

Radical Equations

$$\sqrt[3]{x} = 5$$

$$(\sqrt[3]{x})^3 = 5^3$$

$$x = 125$$

Check $x = 125$

$$\sqrt[3]{125} = 5 \rightarrow \sqrt[3]{5^3} = 5 \rightarrow 5 = 5 \checkmark$$

$$\begin{aligned} x^{\frac{1}{3}} &= 5 \\ (x^{\frac{1}{3}})^3 &= 5^3 \\ x^1 &= 125 \end{aligned}$$

Note: we are multiplying both sides of an equation by an algebraic factor.

A new equation is formed and often, there are new solutions to the equation.

Thus, it is important to check all possible solutions.

$$\sqrt[3]{x} = -5$$

$$(\sqrt[3]{x})^3 = (-5)^3$$

$$\boxed{x = -125}$$

Check $x = -125$

$$\sqrt[3]{-125} = -5$$

$$\sqrt[3]{(-5)^3} = -5$$

$$-5 = -5 \checkmark$$

$$\sqrt{x} = -7$$

$$(\sqrt{x})^2 = (-7)^2$$

$$x = 49$$

Check $x = 49$

$$\sqrt{49} = -7$$

$$7 \neq -7$$

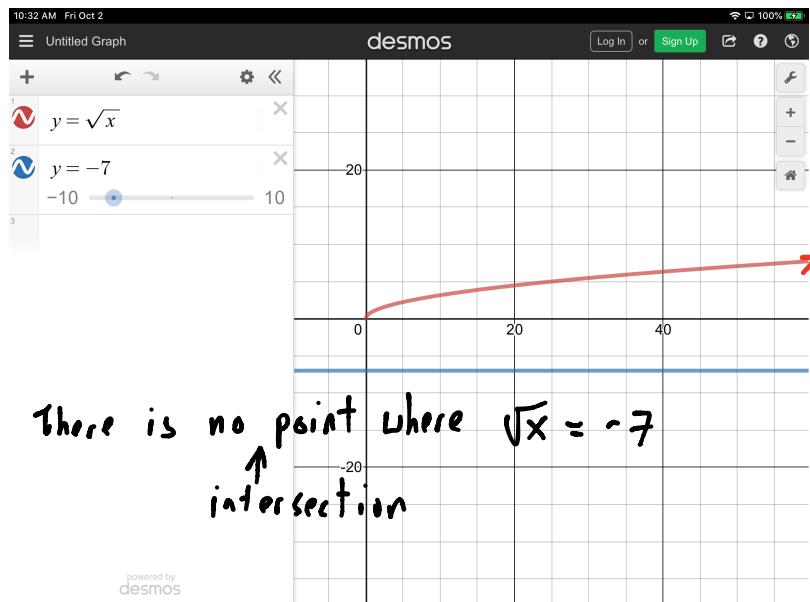
extraneous solution

Since $x = 49$ does not check,

$x = 49$ is not a solution to the equation

∴ There is no solution to this equation

* Note: We are only assuming principal (positive) square root at this time.



$$\begin{array}{r} \sqrt{p} + 5 = 9 \\ -5 \quad -5 \\ \hline \sqrt{p} \quad = 4 \\ (\sqrt{p})^2 = (4)^2 \end{array}$$

Check $p=16$

$$\begin{array}{l} \sqrt{16} + 5 = 9 \\ 4 + 5 = 9 \\ 9 = 9 \checkmark \end{array}$$

$$p = 16$$

∴ $p = 16$ is the solution

$$3\sqrt{9a-18} - 3a = 0 \quad \xrightarrow{* \text{ idea}} \quad \sqrt{9(a-2)} - a = 0$$

$$3(\sqrt{9a-18} - a) = 0$$

$$\begin{array}{c} 3 \neq 0 \\ | \quad \sqrt{9a-18} - a = 0 \\ \hline \sqrt{9a-18} = a \end{array}$$

+a +a

← isolate the radical

$$(\sqrt{9a-18})^2 = a^2$$

$$\begin{array}{r} 9a-18 = a^2 \\ -a^2 \quad -a^2 \\ \hline -a^2 + 9a - 18 = 0 \end{array}$$

$$-(a^2 - 9a + 18) = 0$$

$$-1(+0) (a-6)(a-3) = 0$$

$$\begin{array}{l} a-6=0 \quad \text{or} \quad a-3=0 \\ +6+6 \quad \quad \quad +3+3 \\ \hline a=6 \quad \text{or} \quad a=3 \end{array}$$

Check $a=6$

$$3\sqrt{9(6)-18} - 3(6) = 0$$

$$3\sqrt{54-18} - 18 = 0$$

$$3\sqrt{36} - 18 = 0$$

$$\begin{aligned} 3(6) - 18 &= 0 \\ 18 - 18 &= 0 \\ 0 &= 0 \end{aligned}$$

$\therefore a = 6$ is a solution

Check $a = 3$

$$\sqrt[3]{9(3)} - 18 - 3(3) = 0$$

$$\sqrt[3]{27} - 18 - 9 = 0$$

$$\sqrt[3]{9} - 9 = 0$$

$\therefore a = 3$ is a solution

$$3(3) - 9 = 0$$

$$9 - 9 = 0$$

$$0 = 0 \checkmark$$

Solution set

$$a \in \{3, 6\}$$

* idea

$$\sqrt[3]{9(a-2)} - a = 0$$

$$\sqrt[3]{9(a-2)} = a$$

$$\sqrt[3]{9} \sqrt[3]{a-2} = a$$

$$\frac{\sqrt[3]{9} \sqrt[3]{a-2}}{3} = \frac{a}{3} \rightarrow (\sqrt[3]{a-2})^2 = a^2$$

$$\sqrt[3]{a-2} = \frac{a}{3}$$

$$3^2 (\sqrt[3]{a-2})^2 = a^2$$

$$\star \sqrt[3]{ab} = \sqrt[3]{a} \sqrt[3]{b}$$

$$\star (ab)^n = a^n b^n$$

$$(\sqrt{a-2})^2 = \left(\frac{a}{3}\right)^2$$

$$a-2 = \frac{a^2}{9}$$

$$9(a-2) = a^2$$

$$9a - 18 = a^2$$

$$9(a-2) = a^2$$

$$9a - 18 = a^2$$

⋮
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$$\frac{2\sqrt{8-w} - w = 0}{+w +w}$$

$$2\sqrt{8-w} = w \leftarrow \text{isolate term with radical}$$

$$(2\sqrt{8-w})^2 = w^2$$

$$2^2 \cdot (\sqrt{8-w})^2 = w^2 \quad * (a b)^n = c^n \cdot b^n$$

$$4(8-w) = w^2$$

$$32 - 4w = w^2$$

$$\frac{-32 + 4w}{-32 + 4w} \quad +4w - 32$$

$$0 = w^2 + 4w - 32$$

$$0 = (w+8)(w-4)$$

$$0 = w+8 \quad \text{or} \quad w-4 = 0$$

:

$$w = -8$$

$$w = 4$$

Check $w=4$

$$2\sqrt{8-(4)} - (4) = 0$$

$$2\sqrt{4} - 4 = 0$$

$$2(2) - 4 = 0$$

$$0 = 0 \checkmark$$

$\therefore w=4$ is a solution

Check $w=-8$

$$2\sqrt{8-(-8)} - (-8) = 0$$

$$2\sqrt{16} + 8 = 0$$

$$2(4) + 8 = 0$$

$$16 \neq 0$$

$\therefore w=-8$ not a solution

$$\frac{\sqrt{2x+6} - \sqrt{x+4} = 1}{\sqrt{2x+6} = 1 + \sqrt{x+4}}$$

$\cancel{x(a+b)^2 \neq a^2 + b^2}$

$$(\sqrt{2x+6})^2 = (1 + \sqrt{x+4})^2$$

$$2x+6 = 1^2 + 2(1)(\sqrt{x+4}) + (\sqrt{x+4})^2$$

$$2x+6 = 1 + 2(\sqrt{x+4}) + x+4$$

$$\begin{array}{r} 2x+6 = x+5 + 2\sqrt{x+4} \\ -x-5 \quad -x-5 \\ \hline (x+1)^2 = (2\sqrt{x+4})^2 \end{array}$$

$$(x+1)(x+1) = 4(x+4)$$

$$\begin{array}{r} x^2 + 2x + 1 = 4x + 16 \\ \quad -4x - 16 \quad -4x - 16 \\ \hline x^2 - 2x - 15 = 0 \end{array}$$

(Factor + Check)
 $\cancel{x=1}$ || leave the rest to you.