

Consider  $y = x^2$

$x$	$y = x^2$
-2	4
-1	1
0	0
1	1
2	4
3	9
4	16
5	25

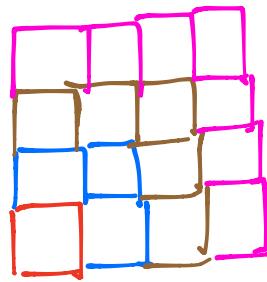
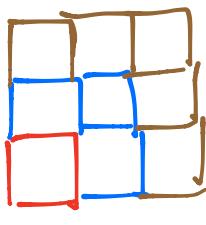
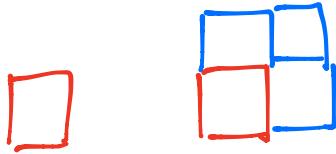
Note:

$y = x^2$  is symmetric

$x = 0$  is axis

of symmetry

if 2nd difference is  
the same for all values,  
the function is quadratic



1

4

9

16

+3  
+5

+5  
+7

To find next  $x^2$  in sequence,

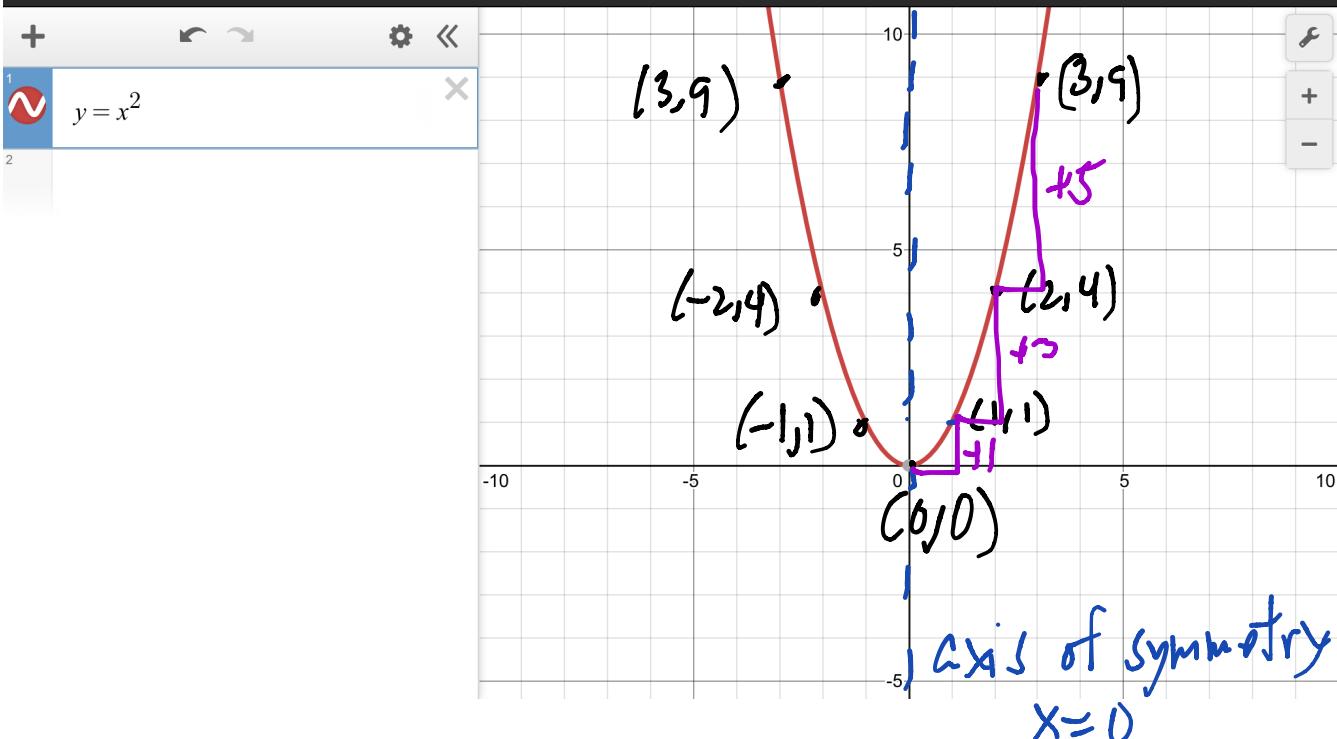
-  $x^2$  (multiply next number by itself)

- add next odd number.

Untitled Graph

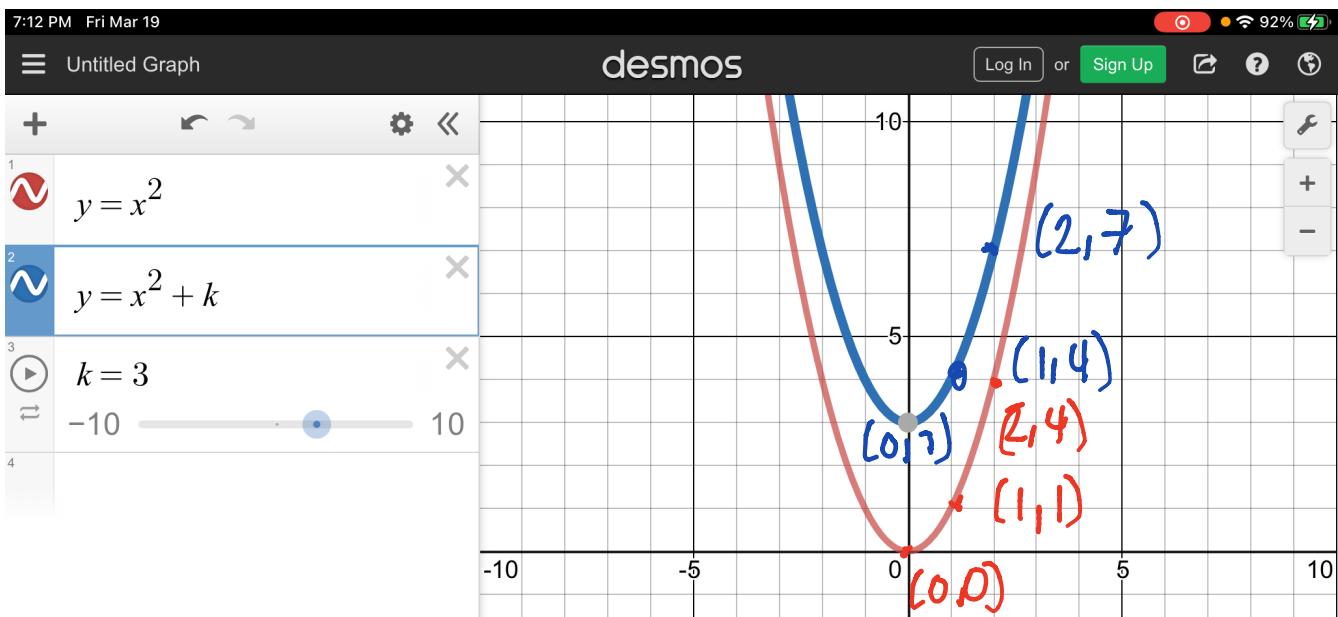
desmos

Log In or Sign Up



Shifting  $y = x^2$

the case of  $y = x^2 + k$



$$y = x^2 + 3$$

All points on  $y = x^2 \uparrow 3$

if  $(x, y)$  is a point on  $y = x^2$

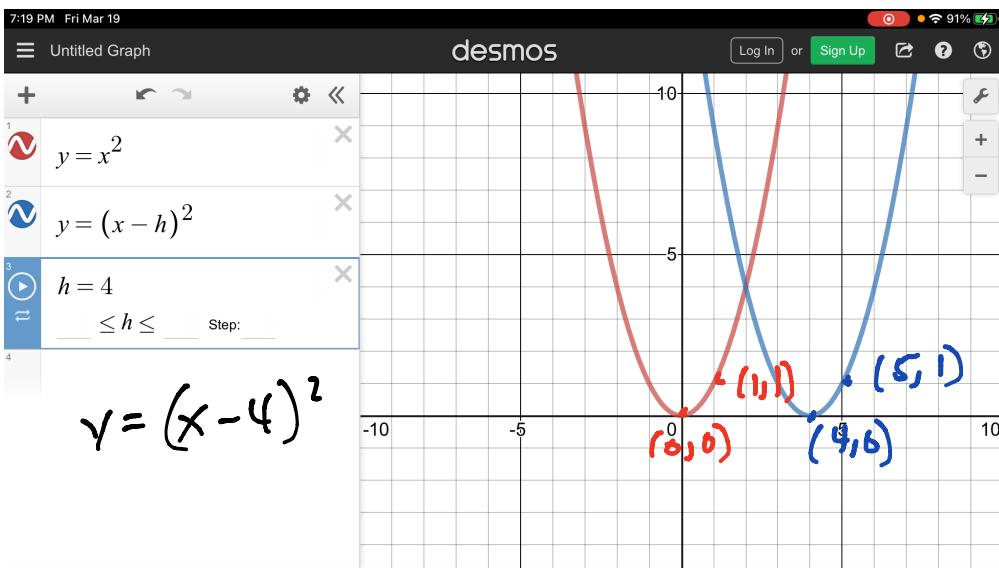
then  $(x, y+3)$  is a point on  $y = x^2 + 3$

∴ then  $(x, y+k)$  is a point on  $y = x^2 + k$

$$y = x^2 + 3 \text{ is } y = x^2 \uparrow 3$$



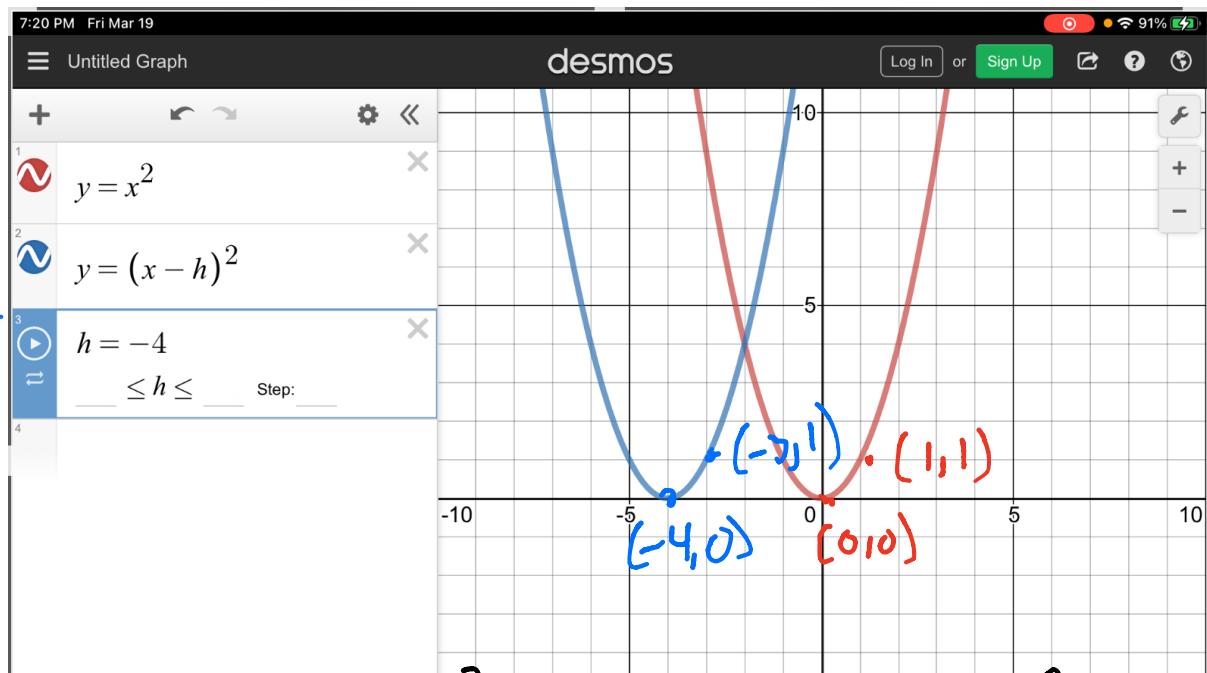
# The case of $(x-h)^2$



\*  $h$  represents horizontal shift

Given  $y = (x - 4)^2$ .

$y = x^2$  is moved  $\rightarrow 4$

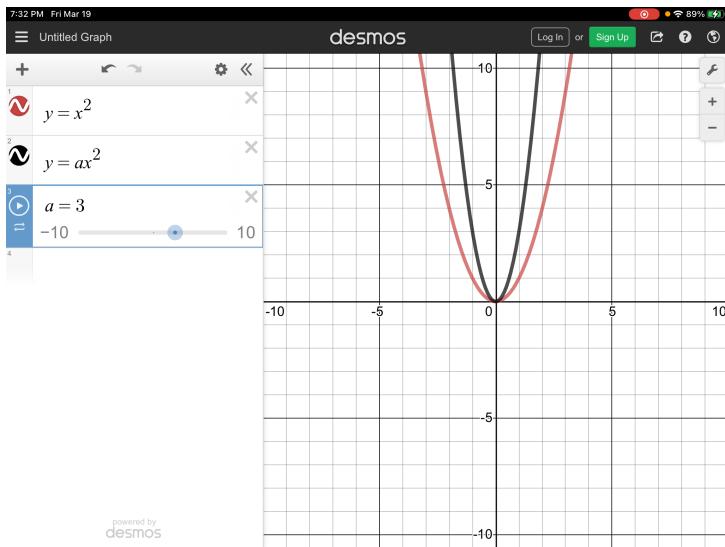


$$y = (x - (-4))^2 = y = (x + 4)^2$$

$y = x^2$  moved  $\leftarrow 4$

Given  $y = (x - h)^2$  if  $h$  is negative  $\leftarrow h$   
 $h$  is positive  $\rightarrow h$

$$y = ax^2$$

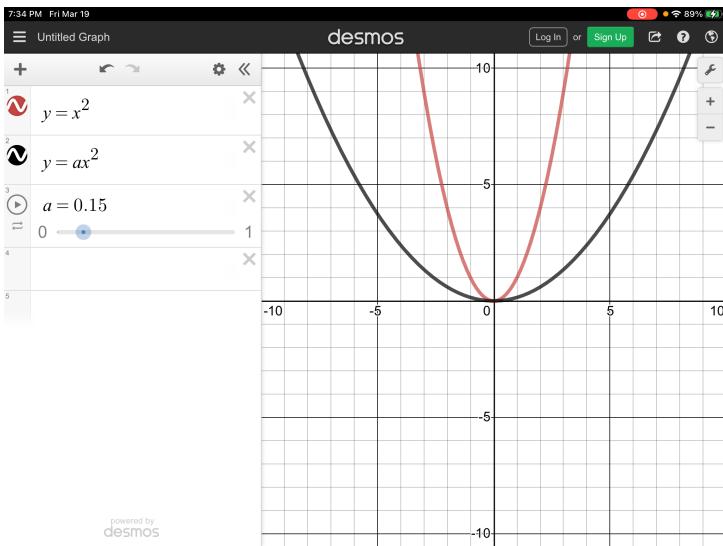


$$y = 3x^2$$

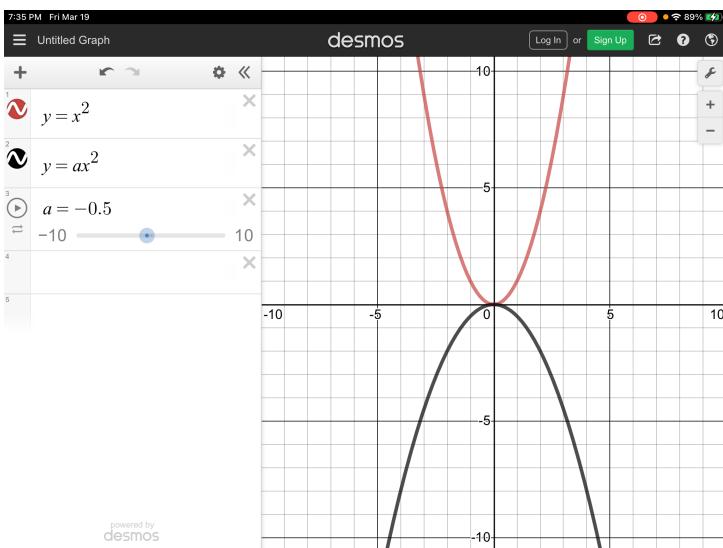
sharper/more acute  
than  $y = x^2$

$y = cx^2$  sharper if  
 $|a| > 1$

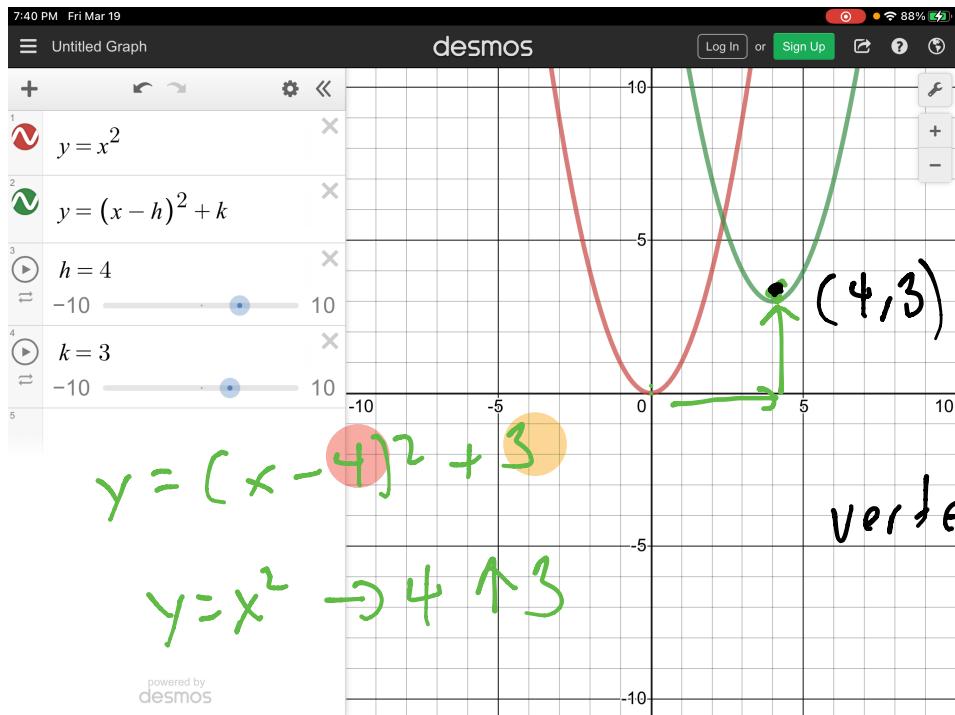
$y = ax^2$  is more obtuse  
if  $0 < |a| < 1$



$y = ax^2$  flips  
if  $a$  is negative.



$$y = (x - h)^2 + k \rightarrow (h, k) = \text{vertex}$$



e.g.  $y = (x + 6)^2 - 4$

$$y = (x - h)^2 + k$$

What is the vertex?

$$y = (x - (-6))^2 + (-4)$$

follows this form

vertex  $(-6, -4)$

Given  $y = (x - 8)^2 - 3$

$$* y = (x - h)^2 + k$$

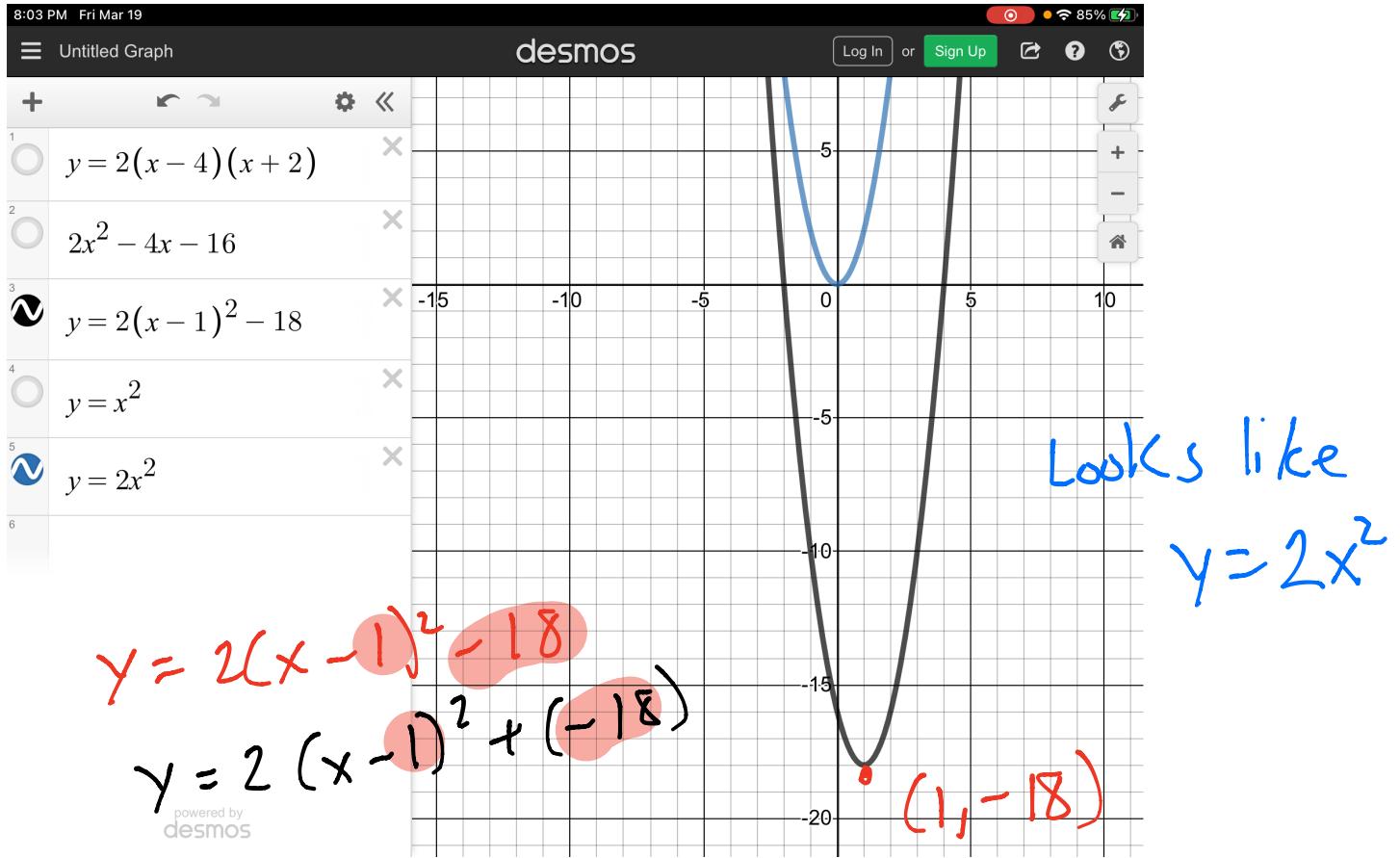
what is the vertex?

$$y = (x - 8)^2 + (-3)$$

vertex  $(8, -3)$

# General Case

$$y = a(x - h)^2 + k$$



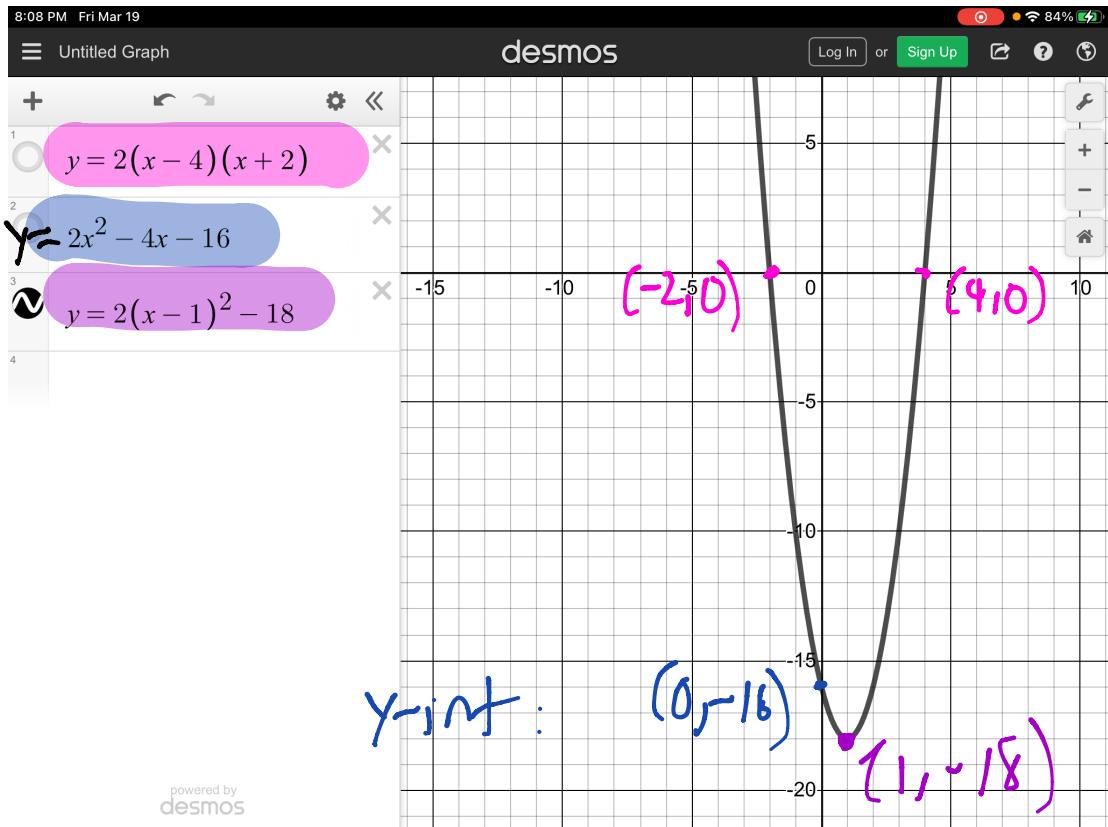
$$y = 2(x - h)^2 + k$$
$$(h, k) = (1, -18)$$

$y = 2(x - 1)^2 - 18$  is " $y = 2x^2$  with vertex  $(1, -18)$ "

# Three forms of a quadratic function

$$y = 2(x-1)^2 - 18 \quad \text{vertex } (1, -18)$$

$$y = a(x-h)^2 + k \leftarrow \text{vertex form vertex } (h, k)$$



$$y = 2x^2 - 4x - 16$$

$$y = ax^2 + bx + c \quad \leftarrow \text{standard form}$$

$y = mx + b \quad (0, b) = \text{y-intercept}$

$(0, c)$  is y-intercept

$$y = 2(x^2 - 2x - 8)$$

x-int; (4,0) and (-2,0)

$$y = 2(x - 4)(x + 2)$$

$$y = a(x - r_1)(x - r_2) \leftarrow \text{roots form}$$

\*  $(r_1, 0)$   $(r_2, 0)$  are x-intercepts

$r_1, r_2$  are solutions to

$$a(x - r_1)(x - r_2) = 0$$

Given =  $y = 2x^2 - 12x + 10$  ← standard form  
 $y = ax^2 + bx + c$

find the vertex  
 "convert to vertex form".

$$y = a(x-h)^2 + k$$

\* Need to complete  
 the square

$$y = 2x^2 - 12x + 10$$

$$y - 10 = 2x^2 - 12x$$

$$\frac{y-10}{2} = \frac{2x^2}{2} - \frac{12x}{2}$$

$$\frac{y-10}{2} = x^2 - 6x$$

$$\frac{y-10}{2} + \left(-\frac{6}{2}\right)^2 = x^2 - 6x + \left(-\frac{6}{2}\right)^2$$

$$\frac{y-10}{2} + 9 = \left(x - \frac{6}{2}\right)^2$$

\* We need to solve for

$$\frac{y-10}{2} = \left(x - 3\right)^2 - 9$$

$$2 \cancel{\left(\frac{y-10}{2}\right)} = 2 \left( \left(x-3\right)^2 - 9 \right) * \text{Multiply both sides by 2.}$$

$$y - 10 = 2 \left(x-3\right)^2 - 18$$

$$y = 2 \left(x-3\right)^2 - 18 + 10$$

$$y = 2(x-3)^2 - 8$$

$$y = 2(x-3)^2 + (-8)$$

vertex

$$(3, -8)$$

Given  $y = ax^2 + bx + c$ , find vertex.

$$y = ax^2 + bx + c \rightarrow y = a(x-h)^2 + k$$

$$y - c = ax^2 + bx \quad \text{assume } a, b, c \in \mathbb{R}$$

$$\frac{y-c}{a} = \frac{ax^2}{a} + \frac{bx}{a} \quad a \neq 0$$

$$\frac{y-c}{a} = x^2 + \frac{b}{a}x$$

$$\frac{y-c}{a} + \left(\frac{b}{2a}\right)^2 = x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 \quad \leftarrow \text{complete the square}$$

$$\frac{y-c}{a} + \left(\frac{b}{2a}\right)^2 = \left(x + \frac{b}{2a}\right)^2$$

$$\frac{y-c}{a} = \left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2$$

$$y - c = a\left(x + \frac{b}{2a}\right)^2 - \frac{ab^2}{4a^2}$$

$$y - c = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a}$$

$$y = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c$$

$$y = a\left(x - \left(-\frac{b}{2a}\right)\right)^2 + \left(c - \frac{b^2}{4a}\right)$$

$$"y = a(x-h)^2 + k"$$

$\frac{4ac - b^2}{4a}$

$$(h, k) = \left(-\frac{b}{2a}, c - \frac{b^2}{4a}\right)$$

\*  $h = -\frac{b}{2a}$   $(h, k) = "(x, y)"$   
vertex

\*  $k = ah^2 + bh + c$

$$y = ax^2 + bx + c$$

Replace  $y$  with  $k$  and  $x$  with  $h$ .

Vertex of  $y = 2x^2 - 12x + 10$   $y = ax^2 + bx + c$

$$h = -\frac{b}{2a}$$

$$= -\frac{(-12)}{2(2)}$$

$$= -\frac{12}{4}$$

$$= -(-3)$$

$$= 3$$

$$k = 2(3)^2 - 12(3) + 10$$

$$k = 2(9) - 36 + 10$$

$$k = 18 - 36 + 10$$

$$k = -18 + 10$$

$$k = -8$$

$$\text{vertex} = (3, -8)$$

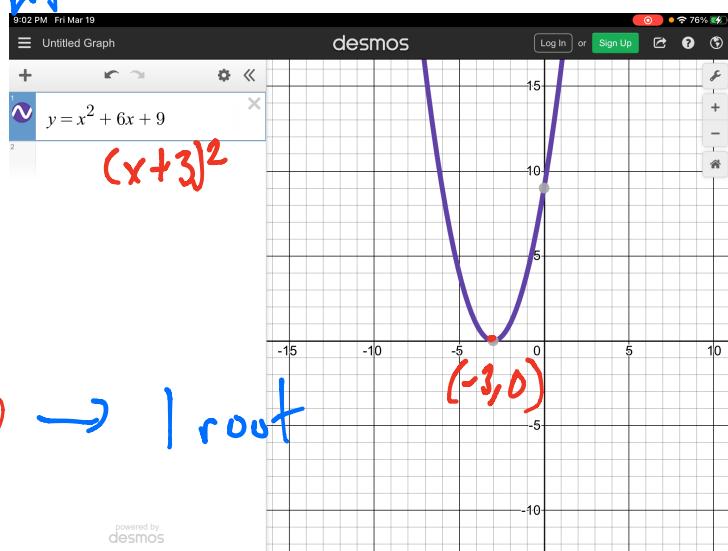
$$b^2 - 4ac$$

"discriminant"

$$y = x^2 + 6x + 9$$

$$b^2 - 4ac$$

$$(6)^2 - 4(1)(9) = 0 \rightarrow 1 \text{ root}$$



$$y = x^2 + 6x + 8$$

$$b^2 - 4ac$$

$$(6)^2 - 4(1)(8) = 4$$

$$\sqrt{4} = 2$$

$b^2 - 4ac$  is a perfect square

$$y = x^2 + 6x + 1$$

$$b^2 - 4ac$$

$$(6)^2 - 4(1)(1) = 32$$

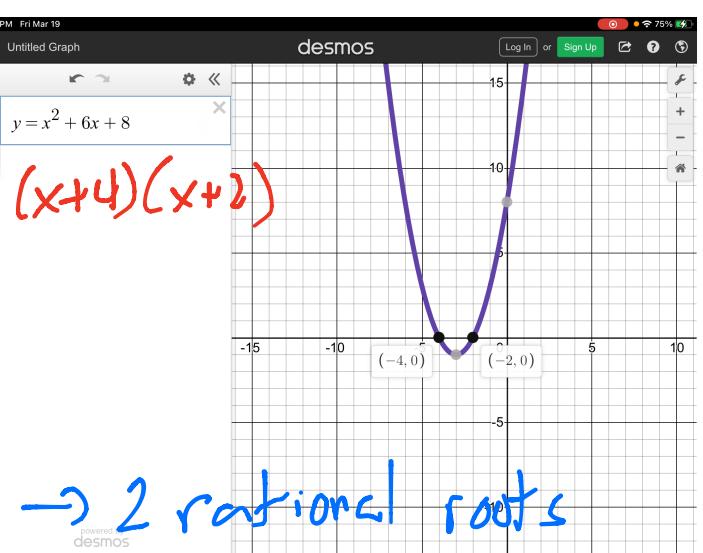
$\rightarrow 2$  roots

irrational

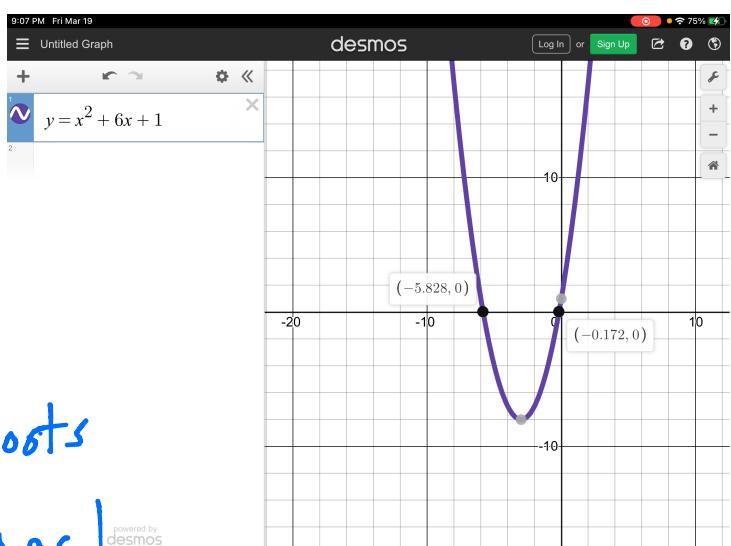
$$\sqrt{32} = \sqrt{16} \sqrt{2}$$

$$= 4\sqrt{2}$$

$b^2 - 4ac$  is not a perfect square  $\rightarrow$  2 irrational roots



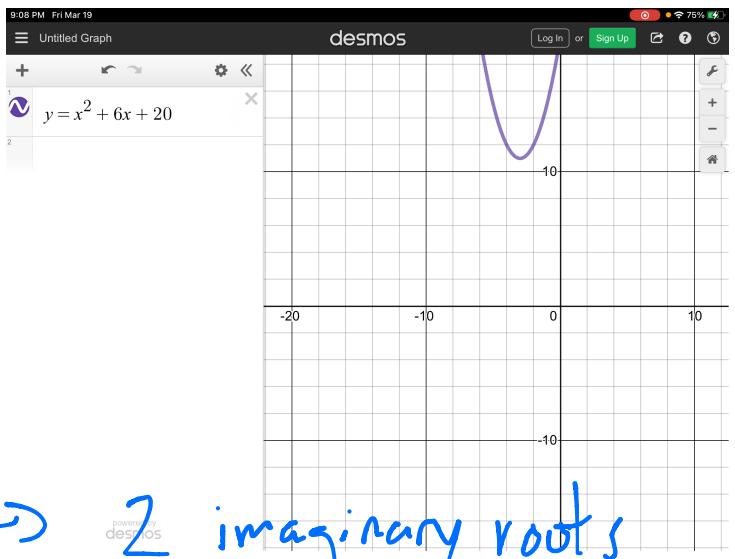
$\rightarrow 2$  rational roots



$$y = x^2 + 6x + 20$$

$$b^2 - 4ac$$

$$(6)^2 - 4(1)(20) = -44$$



$b^2 - 4ac$  is negative  $\rightarrow$  2 imaginary roots

$$\sqrt{-44} = 2\sqrt{11} i$$