \pm \frac{\sqrt{65}}{2}$$

$$x = \frac{5 \pm \sqrt{65}}{2}$$

$$\begin{array}{r} +\frac{5}{2} \quad +\frac{5}{2} \\ \hline x = \frac{5}{2} \pm \frac{\sqrt{65}}{2} \end{array} \Rightarrow x = \frac{5+\sqrt{65}}{2}, \frac{5-\sqrt{65}}{2}$$

$$2b^2 - 12b = 5$$

$$\underline{=}$$
$$\frac{2b^2}{2} - \frac{12b}{2} = \frac{5}{2}$$

$$b^2 - 6b = \frac{5}{2}$$

$$b^2 - 6b + \left(-\frac{6}{2}\right)^2 = \frac{5}{2} + \left(-\frac{6}{2}\right)^2$$

factored $\rightarrow (b - \frac{6}{2})^2 = \frac{5}{2} + \frac{18}{2}$ ← constant we need to take square root of

$$(b - 3)^2 = \frac{23}{2}$$

$$\sqrt{(b-3)^2} = \pm \sqrt{\frac{23}{2}}$$

$$b - 3 = \pm \sqrt{\frac{23}{2}}$$

$$b = 3 \pm \sqrt{\frac{23}{2}}$$
 ← not rationalized

$$b = 3 \pm \frac{\sqrt{23}}{\sqrt{2}}$$

$$b = 3 \pm \frac{\sqrt{23}\sqrt{2}}{\sqrt{2}\sqrt{2}}$$

$$\boxed{b = 3 \pm \frac{\sqrt{46}}{2}}$$

$$b = 3 + \frac{\sqrt{46}}{2}, 3 - \frac{\sqrt{46}}{2}$$

Given a quadratic equation in standard form.

Solve for x by CTC.

$$ax^2 + bx + c = 0 \quad , \text{ Assume } a, b, c \in \mathbb{R}$$

$$ax^2 + bx = -c \quad a \neq 0$$

$$\frac{ax^2 + bx}{a} = \frac{-c}{a}$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a} \left(\frac{b^2}{4a^2}\right)$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

← quadratic formula

$$2b^2 - 10b = 5 \quad \text{solve by } \underline{\text{quadratic formula}}$$

$$2b^2 - 10b - 5 = 0$$

given

$$ax^2 + bx + c = 0$$

$$a = 2$$

$$b = -10$$

$$c = -5$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$b = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(2)(-5)}}{2(2)}$$

$$= \frac{10 \pm \sqrt{100 + 40}}{4}$$

$$\sqrt{140} = \sqrt{2^2 \cdot 35}$$

$$= \frac{10 \pm \sqrt{140}}{4}$$

$$= \frac{10 \pm \sqrt{4 \cdot 35}}{4}$$

$$= \frac{10 \pm 2\sqrt{35}}{4}$$

$$\begin{array}{r} 140 = 7 \cdot 2^2 \\ \backslash \quad \backslash \\ 14 \quad 10 \\ \backslash \quad \backslash \\ 7 \quad 2 \end{array}$$

$$= \frac{2(5 \pm \sqrt{35})}{8(2)} = \frac{5 \pm \sqrt{35}}{2}$$